I BSC SEMESTER-I

MECHANICS, WAVES AND OSCILLATIONS

PHYSICS STUDY MATERIAL

2022-2023

Department of Physics
Sri Y.N.College (A)
Narsapur

B.Sc., First Year, SEMESTER - I

PHYSICS

(MACHANICS, WAVES AND OSCILLATIONS)

(STUDY MATERIAL
&
MODEL PAPERS)

SYLLABUS

B.Sc., First Year, SEMESTER - I

PHYSICS

(MECHANICS, WAVES AND OSCILLATIONS)

Unit - I

1. Mechanics of particles

Review of Newton's Laws of Motion, Motion of variable mass system, Motion of a rocket, Multistage rocket, Concept of impact parameter, scattering cross-section, Rutherford Scattering cross-section, Rutherford scattering - Derivation.

2. Mechanics of Rigid Bodies

Rigid body, rotational kinematic relations, Equation of motion for a rotating body, Angular momentum and Moment of inertia tensor, Euler equations, Precession of a spinning top, Gyroscope, Precession of atom and nucleus in magnetic field, Precession of the equinoxes.

Unit - II

3. Motion in a Central Force Field

Central forces, definition and examples, characteristics of central forces, conservative nature of central forces, Equation of motion under a central force, Kepler's laws of planetary motion-Proofs, Motion of satellites, Basic idea of Global Positioning System (GPS), weightlessness, Physiological effects of astronauts.

Unit - III

4. Relativistic Mechanics

Introduction to relativity, Frames of reference, Galilean transformations, absolute frames, Michelson-Morley experiment, negative result, Postulates of Special theory of relativity, Lorentz transformation, time dilation, length contraction, variation of mass with velocity, Einstein's mass-energy relation.

Unit - IV

5. Undamped, Damped and Forced Oscillations :

Simple harmonic oscillator and solution of the differential equation, Damped harmonic oscillator, Forced harmonic oscillator - Their differential equations and solutions, Resonance, Logarithmic decrement, Relaxation time and Quality factor.

6. Coupled Oscillations

Coupled oscillators - Introduction, Two coupled oscillators, Normal Coordinates and Normal modes - N-coupled oscillators and wave equation.

Unit - V

7. Vibrating Strings

Transverse wave propagation along a stretched string, General solution of wave equation and its significance, Modes of vibration of stretched string clamped at ends, Overtones and Harmonics, Melde's strings.

8. Ultrasonics

Ultrasonics, General Properties of ultrasonic waves, Production of ultrasonics by piezoelectric and magnetostriction methods, Detection of ultrasonics, Applications of ultrasonic waves, SONAR.

PRACTICAL SYLLABUS

B.Sc., First Year, SEMESTER - I

PHYSICS

(MECHANICS, WAVES AND OSCILLATIONS)

PRACTICAL SYLLABUS :

1. Young's modulus of the material of a bar (scale) by uniform bending
2. Young's modulus of the material bar (scale) by non-uniform bending
3. Surface tension of a liquid by capillary rise method
4. Vsicosity of liquid by the flow method (Poiseuille's method)
5. Bifilar suspension - Moment of inertia of a regular rectangular body
6. Fly-wheel - Determination of moment of inertia
7. Rigidty modulus of material of a wire-Dynamic method (Torsional pendulum)
8. Volume resonator experiment
9. Determination of 'g' by compound / bar pendulum
10. Simple pendulum - normal distribution of erros-estimation of time period and the error of the mean by statistical analysis
11. Determination of the force constant of a spring by static and dynamic method.
12. Coupled oscillators
13. VErification of laws of vibrations of stretched string - Sonometer
14. Determination of frequency of a bar - Melde's experiment
15. Study of a damped oscillation using the torsional pendulum immersed in liquid-decay constant and damping correction of the amplitude.

CONTENTS

B.Sc., First Year, SEMESTER - I

PHYSICS

(MECHANICS, WAVES AND OSCILLATIONS)

UNIT – I : MECHANICS

1. MECHANICS OF PARTICLES

LONG ANSWER QUESTIONS

1. Define elastic and inelastic collisions? Derive the equations for the final velocities of the particles in a two dimensional elastic collision?
2. Describe equation of motion of a system of variable mass?
3. Give the theory of motion of a Rocket. Derive equation of motion of Rocket.
4. Derive an expression for Rutherford's scattering cross section.

SHORT ANSWER QUESTIONS

5. State Newton's laws ?
6. Explain the law of conservation of angular momentum?
7. Explain the conservation of Energy?
8. Write short notes on multi stage rockets?
9. Explain the concept of Impact parameter and scattering cross-section.

SOLVED PROBLEMS

10. A rocket of mass 40 kg has 360 kg of fuel. The exhaust velocity of the fuel is 2 km/sec. Find the velocity gained by rocket when rate of consumption of the fuel is 4 kg/sec.
11. A rocket when empty weight 5000 kg and is filled with 40,000 kg of fuel. The exhaust velocity of the burnt gas is 2 kms^{-1}. Find the maximum velocity attained by the rocket.
12. A rocket of mass 20 kg has 180 kg of fuel. The exhaust velocity of the fuel is 1.6 km/sec. Calculate the minimum rate of consumption of fuel so that the rocket may rise from the ground. Also calculate the ultimate vertical speed gained by the rocket, when the rate of consumption of the fuel is 2 kg/sec.
13. An alpha particle of energy 5 Mev approches a copper nucleus (Z = 32) in the head on collision. Calculate the distance of nearest approach.
14. A 0.03 kg mass travelling at 0.08 m/s makes an elastic with a 0.05 kg mass at rest. Find the speed of each mass after .
15. A ball moving at a speed of 2.2 m/sec. Strikes an identical stationary ball. After collision one ball moves at 1.1 m/sec. at 60^0 angle with the original line of motion. Find the velocity of the other ball.
16. α Particle from a polonium source strike a thin gold foil of thickness $4\times10^{-7}m$. Most of the particles scatter in the forward direction but $\left(\frac{1}{6.17\times10^{6}}\right)$ fraction of α -particles are found to be scattered by more than 90^0. Find the cross section of this type of scattering if the number of gold nuclei per unit volume is 5.9×10^{28} per m^3.
17. A particle of mass 5kg moving with velocity of 10m/s collides with another particle of mass 10 kg moving in opposite directions with a velocity of 20m/s. During collision they stick together. Find the common velocity.

2. MECHANICS OF RIGID BODIES

LONG ANSWER QUESTIONS

1. Derive equation of motion of a rigid rotating body.

2. Derive Euler equations of rotational motion for a rigid body fixed at one point. Prove the law of conservation of K.E and angular momentum from them.
3. What is a symmetric top? Derive an expression for the precessional velocity of a symmetric top?
4. Derive an expression for angular momentum and inertia tensor.

SHORT ANSWER QUESTIONS

5. Write short notes on gyroscope?
6. Explain the precession of equinoxes and its consequence?
7. Explain the precession of atom and nucleus in magnetic field ?

SOLVED PROBLEMS

8. A car develops 75 KW power when rotating at a speed of 1000 rpm. What is torque acting ?
9. A sphere of mass 2.5 kg. and diameter 1m rolls without slipping with a constant velocity of 2 m/s. Calculate its total energy.
10. A ballet dancer spins about a vertical axis at the rate of 1 revolution per second with her arms out stretched. When her arms folded her moment of inertia about the vertical axis decreases by 60%.Calculate the new rate of revolution.
11. The speed of a particle moving a circle of radius 20 cms increase at the rate of 10 cm/sec^2. If the mass is 200 gms, find the torque on it.
12. A 500 gm stone is revolved at the end of a 0.4 m long string at the rate of 12.5 rad/s. What is its angular momentum.
13. The kinetic energy of metal disc rotating at a constant speed of 5 revolutions per second is 100 joules. Find the angular momentum of the disc.
14. A circular disc of mass 100 kg and radius 1 m is mounted axially and made to rotate. Calculate the K.E it possesses when executing 120 rotations per minute.
15. A fly wheel when slowed down from 60 rpm to 30 rpm loses 100J of energy. What is its moment of inertia?

UNIT - II : 3. CENTRAL FORCES

LONG ANSWER QUESTIONS

1. Discuss the conservative nature of central forces.
2. Derive the equation of motion of a particle under a central force?
3. State and prove the Kepler's laws of planetory motion?

SHORT ANSWER QUESTIONS

4. What is a Central Force? Give two Examples.
5. Prove that the Areal Velocity is constant under the influence of central force ?
6. What are the characteristics of central force ?
7. Prove that conservative force as a negative gradient of potential energy ?
8. Show that the curl of a central force is zero ?
9. Derive Newton's law of gravitation from Kepler's law.
10. Explain about geostationary satelite and found its height from the surface of the earth (radius of the earth $R = 6.4\times 10^6$ m).
11. Discuss about Motion of satellites ?

SOLVED PROBLEMS

12. If earth is at one half of its present distance from sun, what will the number of days in a year.
13. Estimate the mass of the sun assuming the orbit of earth around the sun is a circle. The distance between the sun and the earth is 1.49×10^{11}m.
14. Show that the force $F = (y^2 - x^2)\, i + 2xyj$ is conservative.

15. If the radius of the earth suddenly changes to half the present value without any change in mass. What would be the change in the duration of the day?

16. The maximum and minimum distances of a comet from the sun are 1.6×10^{10} m and 8×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 m/sec. Calculate the speed at the farthest point.

UNIT - III : 4. RELATIVISTIC MECHANICS

LONG ANSWER QUESTIONS

1. Describe Michelson - Morley experiment and explain the importance of its result?
2. State Postulates of special theory of Relativity ? Derive Lorentz transformations.
3. Derive Einstein's mass-energy relation. (Or)
   Derive Equivalence of mass and energy.
4. Explain variation of mass with velocity ?

SHORT ANSWER QUESTIONS

5. Prove $x^2 + y^2 + z^2 = c^2t^2$ is invariant under Lorentz transformations.
6. Write short notes on length contraction.
7. Write short notes on Time Dilation.

SOLVED PROBLEMS

8. If rod travels with a speed 0.6c along its length claculate the percentage contraction.
9. A rocket ship is 100 metre long on the ground. When it is in flight, its length is 99 metres to an observer on the ground. What is its speed?
10. A clock showing correct time when at rest and loses 2 hours in a day when it is moving. What is its velocity?
11. At what speed the mass of an object will be double of its value at rest.
12. If the total energy of a particle is exactly thrice its rest energy, what is the velocity of the particle?
13. A particle is moving with 90% of the velocity of light. Compare its relativistic mass with rest mass.
14. With what speed shoult it be moved relative to an observer so that it may appear to lose 4 minutes in 24 hours.

UNIT - IV

5. UNDAMPED, DAMPED AND FORCED OSCILLATIONS

LONG ANSWER QUESTIONS

1. What is simple oscillator? Give the equation of motion of a simple oscillator and its solution?
2. Define damped harmonic oscillator. Derive the equation of Motion of damped harmonic oscillator. Discuss different cases.
3. Define forced harmonic oscillators. Derive the differential equation and give its solution. Discuss different cases.

SHORT ANSWER QUESTIONS

4. What is simple-harmonic motion? What are its physical characteristics ?
5. Explain the amplitude and sharpness resonance.
6. Define quality factor. Explain.
7. Write short notes on logarthmic decrement of the oscillator.
8. Define relaxation time? Derive expression for it?

SOLVED PROBLEMS

9. The displacement of a particle making S.H.M is given by $x = 0.5\cos\left(10\pi t + \frac{\pi}{3}\right)$ calculate (1) amplitude, (2) Frequency, (3) Phase, (4) Displacement after 1 sec.

10. A particle vibrates simple harmonically with a period of 2 sec. Find the amplitude if its max. velocity is 10 cm/s.
11. The amplitude of seconds pendulum falls to half of its initial value in 150 sec. Calculate the Q – factor.
12. The Q value of a spring loaded with 0.3 kg is 60. If it vibrates with a frequency of 2 Hz. Calculate the force constant and the mechanical resistance.
13. The quality factor Q of a sonometer wire is 2×10^2. On plucking, the wire emits a note of frequency 120 Hz. Calculate the time in which the amplitude falls to $(1/e^2)$ of the initial value.

6. COUPLED OSCILLATORS

LONG ANSWER QUESTIONS

1. Obtain the normal mode and normal coordinates of two identical pendulums with their bobs connected by means of elastic massless spring ?
2. Obtain the equation of motion, considering the case of N-coupled oscillators and derive the equation for the frequency of the system ?

SHORT ANSWER QUESTIONS

3. What are coupled oscillators and give examples ?
4. Derive the wave equation of N - coupled oscillators ?

SOLVED PROBLEMS

5. Sodium chloride molecule has a natural vibrational frequency = 1.14×10^{13} Hz. Calculate the interatomic force constant. Mass of sodium atom = 23 a.m.u. Mass of cI atom = 35 a.m.u. (1 a.m.u. = 1.67×10^{-27}kg).

UNIT – V

7. VIBRATING STRINGS

LONG ANSWER QUESTIONS

1. Derive wave equation of Transverse wave propagation along a stretched string and give it's general solution
2. Describe the modes of vibrations of a stretched strings clamped at both ends. What are overtones?
3. Derive the expression for velocity of a transverse wave along a stretched string?

SOLVED PROBLEMS

4. A string of length 8 m fixed at both ends has a tension of 49 N and mass of 0.4 kg. Find the speed of transverse wave.
5. A steel wire of diameter 1 c.m. is kept under a tension of 5 KN. The density of steel is 7.8 g/c.c. Calculate the velocity of the transverse wave.
6. A steel wire 50 cm long has mass of 5 gms. It is stretched with a tension of 400 N. Find the frequency of the wire in fundamental mode of vibration.
7. Two similar wires are under same tension. When tension in one wire increased by 6.09% and the two wires vibrate simultaneously, 6 feats are heard per second. Find the original frequency of the two strings.
8. The fundamental frequency of vibration of a stretched string of length 1m is 256 Hz. Find the frequency of the same string of half the original length under identical condition.

8. ULTRASONICS

LONG ANSWER QUESTIONS

1. Write an essay on the production of ultrasonics.

SHORT QUESTION ANSWERS

2. What are ultrasonics ? What are their properties ?
3. What are the methods for the detection of ultrasonics?
4. Write short notes on Acoustic grating.
5. Write a note on applications of ultrasonics.
6. Explain about SONAR.

SOLVED PROBLEMS

7. A magnetostriction oscillator has frequency 20 kHz. If it produces sound wave of velocity 6.2×10^3 m/s, find the length of ferrite rod.
8. Calculate the frequency of fundamental note emitted by a piezo electrtic crystal. Use the following data.
9. A quartz crystal thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency. Given Y for quartz = 7.9×10^{10} newton/m^2 and ρ for quartz = 2650 kg/m^3.
10. A piezo electric crystal has a thickness 0.002 m. If the velocity of sound wave in crystal is 5750m/s, calculate the fundamental frequency of crystal.
11. Calculate the capacitance to produce ultrosonic waves of 10^6 Hz with an inductance of 1 henry.

UNIT – I : MECHANICS

1. MECHANICS OF PARTICLES

LONG ANSWER QUESTIONS

Q. 1. Define elastic and inelastic collisions? Derive the equations for the final velocities of the particles in a two dimensional elastic collision?

Ans : Elastic collision :

1 When the kinetic energy of the particles remains conserved in the collision, the collision is said to be elastic.

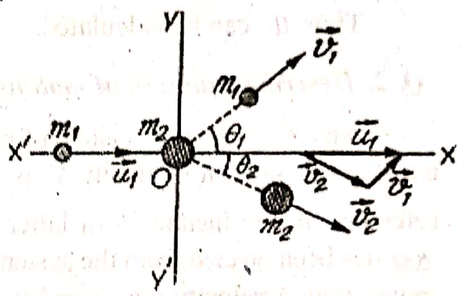
2. The collision between atomic, nuclear and fundamental particles are usually elastic.

Inelastic collision :

1.. When the kinetic energy is changed in the collision, the collision is said to be inelastic.

2. Collisions between gross bodies are always in elastic.

Elastic collision in two Dimensions : As shown in fig, let a particle of mass m_1 moving with velocity u_1 collide with a particle of mass m_2 at rest (u_2 =0). Let after collision, the particle of mass m_1 is deflected or scattered at an angle θ_1 with the original direction. Similarly, the particle of mass m_2 moves in a direction which makes an angle θ_2 with the original direction. Further let v_1 and v_2 be the velocities of masses m_1 and m_2 respectively after collision.



Fig

Applying the law of conservation of linear momentum along X-axis we have

$$m_1u_1 + 0 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{............ (1)}$$

Applying the law of conservation of linear momentum along Y-axis, we have

$$0 + 0 = m_1 v_1 \text{Sin}\,\theta_1 - m_2 v_2 \text{Sin}\,\theta_2 .$$

$$m_1 v_1 \text{Sin}\,\theta_1 = m_2 v_2 \text{Sin}\,\theta_2 . \quad \text{............ (2)}$$

According to the law of conservation of kinetic energy

$$\frac{1}{2} m_1 u_1^2 + 0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{............ (3)}$$

We have $m_1 = m_2$

Framer (1) $u_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2$ .... (4)

Framer (2) $v_1 \text{Sin}\,\theta_1 = v_2 \text{Sin}\,\theta_2$ . ... (5)

Framer (4) $(u_1 - v_1 \cos\theta_1) = v_2 \cos\theta_2$

Squaring both sides we have eq (4)

$(u_1^2 + v_1^2 \cos^2\theta_1 - 2u_1 v_1 \cos\theta_1) = v_2^2 \cos^2\theta_2$ .... (6)

Squaring both sides of eq (5) we have

$v_1^2 \text{Sin}^2\theta_1 = v_2^2 \text{Sin}^2\theta_2$ .... (7)

Adding eqns (6) and (7) we get

$u_1^2 + v_1^2 - 2u_1 v_1 \cos\theta_1 = v_2^2$ .... (8)

From eq (3) $u_1^2 = v_1^2 + v_2^2$ (or) $u_1^2 - v_1^2 = v_2^2$ .... (9)

Subtracting eq (9) from eq (8) we get

$2v_1^2 - 2u_1 v_1 \cos\theta_1 = 0$

$v_1 - u_1 \cos\theta_1 = 0$ (or) $v_1 = u_1 \cos\theta_1$ .... (a)

Thus v_1 can be calculated.

From eq (3) $v_2^2 = u_1^2 - v_1^2$

$$v_2^2 = u_1^2 - u_1^2\cos^2\theta_1 = u_1^2(1-\cos^2\theta_1) = u_1^2 \sin^2\theta_1.$$

(or) $v_2 = u_1 \sin\theta_1$ .... (b)

from eq (a) and (b) it is clear that v_1 and v_2 are perpendicular components of u_1. (or)

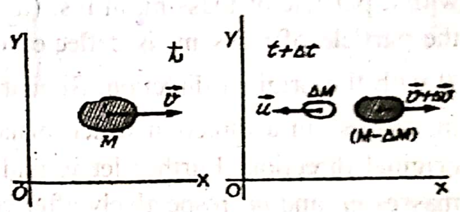
$\theta_1 + \theta_2 = 90^0$

From eq (2) $\sin\theta_2 = \left(\frac{V_1}{V_2}\right)\sin\theta_1$.

Thus θ_2 can be calculated.

Q. 2. Describe equation of motion of a system of variable mass?

Ans : Fig shows a system of mass M whose centre of mass is moving with velocity V as seen from a particular reference at any instant. At a latter instant t + Δt a mass ΔM has been ejected from the system and its centre of mass moves with a velocity u as seen by an observer.



Fig

Now the system mass is reduced to $M - \Delta M$ and the velocity of the the centre of mass of the system is changed $V + \Delta V$.

The system represents a motion like a rocket.

From Newton's second law

$F_{ext} = \frac{dp}{dt}$ $\qquad F_{ext} \cong \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t}$

Consider both the parts of masses ΔM and $M - \Delta M$ as forming one and the same system we can write.

$$F_{ext} = \frac{p_f - p_i}{\Delta t} = \frac{(M-\Delta M)(V+\Delta V) + \Delta M u) - [MV]}{\Delta t} \quad \text{........ (1)}$$

$$F_{ext}\ M\frac{\Delta V}{\Delta t} - V\frac{\Delta M}{\Delta t} - \Delta t\frac{\Delta M}{\Delta t} + u\frac{\Delta M}{\Delta t} \quad \text{........ (2)}$$

If Δt approached zero, So $\frac{\Delta V}{\Delta t}$ approached $\frac{dV}{dt}$ and $\frac{\Delta M}{\Delta t}$ is replaced by $-\frac{\Delta m}{\Delta t}$.

eq (2) becomes,

$$F_{ext} = M\frac{dV}{dt} + V\frac{dM}{dt} - u\frac{dM}{dt} \quad \text{(or)} \quad F_{ext} = \frac{d}{dt}(MV) - u\frac{dM}{dt} \quad \text{........ (3)}$$

eq (3) expresses Newton's second law as applied to a body of variable mass.

eq (3) can also be expressed as

$$F_{ext} = M\frac{dV}{dt} + V\frac{dM}{dt} - u\frac{dM}{dt} \qquad F_{ext} = M\frac{dV}{dt} + (v-u)\frac{dM}{dt}$$

$$\text{(or)} \quad M\frac{dV}{dt} = F_{ext} + (u-v)\frac{dM}{dt} \quad \text{(or)} \quad M\frac{dV}{dt} = F_{ext} + F_{reaction}.$$

Q. 3. Give the theory of motion of a Rocket. Derive equation of motion of Rocket.

Ans : Rocket is an example for mass variable system. When the fuel is burnt, gases are produced. These gases will escape through a fine nozzle, as a result a thrust is produced. Hence the rocket moves. There are solid fuel rockets and liquid fuel rockets. Gun powder is used as solid fuel. Liquid hydrogen or liquid paraffin is used as liquid fuel. The motion of the rocket can be explained by Newton's third law of motion.

Let M be the mass and V be the velocity of rocket at any instant with respect to a Laboratory frame of reference. Let dM be the mass of the gas ejected in a time dt. Let u be the velocity of gas jet with respect to rocket.

$\therefore$ The relative velocity of gas Jet with respect to lab frame $= V_{rel} = V - u$

Rate of change of momentum of Jet coming out of the rocket $= \dfrac{-dM}{dt}(V-u)$

$\therefore$ Thrust acting on the rocket to move it forward $= \dfrac{dM}{dt}(V-u)$

The Net force acting on the rocket in the forward direction $= \dfrac{dM}{dt}(V-u) - Mg$ ......(1)

From Newtons second law, the force on rocket $= \dfrac{d}{dt}(MV)$ ....(2)

$\therefore$ From (1) and (2) $\dfrac{d}{dt}(MV) = \dfrac{dM}{dt}(V-u) - Mg$ ....(3)

$$M\frac{dV}{dt} + V\frac{dM}{dt} = V\frac{dM}{dt} - u\frac{dM}{dt} - Mg \qquad M\frac{dV}{dt} = -u\frac{dM}{dt} - Mg$$

$$\frac{dV}{dt} = \frac{-u}{M}\frac{dM}{dt} - g \qquad dV = -u\frac{dM}{M} - gdt \qquad \text{........(4)}$$

Let M_0 is initial mass and V_0 be the velocity when $t = 0$

Integrating equation (4) $\displaystyle\int_{V_0}^{V} dV = -u\int_{M_0}^{M}\frac{dM}{M} - g\int_0^t dt$

$$V - V_0 = +u\,\text{loge}\left(\frac{M_0}{M}\right) - gt \qquad \therefore V = V_0 + u\log e\left(\frac{M_0}{M}\right) - gt \qquad \text{........(5)}$$

This equation gives the velocity of the rocket at any instant t.

Special Cases :

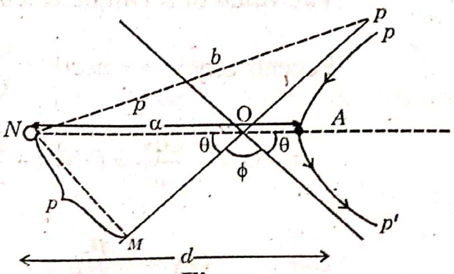
1) If the value of force of gravity (g) is ignored then $V = V_0 + u\log_e\left(\dfrac{M_0}{M}\right)$

2) If the initial velocity of the rocket is zero, then $V = u\log_e\left(\dfrac{M_0}{M}\right)$

Or $V = 2.3024 \log_{10}\left(\dfrac{M_0}{M}\right)$

Q. 4. Derive an expression for Rutherford's scattering cross section.

Ans : Consider an atom of an element of atomic number z. Let Ze be the charge of Nucleus. Let m be the mass of α – particle and $2e$ be the charge. It is approaching the nucleus with a velocity V_0 along the direction pM. In the absence of repulsive force, the α – particle moves in a straight line path. But due to repulsive force it follows a parabolic path with nucleus at its foci. pO and $p'O$ are assymptotes of the hyperbola. If the α – particle is directed straight towards the nucleus (i.e $p = 0$), the α – particle is stopped at a distance b. The value of b can be determined by using conservation of energy.



Fig

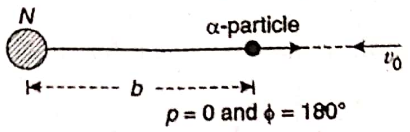
m = mass of α – particle ; ϕ = scattering angle ; Ze = charge of Nucleus; $2e$ = charge of α – particle; p = Impact parameter

Electrostatic potential due to Nucleus at a distance b is $\frac{1}{4\pi \epsilon_0}\left(\frac{ze}{b}\right)$

$\therefore$ The P.E of α – particle at a distance 'b' from the Nucleus $= \frac{Ze}{4\pi \epsilon_0 b}(2e) = \frac{2Ze^2}{4\pi \epsilon_0 b}$

when the α – particle is momentarily stopped at a distance b, its K.E. i.e $\frac{1}{2}mv_0^2$ is totally converted into P.E.

$$\therefore \frac{1}{2}mV_0^2 = \frac{2Ze^2}{4\pi \epsilon_0 b}$$



$$\therefore b = \frac{Ze^2}{\pi \epsilon_0 mv_0^2} \quad \text{........(1)}$$

$$\therefore bV_0^2 = \frac{Ze^2}{\pi \epsilon_0 m} \quad \text{............ (A)}$$

Consider the case i.e when $p \neq 0$ in such case, the α – particle will be deflected through at anangle ϕ. It travels along the path pAp'. Let V be the velocity of the α – particle at the vertex A of hyperbola. The velocity can be calculated by the law of conservation of angular momentum.

Angular momentum of α – particle at $P = mV_0p$

Angular momentum of α – particle at $A = mV(NA) = mVd$

$$\therefore mV_0p = mVd \qquad V = \frac{V_0p}{d} \quad \text{........(2)}$$

K.E. of α – particle at $p = \frac{1}{2}mV_0^2$ K.E. of α – particle at $A = \frac{1}{2}mV^2$

P.E. of α – particle at $A = \frac{1}{4\pi \epsilon_0}\left(\frac{Ze}{d}\right)(2e) = \frac{2Ze^2}{4\pi \epsilon_0 d}$

Applying the law of conservation of energy

$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV^2 + \frac{2ze^2}{4\pi \epsilon_0 d} \qquad V_0^2 = V^2 + \frac{ze^2}{\pi \epsilon_0 md} \qquad V^2 = V_0^2 - \frac{ze^2}{\pi \epsilon_0 md}$$

$$V^2 = V_0^2 - \frac{b}{d}V_0^2 \qquad \left(\because bV_0^2 = \frac{ze^2}{\pi \epsilon_o m}\right) \qquad V^2 = V_0^2\left(1 - \frac{b}{d}\right) \quad \text{........(3)}$$

Substituting the value of V from equation (2) in equ. (3)

$$\frac{V_0^2p^2}{d^2} = V_0^2\left(1 - \frac{b}{d}\right) \qquad \frac{p^2}{d^2} = 1 - \frac{b}{d} \qquad \frac{p^2}{d^2} = \left(\frac{d-b}{d}\right) \qquad p^2 = d^2\frac{(d-b)}{d} \quad \text{........(4)}$$

The value of d can be calculated as follows bythe property of hyperbola

Eccentricity $= e = \sec\theta$; $\frac{ON}{OA} = e$ From the diagram $ON + OA = d$

$$\therefore d = ON + \frac{ON}{e} = ON\left(1 + \frac{1}{e}\right) = ON(1 + \cos\theta)$$

From the Δ^{el} NOM; $\sin\theta = \frac{MN}{ON}$

But $ON = \frac{p}{\sin\theta}$

$$\therefore d = \frac{p}{\sin\theta}(1+\cos\theta) = \frac{p}{2\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}} \times 2\cos^2\frac{\theta}{2} = p\cot\frac{\theta}{2}$$

$$\therefore p^2 = p\cot\frac{\theta}{2}\left(p\cot\frac{\theta}{2} - b\right)$$

$$p^2 = p^2\cot^2\frac{\theta}{2} - b\cot\frac{\theta}{2} \Rightarrow p^2 - p^2\cot^2\theta/2 = -pb\cot\theta/2$$

$$\therefore b\cot\frac{\theta}{2} = p\cot^2\frac{\theta}{2} - p$$

$$b = \frac{p\cot^2\frac{\theta}{2} - P}{\cot\frac{\theta}{2}} = p\left[\cot\frac{\theta}{2} - Tan\frac{\theta}{2}\right] = p\left[\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right]$$

$$b = p\left[\frac{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right] = 2p\frac{\cos\theta}{\sin\theta} = 2p\cot\theta = 2p\cot\left(\frac{\pi-\phi}{2}\right) = 2\,Tan\frac{\phi}{2}\times p$$

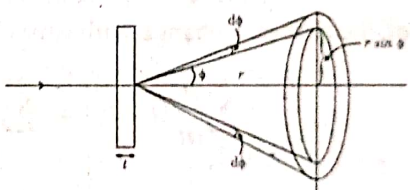
$$\therefore Tan\frac{\phi}{2} = \frac{b}{2p} \qquad \text{........(5)}$$

Substituting the value of b from equ (1) in equ. (5)

$$Tan\frac{\phi}{2} = \frac{ze^2}{2\pi\epsilon_0 m v_0^2 p} \qquad \text{........(6)}$$

Rutherford's scattering cross section :

Let n be the no. of atoms per unit volume of the scatterer of thickness t. The scattered particles are striking the screens. Let r be the distance of the screen from scatterer. Let θ be the total no. of particles that strike the unit area of the scatterer. Let us imagine that the α - particles having an impact parameter p are scattered through an angle ϕ.



Fig

$\therefore$ The probable no. of α – particles coming within the distance of an impact parameter striking per unit area $= \pi p^2 nt\theta$

The no. of α – particles scattred between angles ϕ and $(\phi + d\phi)$ is $2\pi pnt\theta dp$.

Solid angle between ϕ and $(\phi + d\phi) = 2\pi\sin\phi d\phi$

$\therefore$ Scattering cross-section

$$\sigma = \frac{No.of\ particles\ scattered\ int\ o\ the\ solid\ angle\ per\ unit\ time}{Incident\ int\ ensity}$$

But the no. of scattered particles into solid angle between ϕ and $(\phi + d\phi)$ = Number of incident particles having impact parameters lying between p and $p + dp$

$2\pi\sin\phi d\phi\sigma I = -2\pi p dp I$

The $-ve$ sign indicates that as p increases ϕ decreases.

$$\therefore \sigma = \frac{-2\pi p dp I}{2\pi \sin\phi d\phi I} = \frac{-pdp}{\sin\phi d\phi}$$

But from (6) $p = \frac{ze^2}{2\pi \in_0 mV_0^2} = \cot\frac{\phi}{2} = \frac{b}{2}\cot\frac{\phi}{2}$

$$\therefore dp = \frac{-b}{2}\operatorname{cosec}^2\frac{\phi}{2}\times\frac{1}{2}.d\phi = \frac{-b}{4}\operatorname{cosec}^2\frac{\phi}{2}.d\phi$$

$$\therefore \sigma = \frac{-\left(\frac{b}{2}\right)\cot\frac{\phi}{2}\left(\frac{-b}{4}\right)\operatorname{cosec}^2\frac{\phi}{2}.d\phi}{2\sin\frac{\phi}{2}.\cos\frac{\phi}{2}.d\phi} = \frac{b^2}{16}\frac{1}{\left(\sin^4\frac{\phi}{2}\right)}$$

Substituting the value of b from equ (1)

$$\sigma = \frac{z^2e^4}{16\pi^2 \in_0^2 m^2V_0^4 \sin^4\frac{\phi}{2}}$$

This is the formula for Rutherfords scattering cross-section.

SHORT ANSWER QUESTIONS

Q. 5. State Newton's laws ?

Ans : Newton's First law : "Every body continues in the state of rest or of uniform motion in a straight line unless an external force acts on it to change that state".

Newton's Secondlaw : "The product of the mass m of a mass point by its acceleration a is equal to the force acting on it i,e.

$$F = m a$$

Newton's thirdlaw : "To every action, there is always an equal and opposite reaction".

Q. 6. Explain the law of conservation of angular momentum?

Ans : The law of conservation of angular momentum states that if no external torque acts an a body rotating about a fixed point, the angular momentum of the body remains constant.

When external forces acts on the particles they exert torque on them. Now the angular momenta of the particles change with time. The rate of change of angular momenta is given by

$$\frac{dL}{dt} = \sum_{i=1}^{n}\frac{d}{dt}(r_i\times p_i) = \sum_{i=1}^{n}\left[\frac{dri}{dt}\times p_i + r_i\times\frac{dp_i}{dt}\right] = \sum_{i=1}^{n} r_i\times\frac{dp_i}{dt}$$

because $\frac{dri}{dt}\times p_i = V_i\times m_iV_i = m_i(V_i\times V_i) = 0$ $\quad\therefore \frac{dL}{dt} = \sum_{i=l}^{n} ri\times F_i = \sum_{i=l}^{n} T_i = T_{ext}$

$\left(\therefore F_i = \frac{dp_i}{dt}\right)$ If $T_{ext} = 0$ $\frac{dL}{dt} = 0$ (or) $L =$ Constant.

Q. 7. Explain the conservation of Energy?

Ans : According to law of conservation of energy.

"Energy can neither be created nor destroyed". It can be transformed from one form to another form.

Let us consider the case of a body of mass m at a height h above the ground.

At point A kinetic energy of the body is zero. Potential energy is mgh

at point B kinetic energy of the body is mgx.

Potential energy is mg $(h - x)$, T.E = mgh.

At point c. Kinetic energy is mgh potential energy is 0.

$$T.E = mgx$$

At all the points T.E remains constant. So "The total energy in any system, always remains constant".

Q. 8. Write short notes on multi stage rockets?

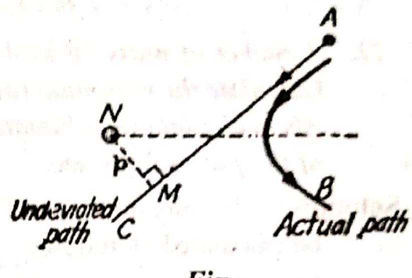
Ans : The motion of a rocket is based on the law of conservation of linear momentum.

Generally the velocity attained by a rocket is approximately 4 km/s. To obtain higher velocities multistage rockets are used. Generally a multistage rocket has 3 stages. The first stage of the rocket is used to acquire the acceleration of the rocket. When the fuel of first stage is completed, it detaches from the rocket. The velocity at this stage becomes the initial velocity of the second stage. The second stage starts its functioning. When the fuel of the second stage is completed it detaches from the main rocket. Finally the third stage rocket starts.

Q. 9. Explain the concept of Impact parameter and scattering cross-section.

Ans : Impact parameter : Consider a positive particle, like a proton or an α -particle approaching a massive nucleus N of an atom, as shown in fig.

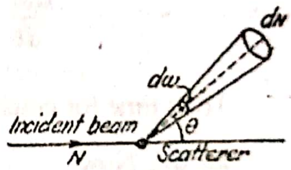
Due to coulombic force of repulsion, the particle follows, a hyperbolic path AB with nucleus N as its focus. In the absence of the repulsive force. The particle would have followed the straight line path Ac. As shown in fig, P is the perpendicular distance from the nucleus N to the original direction AC of the particle. The distence (NM = P) is called the impact parameter.



Fig

Thus impact parameter is defined as the closest distence between nucleus and positive charged particle projected towards it.

Scattering cross-section : When α - particles are projected into a thin metal foil. They are deflected or scattered in different directions. Let N be the incident intensity. Suppose dN be the number of particles scattered per unit time into solid angle dω located in the direction θ *and* ϕ with respect to the bombarding direction.



Fig

The ratio dN/N is called scattering cross section. Thus it is defined as in a given direction the ratio of number of scattering particles into solid angle dω per unit time to the incident intensity.

SOLVED PROBLEMS

10. *A rocket of mass 40 kg has 360 kg of fuel. The exhaust velocity of the fuel is 2 km/sec. Find the velocity gained by rocket when rate of consumption of the fuel is 4 kg/sec.*

Solution :

$$V_{max} = V_0 + u \log_e\left(\frac{M_0}{M}\right) - gt$$

Initial velocity $V_0 = 0$ and $g = 9.8\ m/s^2 = \dfrac{9.8}{1000}\ km/s^2$

Exhaust velocity of fuel $u = 2\ km/s$.

(Mass of rocket + Mass of fuel) $M_0 = 40 + 360 = 400\ kg$.

Rate of consumption of fuel $\dfrac{dM}{dt} = 4\ kg/s$.

At this rate the time taken to burnt the whole fuel of mass

$$360 \text{ kgs is } t = \frac{360}{4} = 90s$$

Mass of the rocket $M = 40\ kg$

$$\therefore V = 0 + 2\log_e\left(\frac{400}{40}\right) - \frac{9.8}{1000} \times 90 = 3.72 km/s$$

11. *A rocket when empty weight 5000 kg and is filled with 40,000 kg of fuel. The exhaust velocity of the burnt gas is 2 kms^{-1}. Find the maximum velocity attained by the rocket.*

Solution : Weight of the empty rocket $(M) = 5 \times 10^3$ kg

Weight of the Rocket + Fuel $(M_0) = (5 \times 10^3 + 40 \times 10^3)$ kg = 45000 kg

Exhaust velocity $(u) = 2\ kms^{-1} = 2000\ ms^{-1}$

If initial velocity is zero, the maximum velocity of the rocket.

$$V = u\log_e\left(\frac{M_0}{M}\right) = 2.303u\log\left(\frac{M_0}{M}\right) = 2.303 \times 2000 \times \log_{10}\left(\frac{45000}{5000}\right)^{10}$$

$$= 2.303 \times 2000 \times \log_{10}{}^{9} = 2.303 \times 2000 \times 0.9542 = 4395.04\ ms^{-1}$$

$v = 4.395\ kms^{-1}$

12. *A rocket of mass 20 kg has 180 kg of fuel. The exhaust velocity of the fuel is 1.6 km/sec. Calculate the minimum rate of consumption of fuel so that the rocket may rise from the ground. Also calculate the ultimate vertical speed gained by the rocket, when the rate of consumption of the fuel is 2 kg/sec.*

Solution : The minimum rate of consumption of fuel so that the rocket may rise from the ground may be calculated as follows.

Magnitude of thurst = Weight of rocket or

$$u\frac{dM}{dt} = Mg$$

$$\frac{dM}{dt} = \frac{Mg}{u} = \frac{200 \times 9.8}{1.6 \times 1000} = 1.225\ kg/sec.$$

Total time for consumption of all fuel = $\frac{180}{2} = 90$ *secs* since the rate of consumption of fuel is 2 *kg/sec.* Now

$$V_{max} = u\log_e = \frac{M_0}{M} - gt = 1.6 \times 10^3 \log_e\frac{200}{20} - 9.8 \times 90$$

$= 1.6 \times 10^3 \times 2.303 - 882 = 2802.8$ *m/sec* = 2.8 *km/sec.*

13. *An alpha particle of energy 5 Mev approches a copper nucleus (Z = 32) in the head on collision. Calculate the distance of nearest approach.*

Solution : Energy of the α - particle

$E = 5$ *Mev* $= 5 \times 10^6 \times 1.6 \times 10^{-19}$ *Joules.*

Atomic number of copper Z = 32.

Distance of nearest approach = b

$$b = \frac{Ze2e}{4\pi\epsilon_0(E)} = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{E}$$

$$= \frac{9 \times 10^9 \times 2 \times 32 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}} = 184.32 \times 10^{-16} m.$$

14. *A 0.03 kg mass travelling at 0.08 m/s makes an elastic with a 0.05 kg mass at rest. Find the speed of each mass after .*

Solution .: Given $u_1 = 0.08$ m/s, $m_1 = 0.03$ kg, $u_2 = 0$ and $m_2 = 0.05$ kg

Here we have $m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$\therefore 0.03 \times 0.08 + 0.05 \times 0 = 0.03\ v_1 + 0.05\ v_2$

Solving we get $3V_1 + 5V_2 = 0.24$ ....(1)

From Newton's experimental law $\dfrac{v_2 - v_1}{u_2 - u_1} = -e = -1$

$$\therefore \frac{v_2 - v_1}{0 - 0.08} = -1$$

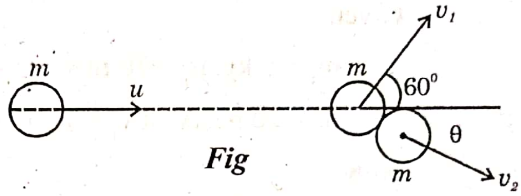
$3\,v_2 - 3v_1 = 0.24$ ....(2)

From ern's (1) be (2), solving , We get

$V_1 = -0.2$ m/sec $\qquad V_2 = 0.6$ m/sec.

15. ***A ball moving at a speed of 2.2 m/sec. Strikes an identical stationary ball. After collision one ball moves at 1.1 m/sec. at 60⁰ angle with the original line of motion. Find the velocity of the other ball.***

Solution : Let m be the mass of each ball and u be the initial velocity of the first ball. Let V_1 and V_2 be the final velocities of the balls respectively after collision as shown in fig.



Fig

Applying the law conservation of momentum also original direction of motion have

$mu = mV_1 \cos 60^0 + mV_2 \cos\theta$ or $u = V_1 \cos 60^0 + V_2 \cos\theta$

$2.2 = 1.1\,(0.5) + V_2 \cos\theta$ $\quad (\therefore \cos 60^0 = \theta)$

$\therefore V_2 \cos\theta = 2.2 - 0.55 = 1.65$ .... (1)

Now applying conservation of momentum perpenducular to the original direction of motion, we have

$0 = mV_1 \sin 60^0 - mV_2 \sin\theta$ or $0 = V_1 \sin 60^0 - V_2 \sin\theta$

or $V_2 \sin\theta = 1.1\,(0.866)$ $\quad (\sin 60^0 = 0.866)$

or $V_2 \sin\theta = (0.953)$ .... (2)

Squaring and adding eqs. (1) and (2) we get

$$V_2^2 = (1.65)^2 + (0.953)^2$$

or $$v_2 = \sqrt{\left[(1.65)^2 + (0.953)^2\right]} = 1.9 \text{ m/sec.}$$

Dividing eq. (2) by eq. (1) we get, $\tan\theta = \dfrac{0.953}{1.65} = 0.577$ $\qquad \theta = \tan^{-1}(0.577) = 30^0$.

16. ***α Particle from a polonium source strike a thin gold foil of thickness $4 \times 10^{-7}\,m$. Most of the particles scatter in the forward direction but $\left(\dfrac{1}{6.17 \times 10^6}\right)$ fraction of α -particles are found to be scattered by more than 90⁰. Find the cross section of this type of scattering if the number of gold nuclei per unit volume is 5.9 × 10²⁸ per m³.***

Solution : Given thickness of scattering material $t = 4 \times 10^{-7}$ m. $\qquad \phi = 90^0$

atomic no.of gold $z = 99$

charge of α particle e = 1.6×10^{-19} coulomb

Q = 5.9×10^{28} per m³.

V_0 velocity of α particle = 3×10^8 m.

m = $(2 \times 5 \times 10^6)$.

Scattering cross section of α particles $= \dfrac{z^2 = e^4}{16\pi^2 \in^2 m^2 V_0^2 \sin^2\phi/2}$

$$= \frac{(79)^2 \times (1.6 \times 10^{19})^2}{16 \times (3.14)^2 \times (8.85 \times 10^{-12})^2 (2 \times 5 \times 10^6)^2 \times (1.6 \times 10^{-19})^2 \times Sin^2\left(\frac{90}{2}\right)} = 6.9 \text{ barns}$$

where $m_0{}^2 V_0{}^4 = (2 \times 5 \times 10^6)^2 \times (1.6 \times 10^{-19})^2$

17. ***A particle of mass 5 kg moving with velocity of 10 m/s collides with another particle of mass 10 kg moving in opposite directions with a velocity of 20m/s. During collision they stick together. Find the common velocity.***

Solution : Applying law of conservation of momentum we have

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2.$$

Given

$m_1 = 5$ kg, $u_1 = 10$ m/s, $m_2 = 10$ kg

$u_2 = 20$ m/sec. it $v_1 = v_2 = v$

Now

$$5 \times 10 + 10(-20) = 5v + 10v$$

$$50 - 200 = 15v$$

$$v = \frac{-150}{15} = -10 \text{ m/sec.}$$

The combination thus moves with a velocity of 10 m/sec in the opposite direction.

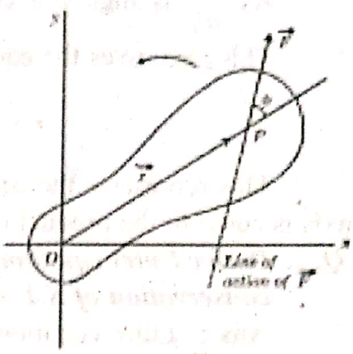
& & &

2. MECHANICS OF RIGID BODIES

LONG ANSWER QUESTIONS

Q. 1. Derive equation of motion of a rigid rotating body.

Ans : Consider a section of a rigid body free to rotate about fixed (Z-axis) on an inertial frame as shown in fig. A force $F\rightarrow$, taken in X-Y plane of the section, acts on a particle P of the body.



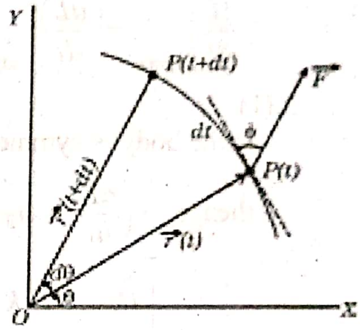
Fig

The position of point P with respect to rotational axis (Z–axis) is defined by vector r. This force produces an angular acceleration in the body and hence it rotates about the Z – axis. The torque acting on the particle P (which may also be taken as a torque acting on the rigid body as a whole) is defined as

$$\tau = r \times F$$

The direction of τ being along the Z–axis up (right hand rule)

Now we shall investigate the relationship between torque applied to the rigid body and angular acceleration of the body. This is also known as the equation of motion of rigid rotating body. Suppose the body rotates through an infinitesimal angle $d\theta$, in an infinitesimal time dt. During time dt, the particle P moves from a position $P(t)$ to a new position $P(t + dt)$ along a circular arc of radius r. The distance convered by P is ds.



Fig

From fig. $ds = r\,d\theta$

The workdone by the force F during rotation is given by

$$dW = F\,.\,ds = F(ds)\cos\phi = F\cos\phi\,.\,ds$$

where $F\cos\phi$ is the component of F in the direction of ds.

$$= (F\cos\phi)\,r\,d\theta \qquad (\because ds = r\,d\theta)$$

Here $(F\cos\phi)\,r$ is the magnitude of the instantaneous torque exerted by F on the rigid body about the axis of rotation (Z – axis).

$$dW = \tau.d\theta \qquad \text{or} \qquad \frac{dW}{dt} = \tau.\frac{d\theta}{dt}$$

$$\text{or} \qquad P = \tau.\frac{d\theta}{dt} = \tau\omega \qquad \left(\because \frac{dW}{dt} = P\right)$$

This expression gives the instantaneous power P.

As a result of this work, the rotational kinetic energy of the body $\left(\frac{1}{2}I\omega^2\right)$ is increasing. Now we equate the rate of work done on the body to the rate of increase in kinetic energy i.e.,

$$\frac{dW}{dt} = P = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) \qquad \tau\omega = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right)$$

$$= \frac{1}{2}I\frac{d}{dt}\left(\omega^2\right) \qquad (I = constant)$$

$$= I\omega\frac{d\omega}{dt} = I\omega\alpha \qquad \left(\because \frac{d\omega}{dt} = \alpha\right) \qquad \therefore\ \tau = I\frac{d\omega}{dt} = I\alpha\,.$$

where α is angular acceleration. This is known as equation of motion.

Relation between torque and angular momentum.

We know that $L = I\omega$ $\therefore \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt}$

As $\frac{d\omega}{dt}$ is angular acceleration and so $I\frac{d\omega}{dt} = \tau$ $\therefore \frac{dL}{dt} = \tau$

This also gives the equation of motion of the rigid rotating body.

$$\tau = I\frac{d\omega}{dt}$$

This represents the equation of motion of a rigid body. This states that the torque about a fixed axis is equal to the product of moment of inertia and angular acceleration about that axis.

Q. 2. Derive Euler equations of rotational motion for a rigid body fixed at one point. Prove the law of conservation of K.E and angular momentum from them.

Ans : Euler equation of motion is used to Transform the equations of motion of rotating body from body coordinates to space co-ordinates.

$$\left(\frac{dL}{dt}\right)_{space} = \left(\frac{dL}{dt}\right)_{body} + \omega \times L \qquad \tau = \left(\frac{dL}{dt}\right) + \omega \times L$$

........(1)

If the body is symmetric, and its axes of rotating coincide with the principle axes,

then $\tau_1 = \left(\frac{dL}{dt}\right) + \omega_2 L_3 - \omega_3 L_2$ ....(2)

$$\omega \times L = \begin{vmatrix} i & j & k \\ \omega_1 & \omega_2 & \omega_3 \\ L_1 & L_2 & L_3 \end{vmatrix}$$

sine $L = iL_1 + jL_2 + kL_3$ and $\omega = i\omega_1 + j\omega_2 + k\omega_3$

$\therefore \omega \times L = i(\omega_2 L_3 - \omega_3 L_2) - j(\omega_1 L_3 - \omega_3 L_1) + k(\omega_1 L_2 - \omega_2 L_1)$

$= i(\omega_2 L_3 - \omega_3 L_2) + j(\omega_3 L_1 - \omega_1 L_3) + k(\omega_1 L_2 - \omega_2 L_1)$

$\therefore$ *Since* $L = I\omega$

$$\tau_1 = I_1\left(\frac{d\omega_1}{dt}\right) + \omega_3 I_3 \omega_2 - \omega_3 I_2 \omega_2 \qquad \tau_1 = I_1\left(\frac{d\omega_1}{dt}\right) + \omega_2\omega_3(I_3 - I_2) \quad ....(3)$$

Similarly $\tau_2 = I_2\left(\frac{d\omega_2}{dt}\right) + \omega_1\omega_3(I_1 - I_3)$ ....(4)

$$\tau_3 = I_3\left(\frac{d\omega_3}{dt}\right) + \omega_1\omega_2(I_2 - I_1) \quad ....(5)$$

Equations (3), (4), (5) are known as Euler equations of rotational motion for a rigid body.

To prove the law of conservation of energy : If no external Torque acts on a body the *K.E* of a rigid rotating body is constant.

If external Torque is zero

The Euler equations can be written as

$$I_1\frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3 = 0 \quad ....(1)$$

$$I_2\frac{d\omega_2}{dt} + (I_1 - I_2)\omega_1\omega_3 = 0 \quad ....(2)$$

$$I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2 = 0 \quad ....(3)$$

Multiplying equations (1), (2), (3) by ω_1, ω_2 and ω_3, respectively and adding.

$$I_1\omega_1\frac{d\omega_1}{dt}+(I_3-I_2)\omega_1\omega_2\omega_3=0 \quad \text{........(4)}$$

$$I_2\omega_2\frac{d\omega_2}{dt}+(I_1-I_3)\omega_1\omega_2\omega_3=0 \quad \text{........(5)}$$

$$I_3\omega_3\frac{d\omega_3}{dt}+(I_2-I_1)\omega_1\omega_2\omega_3=0 \quad \text{........(6)}$$

adding the equations (4), (5) and (6)

$$I_1\omega_1\frac{d\omega_1}{dt}+I_2\omega_2\frac{d\omega_2}{dt}+I_3\omega_3\frac{d\omega_3}{dt}=0 \quad \text{........(7)}$$

The above equation can be written as $\frac{1}{2}\left[\frac{d}{dt}\left\{I_1\omega_1^2+I_2\omega_2^2+I_3\omega_3^2\right\}\right]=0$

$$\therefore \frac{d}{dt}\left[\frac{1}{2}I_1\omega_1^2+\frac{1}{2}I_2\omega_2^2+\frac{1}{2}I_3\omega_3^2\right]=0 \qquad \therefore \frac{d}{dt}[K.\in]=0$$

$\therefore$ Rotational K. E = constant

$\therefore$ If no external Torque acts on a body, its rotational $K.E$ constant. This is known as law of conservation of $K.E.$

To prove the law of conservation of angular momentum :

Statement : If no external Torque acts on a body its angular momentum is constant.

Proof : Multiplying equations (1), (2), (3) with $I_1\omega_1$, $I_2\omega_2$, $I_3\omega_3$, respectively and adding

$$I_1^2\frac{d\omega_1}{dt}\omega_1+I_2^2\frac{d\omega_2}{dt}\omega_2+I_3^2\frac{d\omega_3}{dt}\omega_3=0 \qquad \frac{1}{2}\frac{d}{dt}\left[I_1^2\omega_1^2+I_2^2\omega_2^2+I_3^2\omega_3^2\right]=0$$

$$\therefore \frac{1}{2}\frac{d}{dt}\left[L^2\right]=0 \text{ or } L\frac{d(L)}{dt}=0$$

$\therefore$ L = constant. When external Torque is zero.

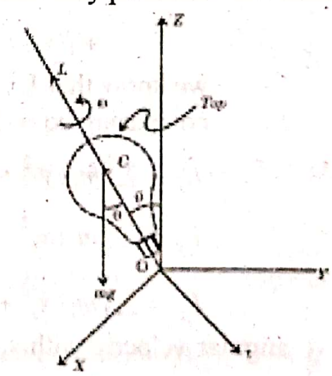
Q. 3. What is a symmetric top? Derive an expression for the precessional velocity of a symmetric top?

Ans : Precession : The rotation of the axis of rotation of the spinning top is called *precession.*

Axis of Precession : The axis about which the direction of rotation of the body precesses is called the *axis of Precession.*

Expression for the angular velocity of Precession of a Top : A symmetrical body rotating about an axis which is fixed at one point is called *Top*. The axis of spinning top moves around the vertical axis and sweeps out a cone.

Consider a symmetric top rotating about the verticl axis OZ. At any instant the axis of the top makes an angle θ with OZ. Let L be the angular momentum. Let r be the posion vector of centre of mass. Let the weight *'mg'* of the top acts vertically down wards from the centre of mass. The torque acting on the top due to weight about



'O' is $\tau=r\times F$ $\qquad \tau=rF\sin\theta \qquad \tau=rmg\sin\theta$

Fig

The direction of Torque is $\perp er$ to the plane containing r and mg. Hence the torque is $\perp er$ to L i.e $\perp er$ to the axis of rotation of the top.

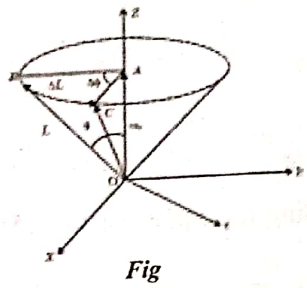
This torque produces angular acceleration $\perp er$ to ω. Hence ω changes in direction by not in magnitude. Hence the axis of Top ω, L, r all precesses about OZ.

Since there is external torque acting on the top, the conservation of angular momentum is not obeyed. The angular momentum value changes in a direction $\perp er$ to L.

In a short time Δt, the torque τ produces a change Δt in angular momentum.

$$\therefore \tau=\frac{\Delta L}{\Delta t} \qquad \text{or} \qquad \Delta L=\tau\Delta t$$

Precessional Velocity (ω_p) **:** Let $\Delta\phi$ be the angle turned by the head of angular momentum in a time Δt. From sectors ABC



Fig

$BC = AC \times \Delta\phi$ $\quad \therefore \Delta\phi = \frac{BC}{AC} = \frac{\Delta L}{OC\sin\theta}$

$\therefore$ Precessional velocity $= \omega_p = \frac{\Delta\phi}{\Delta t}$

From the diagram $\Delta\phi = \frac{BC}{AC} = \frac{\Delta L}{OC\sin\theta} = \frac{\Delta L}{L\sin\theta}$

$\omega_P = \frac{\Delta L}{L\sin\theta \times \Delta t}$ $\quad \omega_p = \frac{\tau}{L\sin\theta}$ $\quad \left(\therefore \frac{\Delta L}{\Delta t} = \tau\right)$ $\quad \omega_P = \frac{mgr\sin\theta}{L\sin\theta}$

$\omega_P = \frac{mgr}{L}$ $\quad$ But $L = I\omega$ $\quad \omega_p = \frac{mgr}{I\omega}$ This is the required relation.

Q. 4. Derive an expression for angular momentum and inertia tensor.

Ans : The relation between angular momentum and angular velocity L = Iω is very simple to explain the motion of rigid rotating body. But when the axis of rotation does not coincide with any of the principal axes of inertia, the relationship between L and ω becomes complicated. Now the motion can be expressed in terms of the components of the angular velocity in the direction of the three axes of a coordinate system which is attached to the rotating body.

In general for i[th] particle may be written as

$$L = \Sigma m_i[(r_i . r_i)\,\omega - (r_i . \omega)\, r_i] \qquad \text{.................. (1)}$$

in general case $r_i . \omega \neq 0$. For this we concider the xyz coordinate system fixed in the body.

Now eq (1) can be written as

$r = ix + jy + kz$ and $\omega = i\omega_x + j\omega y + k\omega_z$

$(r_i .. r_i) = (ix_i + jy_i + kz_i).(ix_i + jy_i + kz_i) = (x_i^2 + y_i^2 + z_i^2)$

$\therefore L = \Sigma m_i[(x_i^2 + y_i^2 + z_i^2)(i\omega x + j\omega y + k\omega z) - (x_i\omega x + y_i\omega y + z_i\omega z)(ix_i + jy_i + kz_i)]$

$= \Sigma m_i\, [\{(y_i^2 + z_i^2)\,\omega x - x_iy_i\omega y - x_iz_i\omega z\}]\, i + [(x_i^2 + z_i^2)\,\omega y - x_iy_i\omega x - y_iz_i\omega z]\, j$

$$+ [(x_i^2 + y_i^2)\,\omega z - x_iz_i\omega x - y_iz_i\omega y]\, k \qquad \text{.................. (2)}$$

we know that $L = iL_x + jL_y + kL_z$ .... (3)

comparing eq n (2) and (3) we get

$L_x = \Sigma\left[m_i(y_i^2 + z_i^2)\,\omega x + [-\Sigma m_ix_iy_i]\,\omega y + [-\Sigma m_ix_iz_i]\right]\omega z$

$L_y = \Sigma\left[m_i(x_i^2 + z_i^2)\,\omega y + [-\Sigma m_ix_iy_i]\,\omega x + [-\Sigma m_iy_iz_i]\right]\omega z$

$$L_z = \Sigma\left[m_i(x_i^2 + y_i^2)\,\omega z + [-\Sigma m_ix_iz_i]\,\omega x + [-\Sigma m_iy_iz_i]\right]\omega y \qquad \text{.................. (4)}$$

angular velocity with x, y, z axes can be written as

$I_{xx} = \Sigma m_i\,(y_i^2 + z_i^2) = \Sigma m_i\,(r_i^2 - x_i^2)$

$I_{xy} = -\Sigma m_ix_iy_i$

$$I_{xz} = -\Sigma m_ix_iz_i \qquad \text{.................. (5)}$$

These are called Inertial coefficients.

In terms of inertial coefficients eq (4) can be written as

$L_x = I_{xx}\,\omega x + I_{xy}\,\omega y + I_{xz}\,\omega z$

$L_y = I_{yx}\,\omega_x + I_{yy}\,\omega_y + I_{yz}\,\omega_z$

$$L_z = I_{zx}\,\omega_x + I_{zy}\,\omega_y + I_{zz}\,\omega_z \qquad \text{.................. (6)}$$

The matrix form of eq (6) is given by

$$\begin{bmatrix} Lx \\ Ly \\ Lz \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \quad \text{.................. (7)}$$

The diagonal elements I_{xx}, I_{yy} and I_{zz} are called as principal moments of inertia around x, y and z axes respectively. The other six terms i,e I_{xy}, I_{xz}, I_{yx}, I_{yz}, I_{zx} and I_{zy} are called as off diagonal terms or products of inertia.

eq (7) can be expressed in a more compact form by using symbols 1, 2, 3 for x, y, z respectively. Thus

$$L_\mu = \sum_{V=1}^{3} I_{\mu\nu}\omega_\nu \qquad \mu = 1, 2, \text{ and } 3 \quad \text{.................. (8)}$$

The more elegant vector form of eq (8) is

$$L = \vec{I}\,\vec{\omega} \quad \text{.................. (9)}$$

where ω is the vector with three components ω_x, ω_y and ω_z and $\vec{I}$ stands for an operator called as tensor.

Properties of Inertia Tensor :

1. Inertial tensor is symmetric i,e the elements of Inertia tensor for all μ and ν obey the relation.

$$I_{\mu\nu} = I_{\nu\mu} \quad \text{.................. (10)}$$

2. We can define xyz axes in the body in such a way that the products of inertia $I_{\mu\nu}$ are zero for all μ, ν. Such axes are called principal axes of Inertia.

$$\vec{I} = \begin{bmatrix} I_x & O & O \\ O & I_y & O \\ O & O & I_z \end{bmatrix} \quad \text{.................. (11)}$$

3. For a sphere all the three axes are symmetric.

   $I_{xx} = I_{yy} = I_{zz}$ .... (12)

   (or) $I_x = I_y = I_z$

   such a body is called spherical top
4. If $I_x = I_y = I_z$, The body is called a symmetric top.
5. A body for which $I_x = I_y$ and $I_z = o$ is called a rotor.
6. For a body with cylindrical symmetry. The axis of the cylinder may be taken as principal z - axis and x and y axes are symmetrical. Then

   $I_{xx} = I_{yy}$ and $I_x = I_y$ .... (13)

Any rigid body other than that having cylindrical shapes and satisfy eq (13) is called a symmetrical top.

7. Consider a solid body rotates about one of its principal axes then $\omega_z = \omega$ and $\omega_x = \omega_y = 0$

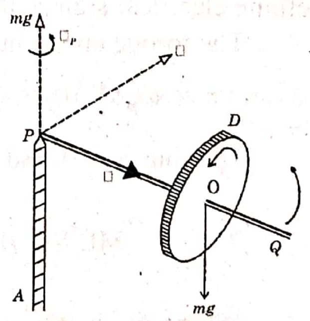
Now $L_x = L_y = 0$ and $L_z = I_z\,\omega$ .... (14)

SHORT ANSWER QUESTIONS

Q. 5. Write short notes on gyroscope?

Ans : The motion of a gyroscope consists of rotation precession and a nutation.

Description : It consists of a heavy disc revolving with a high angular velocity 'ω' about an axis *POQ* passing through the disc, parallel to the horizontal and supported at P on a vertical pivot *AP.* The weight of the wheel acts vertically down wards from the centre of gravity *'O'* of the wheel. Torque acts on the wheel due to gravity.



Fig

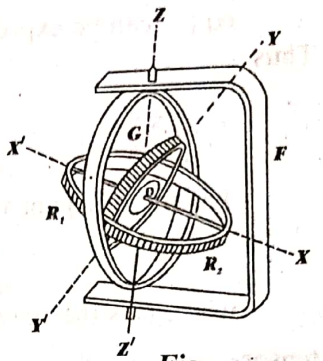
$\tau = mg \times OP$

$\tau = mgr$

Working : When the wheel is in rotatory motion about its axis of symmetry, the gravitational force supplies the torque necessary for the precessional motion. So the body precesses about the axis *AP* with an angular velocity ω_p. The torque τ changes the angular momentum in direction producing the precession of angular velocity ω_p.

If the precessional rate is more, the axis *POQ* rises, if it is low, the axis of rotation fall. The rise and fall of axis of rotation about the equillibrium is called *nutation*.

Gyrostat Description : It consists of a heavy disc *G* capable of rotating about its axis of symmetry. Its axis is mounted such that the disc can turn freely about any one of the three mutually perpendicular axes. Suppose the axis of the disc coincides with *X*–axis. It is carried by a ring R_1. This ring can rotate about the *Y*–axis, held by another ring R_2. R_2 can rotate about the *Z*–axis, held by frame *F*. These 3 axes have the origin coinciding with the *Centre of mass* of the gyrostat. So a gyrostat can rotate about any axis.



Fig

When the disc is made to rotate at high speed, the rate of precession (ω_p) is small. This instrument is used a gyrocampass with its axis of rotation in the magnetic meridian. These are used in ships and submarines.

Q. 6. Explain the precession of equinoxes and its consequence?

Ans : Earth is not a sphere. The lower part of the earth is slightly closure than the upper half. Due to this there is a difference in the force of attraction. This difference acts as an external torque.

In the presence of external torque the axis of rotation precess about an axis. The equitorial plane of the earth make 23.5^0 with the plane of rotation around sun. The line joining the intersection of these two planes is called the line of equinoxes. During one complete rotation of the earth crosses the line of equinoxes twice in a year i.e., on 21 March and 22nd September. The first point is called vernal equinox and second point is called autumnal equinox.

Q.7. Explain the precession of atom and nucleus in magnetic field ?

Ans : We know that the elementary particles possess an intrinsic angular momentem called spin. Most of the atomic nuclei and atoms also possess this spin. The spin of the angular momentam of atoms or nucleus is always have the intrinsic magnetism. So that the atoms or nucleus behaves like a small bar magnet or magnetic dipole. The magnetic moment of the dipole points along the direction in which the atom or nucleus is spinning.

When a bar magnet is placed in external magnetic field, it experiences a torque (τ). The bar magnet shows torque to allign the external field direction. In the same way the atoms and nucleus also shows torque when placed in an external magnetic field.

Due to this torque, the atom or nucleus undergoes a precessional motion just like a symmetric top in the gravitational field.

The magnetic moment of the bar is $M = m \times 2l$

m is pole strength and $2l$ is length of the magnet.

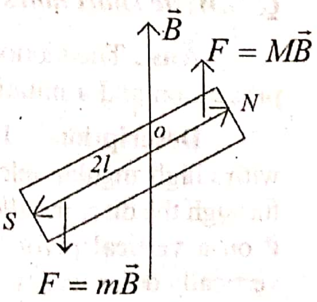
Torque experience by the atom or nucleus is $T - MB \sin\theta$ ....(1)

Where θ is the angle between direction of magnetic induction B and the axis of the magnet.

The rate of precession of atom or nucleus is 'ω_p' can be detected by picking up the very small but definite electrical signals that are produced by the rotating atomic or nuclear magnets.

The torque on the nuclear or atomic magnets are

$$\tau = \omega_p J \sin\theta \quad \text{.........(2)}$$



equating eqns 1 and 2 we get

$$MB \sin\theta = \omega_p J \sin\theta \text{ (or). } \omega_p = \frac{MB}{J}$$

Where ω_p is the speed of precession of the atomic or nuclear magnet.

SOLVED PROBLEMS

8. ***A car develops 75 KW power when rotating at a speed of 1000 rpm. What is torque acting ?***

Solution : The power P developed by torque τ exerted on a rotation body is given by

$P = \tau\,\omega$ or $\tau = P/\omega$

Given that, $P = 75\ KW = 75 \times 10^3\ W = 75 \times 10^3$ *joules/sec.*

and $\omega = 2\pi n = 2\pi\ (1000/60) = 100\ \pi/3$ *rad.sec.*

$$\therefore \quad \tau = \frac{75 \times 10^3}{(100\pi/3)} = 716.3 \text{ joule.}$$

9. ***A sphere of mass 2.5 kg. and diameter 1m rolls without slipping with a constant velocity of 2 m/s. Calculate its total energy.***

Solution : Total kinetic energy is given by

$$K_{total} = K_{trans} + K_{rot,} \quad = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 \quad = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$$

($\because$ for sphere $I = (2/5)\ MR^2$ about diameter and $\omega = V/R$)

$$\therefore \quad K_{total} = \frac{1}{2}MV^2 + \frac{1}{5}MV^2 = \frac{7}{10}MV^2$$

Substituting the given values, we get

$$K_{total} = \frac{7}{10} \times (2.5)\ (2)^2 = 7 \text{ joule}$$

10. ***A ballet dancer spins about a vertical axis at the rate of 1 revolution per second with her arms out stretched. When her arms folded her moment of inertia about the vertical axis decreases by 60%. Calculate the new rate of revolution.***

Solution :

Given that $I_2 = 0.4\ I_1$ (I is reduced 60%)

$\omega_1 = 2\pi \times 1$ *radian / sec.*, $\omega_2 = 2\pi n = ?$

According to the law of conservation of angular momentum

$I_2\omega_2 = I_1\omega_1$ $\quad$ $0.4\ I_1 \times 2\ \pi n = I_1 \times 2\pi \times 1$ $\quad$ $n = 1/0.4 = 2.5$ *Rev/sec.*

11. ***The speed of a particle moving a circle of radius 20 cms increase at the rate of 10 cm/sec². If the mass is 200 gms, find the torque on it.***

Solution :

We know that $L = m\,V\,r$ and $\tau = \dfrac{dL}{dt} = mr\dfrac{dV}{dt}$

Here $m = 200\ gms = 2.2\ k\,gm$, $r = 20\ cm = 0.2\ m$ and $\dfrac{dv}{dt} = 10\ cm/sec^2 = 0.1\ m/sec^2$

$\tau = 0.2 \times 0.2 \times 1.0 = 0.0004\ N\text{-}m.$

12. ***A 500 gm stone is revolved at the end of a 0.4 m long string at the rate of 12.5 rad/s. What is its angular momentum.***

Solution :

We know that, $L = m\,r^2\,\omega$

Given that, $m = 0.5$ *kg.*, $r = 0.4\ m$ and $\omega = 12.5$ *rad/s.*

$\therefore$ $L = 0.5 \times (0.4)^2 \times 12.5 = 1$ *Joule–second*

13. *The kinetic energy of metal disc rotating at a constant speed of 5 revolutions per second is 100 joules. Find the angular momentum of the disc.*

Solution : $L = I\omega$

$\omega = 2\pi n = 2 \times 3.14 \times 5 = 31.4\ sec^{-1}$

$$K.E. = \frac{1}{2}\omega^2 I \quad \text{or} \quad I = \left(\frac{2K.E.}{\omega^2}\right) \quad \text{or} \quad I = \frac{2 \times 100}{(31.4)^2} = 0.2028\ kg\text{–}m^2$$

$\therefore L = I\omega = L = 0.2028 \times 31.4 = 6.368\ kg\text{–}m^2\ sec^{-1}$

14. *A circular disc of mass 100 kg and radius 1m is mounted axially and made to rotate. Calculate the K.E it possesses when executing 120 rotations per minute.*

Solution : Given n = 120 rot / min M = 100 kg, R = 1 m

$$\therefore \omega = \frac{2\pi n}{t} = \frac{2\pi \times 120}{60} = 4\pi\ radi/sec$$

Moment of Inertia $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 100 \times (1)^2$

$\therefore K.E = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2} \times 100 \times (1)^2 \times (4\pi)^2$ K.E = 9349 Joule.

15. *A fly wheel when slowed down from 60 rpm to 30 rpm loses 100J of energy. What is its moment of inertia?*

Solution :

given $\omega_1 = 60\ \text{rpm} = \frac{60}{60} \times 2\pi\ rad/sec = 1 \times 2\pi\ rad/sec.$

$$\omega_2 = \frac{30}{60} \times 2\pi\ \text{rad/sec} = \pi\ \text{rad/sec.}$$

$$\text{loss in K.E} = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_2^2 = \frac{1}{2}I(\omega_1^2 - \omega_2^2) = \frac{1}{2}I((2\pi)^2 - \pi^2)$$

$$100 = \frac{1}{2}I(4\pi^2 - \pi^2).$$

$$I = \frac{2 \times 100}{(4-1)\pi^2} = \frac{200}{3\pi^2}\ kg - m^2$$

& & &

UNIT - II

3. CENTRAL FORCES

LONG ANSWER QUESTIONS

Q. 1. Discuss the conservative nature of central forces.

Ans : Conservative force : A force is said to be conservative, when the work done by the force in moving a particle from a point A to a point B is independent of the path followed between A and B.

Central force is conservative force : The work done depends only on the particle's initial and final positions. In addition, the workdone by a conservative force along a closed path is zero.

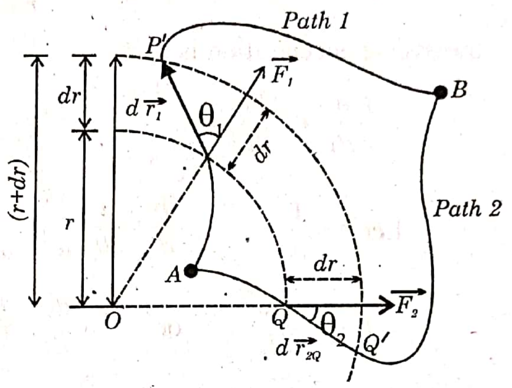
Explanation : Consider a particle is taken from point A to point B through the path $APP^{I}B$ (path1) or $AQQ^{I}B$ (path2) as shown in fig. The amount of workdone by a force F is given by

If the workdone along the two paths is the same, then the force is known as conservative. Thus for a conservative force

$$\int_A^B F.dr = \int_A^B F.dr$$

Path I *Path II*

Consider two points A and B as shown in the diagram. Let a particle moves from point A to B along any path under a central force which directed away from a point O. Taking O as centre draw two arcs of radii r and $(r + dr)$. These are shown in the diagram These arcs will cut the paths at P and P_1, Q and Q_1. Let dr_1 and dr_2 be the displacements of the particle between the arcs along path (I) and (II) respectively. Let F_1 and F_2 be the central forces acting on the particle at points P and Q. Let θ_1 and θ_2 be the angles between F_1 and dr_1 and between F_2 and dr_2.



Fig

$\therefore$ The projections of vectors dr_1 and dr_2 on F_1 and F_2 will be $dr_1 \cos\theta_1$ and $dr_2 \cos\theta_2$.

Since $\vec{F}_1 = \vec{F}_2\ d\vec{r}_1 \cos\theta_1 = d\vec{r}_2 \cos\theta_2$ $\quad \vec{F}_1.d\vec{r}_1 = \vec{F}_2.d\vec{r}_2$

In the same way we can obtain the same result by considering every path segment taken along path I and path II. so, in general

$$\int_A^B \vec{F}.d\vec{r} = \int_A^B F.d\vec{r}$$

Path I. *Path II*

$\therefore$ The work done by the forces along the two paths is equal.

W (Path I) = W (Path II)

In this way, the workdone by a central force acting on a particle moving from point A to point B is independent of path. Hence the central force is conservative.

Further we shall show that for conservative forces, the work done around a close path is zero.

The workdone in moving the particle from point A to point B through path I is given by

$$W(A \to B) = \int_A^B F_1.dr_1 = \int_A^B F.dr$$

(Path I) *(Path II)*

The workdone in moving the particle from point B to point A through path II is given by

$$W(A \to B) = \int_A^B F_2.dr_2 = \int_A^B F.dr$$

(Path I) *(Path II)*

$$\left[\because \int_A^B F_1.dr_1 = \int_A^B F_2.dr_2 = \int_A^B F.dr\right]$$

$\because\ W(A \to B) = -W(B \to A)$ or $W(A \to B) + (B \to A) = 0$

Thus the total around the closed path $A \to B \to A$ is zero.

Q. 2. Derive the equation of motion of a particle under a central force?

Ans : When a body moves under the action of a central force, the force is radial and is always towards a fixed point. The radial accelaration is given by $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$ ....(1)

The transverse accelaration is $\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)$. Since there is no force acting on the body $\perp^{er}$ *to r*, the transverse accelaration is zero.

$$\therefore \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0 \qquad \therefore r^2\frac{d\theta}{dt} = \text{constant} = h \qquad \text{..........(2)}$$

$$\text{Let } r = \frac{1}{u} \qquad \frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\times\frac{d\theta}{dt} = -\left(r^2\frac{d\theta}{dt}\right)\frac{du}{d\theta} = -h\frac{du}{d\theta}$$

$$\therefore h = r^2\frac{d\theta}{dt} \quad \text{or} \quad \frac{d\theta}{dt} = \frac{h}{r^2}$$

$$\text{Further} \qquad \frac{d^2r}{dt^2} = \frac{d}{dt}\left[\frac{dr}{dt}\right] = \frac{d}{dt}\left[-h\frac{du}{d\theta}\right] = -h\frac{d}{d\theta}\left(\frac{du}{dt}\right) = -h\frac{d}{d\theta}\left[\frac{du}{d\theta}\times\frac{d\theta}{dt}\right]$$

$$= -h\frac{d^2u}{d\theta^2}\times\frac{d\theta}{dt} = -h\frac{d^2u}{d\theta^2}\frac{h}{r^2} = -h^2u^2\frac{d^2u}{d\theta^2}$$

$$\therefore \frac{d^2r}{dt^2} = -h^2u^2\frac{d^2u}{d\theta^2} \qquad \text{..........(3)}$$

Substituting the value of $\frac{d^2r}{dt^2}$ from equation (3) in equation (1) Radial accelaration

$$\text{Radial acceleration} = -h^2u^2\frac{d^2u}{d\theta^2} - r\left(\frac{d\theta}{dt}\right)^2 = -h^2u^2\frac{d^2u}{d\theta^2} - r\frac{h^2}{r^4} = -h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3$$

Force acting on the particle = Mass x Radial accelaration

$$F = -m\left[-h^2u^2.\frac{d^2u}{d\theta^2} - h^2u^3\right] = m\left[h^2u^2.\frac{d^2u}{d\theta^2} + h^2u^3\right] \quad \frac{F}{m} = p = \left[h^2u^2.\frac{d^2u}{d\theta^2} + h^2u^3\right]$$ (p is called force per unit mass)

$$\therefore p = h^2u^2.\left[\frac{d^2u}{d\theta^2} + u\right] \qquad \therefore \frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2} \qquad \text{.........(4)}$$

This is the differential equation when a body moves under central force.

Q. 3. State and prove the Kepler's laws of planetory motion?

Ans : There are three laws regarding the motion of planets.

1st Law : Every planet revolves around the sun in elliptical orbit. The sun will be at one of its focii.

2nd Law : The areal velocity of radius vector is constant.

3rd Law : The square of the time period is directly proportional to the cube of the average distance between the sun and the earth.

Deduction of Kepler's Laws :

1st Law : Consider a planet of mass m rotating around the sun of mass M in an orbit of radius r. According to Newtons law of gravitation, the force acting on the planet due to sun is

$$F = -\frac{GMm}{r^2} \quad ....(1)$$

This force is directed towards the centre of the sun.

According to Newton, the radial force

$$F = m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] \quad ....(2)$$

From equations (1) and (2)

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\left(\frac{GM}{r^2}\right) \qquad \frac{d^2r}{dt^2} - r\omega^2 = -\frac{GM}{r^2} \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$$

Multiplying with r^3

$$r^3\frac{d^2r}{dt^2} - r^4\omega^2 = -GMr \qquad r^3\frac{d^2r}{dt^2} - h^2 = -GMr \qquad \left(\because \frac{d\theta}{dt} = \frac{h}{r^2}\right) \quad ....(3)$$

Put $r = \frac{1}{u}$; $\therefore \frac{h}{r^2} = hu^2 = \frac{d\theta}{dt} = \omega$

$$\frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = \frac{-1}{u^2}\frac{du}{dt} = \frac{-1}{u^2}\frac{du}{d\theta} \times \frac{d\theta}{dt} = \frac{-1}{u^2}\frac{du}{d\theta} \times hu^2 \quad \left(\because \frac{d\theta}{dt} = hu^2\right)$$

$$\therefore \frac{dr}{dt} = -h\frac{du}{d\theta} \qquad \frac{d^2r}{dt^2} = -h\frac{d}{dt}\left(\frac{du}{d\theta}\right) = -h.\frac{d}{d\theta}\left(\frac{du}{dt}\right) = -h.\frac{d}{d\theta}\left[\frac{du}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= -h\frac{d}{d\theta}\left(\frac{du}{d\theta}\right) \times hu^2 = -h^2u^2\frac{d^2u}{d\theta^2}$$

$\therefore$ Equation (3) can be written as $\quad r^3\left[-h^2u^2\frac{d^2u}{d\theta^2}\right] - h^2 = -GMr$

Dividing with $-h^2u^2r^3$; $\quad \frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2} \quad ....(4)$

$$\frac{d^2u}{d\theta^2} + \left(u - \frac{GM}{h^2}\right) = 0 \qquad \frac{d^2}{d\theta^2}\left(u - \frac{GM}{h^2}\right) + \left(u - \frac{GM}{h^2}\right) = 0 \qquad \left(\because \frac{GM}{h^2} = constant\right)$$

It's solution is $u - \frac{GM}{h^2} = A \; Cos\, \theta \qquad u = \frac{GM}{h^2} + A \; Cos\, \theta$

$$\therefore \frac{1}{r} = \frac{GM}{h^2} + A\cos\theta$$

Multilplying with $\frac{h^2}{GM}$ $\qquad \frac{\frac{h^2}{GM}}{r} = 1 + \frac{h^2A}{GM}\cos\theta \quad ....(5)$

This equation is similar to polar equation of conicsection

i.e $\frac{l}{r} = 1 + \in Cos\theta \quad ....(6)$

where $l = \frac{h^2}{GM}; \in = \frac{h^2A}{GM}$ $\qquad l$ = semilatus rectum $\qquad \in$ is eccentricity of the conic

If $\in > 1$ the conic is hyperbola $\in = 1$ the conic is parabola $\in < 1$ the conic is ellipse

The total energy of the planet $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$

$$v = r\omega = r\frac{d\theta}{dt} = \frac{h}{r} = \frac{j}{mr} \quad \left(\because\ h = \omega r^2, h = r^2\frac{d\theta}{dt}\left(\frac{h}{r} = r\frac{d\theta}{dt}\right)\right)$$

From equation (6) $\frac{l}{r} = 1 + \in \cos\theta$

When $\theta = 180^0$: $\cos 180^0 = -1;\ r = r_{Max}$, $\therefore \frac{l}{r_{Max}} = 1 - \in$ ....(7)

Similarly when $\theta = 0^0$; $Cos\ 0^0 = 1$, $r = r_{min}$ $\therefore \frac{l}{r_{min}} = 1 + \in$ ....(8)

$$\frac{(7)}{(8)} = \frac{r_{Min}}{r_{Max}} = \frac{1-\in}{1+\in};\ \frac{r_{max}}{r_{min}} = \frac{1+\in}{1-\in}$$

$$\therefore \frac{r_{Max} - r_{Min}}{r_{Max} + r_{Min}} = \frac{1+\in-1+\in}{1+\in+1-\in} = \frac{2\in}{2} = \in$$

$$E = \frac{J^2}{2m}\left(\frac{1}{r_{Max}}\right)^2 - \frac{GMm}{r_{Max}} = \frac{J^2}{2m}\left(\frac{1-\in}{l}\right)^2 - GMm\left[\frac{1-\in}{l}\right] \qquad E = \frac{J^2}{2m}\frac{(1-\in)^2}{l^2} - GMm\left[\frac{1-\in}{l}\right]$$

But $l = \frac{h^2}{GM}$ $\because h = \frac{J}{m}$ $\therefore l = \frac{J^2}{GMm^2}$ But $l^2 = \frac{J^4}{G^2M^2m^4}$; $\frac{1}{l^2} = \frac{G^2M^2m^4}{J^4}$

$$E = \frac{J^2}{2m} \times \frac{G^2M^2m^4}{J^4}(1-\in)^2 - GMm \times \frac{GMm^2}{J^2}(1-\in)$$

$$E = \frac{G^2M^2m^3}{2J^2}(1-\in)^2 - \frac{G^2M^2m^3}{J^2}(1-\in)$$

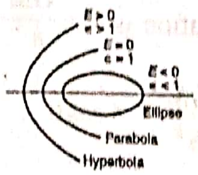
$$2E = \frac{G^2M^2m^3}{J^2}\left[(1-\in)^2 - 2(1-\in)\right] \qquad \frac{2EJ^2}{G^2M^2m^3} = (1-\in)[(1-\in-2)] = -(1-\in)(1+\in)$$

$$\frac{2EJ^2}{G^2M^2m^3} = (1-\in^2) \qquad \therefore E = -\frac{G^2M^2m^3}{2J^2} \times (1-\in^2)$$ Since E is negative $E<1$.

$\therefore$ From this equation it can be shown that every planet revoles around the sun in elliptical orbit.

2nd Law : Let the planet moves from P to P in a time dt. Let $d\theta$ be the angulars displacement.

Let the area covered in a time $dt = \frac{1}{2} \times h^2 d\theta$



Areal velocity $= \frac{1}{2}r^2\frac{d\theta}{dt} = constant = \frac{h}{2}$

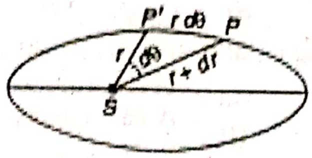
$\therefore$ The areal velocity of radius vector is constant

3rd Law : If a and b are the semi major and semi minor axes of the ellipse,

$$l = \frac{b^2}{a} = \frac{h^2}{GM} \qquad \therefore \frac{b^2}{h^2} = \frac{a}{GM}$$

If T is the period of revolution of the planet around the sun

$$T = \frac{Area\ of\ the\ ellipse}{Areal\ velocity} = \frac{\pi ab}{h/2} = \frac{2\pi ab}{h} \qquad T^2 = \frac{4\pi^2a^2b^2}{h^2} = \frac{4\pi^2a^2 \times ah^2}{GM \times h^2} = \frac{4\pi^2a^3}{GM}$$



$T^2 \alpha\, a^3$ This is 3rd Law.

$\therefore$ The square of the time period is directly proportional to the cube of semi major axis.

$$\therefore \frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3} \text{ or } \frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} \quad \text{.........(4)}$$

SHORT ANSWER QUESTIONS

Q. 4. What is a Central Force? Give two Examples.

Ans : Central force : A central force is defined as a force with always acts on a particle or body towards or away from a fixed point and whose magnitude depends upon only on the distance from the fixed point. The central force on particle is expressed by F can be expressed as

$$F = rf(r) \qquad \text{—(1)}$$

Where f (r) is a function of the distance of the particle from the fixed point and $\hat{r}$ is unit vector.

Examples : 1. The gravitational force excerted on a particle by another particle which is stationary. Consider the gravitational attraction force between two masses m_1 and m_2 separated at a distance r. The force experienced can be written as.

$$F_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}$$

Negative sign indicates that force is attractive.

But $F = f(r)\,\hat{r}$. $\qquad \therefore \; f(r) = -G\frac{m_1 m_2}{r^2} = -C/r^2$.

where $C = G\,m_1\,m_2$ (or) $f(r)\;\alpha\;\frac{1}{r^2}$.

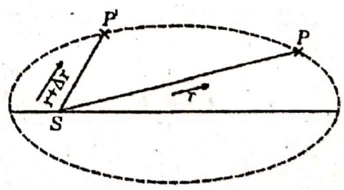
2. The electrostatic forced exerted on a charged particle by another. Stationary charged particle is central force. The electrostatic force between two charges is given by

$$F_{12} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}.\hat{r} \qquad \therefore\; f(r) = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2} = \frac{C}{r^2} \text{ where } C = \frac{1}{4\pi\epsilon_0}\cdot q_1 q_2$$

$\therefore F = f(r).\hat{r}$ (or) $f(r)\;\alpha\;\frac{1}{r^2}$

Q. 5. Prove that the Areal Velocity is constant under the influence of central force ?

Ans : Consider the motion of the earth around the sun. At any instant r is the radius vector of the earth with respect to sun. In a short time interval dt the earth moves from position. P to P^I where the radius vector is $r + \Delta r$. Let ΔA be the area swept by the radius vector in a time interval Δt



$\therefore \Delta A =$ Area of the triangle $\quad SPP^I = \frac{1}{2} \times base \times height$ **Fig**

Area of the Triangle .

$$\Delta A = \frac{1}{2} r \times \Delta r \qquad \frac{\Delta A}{\Delta t} = \frac{1}{2} r \times \frac{\Delta r}{\Delta t} \text{ or } \frac{dA}{dt} = \frac{1}{2} r \times \frac{dr}{dt} = \frac{1}{2} r \times \upsilon = \frac{1}{2m} r \times m\upsilon$$

$$\frac{dr}{dt} = \frac{L}{2m}$$

$\therefore$ Under central force angular momentum remains constant. $L =$ const.

$\therefore \frac{dr}{dt} = const$

$\therefore \frac{dA}{dt} =$ Areal velocity = constant. This means radius vector sweeps out equal areas in equal time.

Q. 6. What are the characteristics of central force ?

Ans : 1. The general form of central force is represented by $F = \hat{r}\ f(r)$

2. Central force is conservative force.
3. Under a central force, the torque acting on the particle is always zero.
4. Under a central force, the angular momentum of the particle remains conserved.
5. Under a central force the areal velocity of the particle remains constant.
6. The central force is attractive when $f(r) < 0$ i,e negative and repulsive $f(r) > 0$ i,e positive.

Q. 7. Prove that conservative force as a negative gradient of potential energy ?

Ans : When a particle acted upon by a conservative force F moves from space point (x_0, y_0, z_0) to another space point (x, y, z) then potential energy at r is given by.

$$U(r) = -\int_{r_0}^{r} F.dr$$

Now we express F and dr in rectangular coordinates as follow.

$$F = i\,F_x + i\,F_y + k\,F_z$$

and $dr = i\,dx + j\,dy + k\,dz$

$$F.\,dr = (i\,F_x + j\,F_y + k\,F_z)\,.\,(i\,dx + j\,dy + k\,dz)$$

$$= F_x\,dx + F_y\,dy + F_z\,dz.$$

$$U(r) = \int_{r_0}^{r} F.dr = -\int_{x_0}^{x} F_x d_x - \int_{y_0}^{y} F_y d_y - \int_{z_0}^{z} F_z d_z = \int U\,(x_1\,y_1\,z)$$

Differentiating the equation partially with respect to x, y, z we get.

$$F_x = -\frac{dU}{dx},\ F_y = -\frac{dU}{dy},\ \text{and } F_z = \frac{-dU}{dz}$$

$$\therefore F = i\,F_x + j\,F_y + k\,F_z = -i\frac{dU}{dx} - j\frac{dU}{dy} - k\frac{dU}{dz} = -\left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)U$$

$$F = -(\nabla U) = -\text{grad } U.$$

Q. 8. Show that the curl of a central force is zero ?

Ans : We know that $F = \nabla U$

$$\therefore \text{Curl } F = \nabla\times F = \nabla\times(\nabla Y) = \nabla\times\left(i\frac{dU}{dx} + j\frac{dU}{dy} + k\frac{dU}{dz}\right) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{dU}{dx} & \frac{dU}{dy} & \frac{dU}{dz} \end{vmatrix}$$

$$= i\left(\frac{d^2U}{\partial y\,\partial z} - \frac{d^2U}{\partial z\,\partial y}\right) + j\left(\frac{d^2U}{\partial z\,\partial x} - \frac{d^2U}{\partial x\,\partial z}\right) + k\left(\frac{d^2U}{\partial x\,\partial y} - \frac{d^2U}{\partial y\,\partial x}\right)$$

As U is a perfect differential, hence

$$\frac{d^2U}{\partial y\,\partial z} = \frac{d^2U}{\partial z\,\partial y}\quad \frac{d^2U}{dx\,\partial z} = \frac{d^2U}{dz\,dx}\ \text{and soon}$$

$$\therefore \text{Curl } F = \nabla\times F = 0$$

Thus the curl of a conservative force is zero.

Q. 9. Derive Newton's law of gravitation from Kepler's law.

Ans : Consider the case two planets of masses M_1 and M_2 revolving around the sun in circular orbits of radii r_1 and r_2 respectively. Let T_1 and T_2 be their respective periods of revolution.

The centripetal force acting on first planet.

$$F_1 = m_1 r_1 \omega_1^2 = m_1 r_1 \left(\frac{2\pi}{T_1}\right)^2 = 4\pi^2 \frac{m_1 r_1}{T_1^2} \qquad \text{—(1)}$$

Similarly, the centripetal force acting on second planet.

$$F_2 = m_2 r_2 \omega_2^2 = m_2 r_2 \left(\frac{2\pi}{T_2}\right)^2 = 4\pi^2 \frac{m_2 r_2}{T_2^2} \qquad \text{—(2)}$$

Dividing eqn's 1 by 2 we get

$$\frac{F_1}{F_2} = \frac{m_1}{m_2} \cdot \frac{r_1}{r_2} \cdot \frac{T_2^2}{T_1^2} \qquad \text{—(3)}$$

According to kepler's 3rd law $T_1^2 \alpha r_1^3$ and $T_2^2 \alpha r_2^3$

$$\therefore \frac{F_1}{F_2} = \frac{m_1}{m_2} \cdot \frac{r_1}{r_2} \cdot \frac{r_2^3}{r_1^3} = \frac{m_1 r_2^2}{m_2 r_1^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{m_1 / r_1^2}{m_2 / r_2^2} \qquad \text{(or)} \qquad F \alpha \frac{m}{r^2} \qquad \text{—(4)}$$

If one planet is sum of Mass M, them according to Newton's 3rd law of Motion.

$$F \alpha \frac{M}{r^2} \qquad \text{—(5)}$$

$F \alpha \dfrac{m M}{r^2}$ which is Newton's law of gravitation.

Q. 10. Explain about geostationary satelite and found its height from the surface of the earth (radius of the earth $R = 6.4 \times 10^6$ m).

Ans : Geo – Stationary satellite : We have seen that the time period of a satellite increases with increasing distancer. Thus for some orbital radius. The time period T will be exactly equal to the time period of rotation of earth i,e., 24 hours. Let us consider that a satellite with T = 24 hours. Let us consider that a satellite with T = 24 hours has been put in a circular orbit with its plane coinciding with equitorial plane. Further let the satellite will move from west to east. Now for an observer on the earth. The satellite will appear sationary. Such a satellite is called Geo-Centric satellite.

Now we shall calculate the altitude of the Geo-stationary satellite.

We know that $T = 2\pi \sqrt{\dfrac{r}{g}}$

The value gat a distence r from the centre of the earth is given by $9.8 \times R^2/r^2$.
Where R is the Radius of the earth.

$$T = 2\pi \sqrt{\frac{r^3}{9.8 R^2}}$$

Now T = 24 hours = 86,400 sec. and R = 6.4×10^6 m.

$$\therefore 86{,}000 = 2 \times 3.14 \sqrt{\frac{r^3}{9.8 \times (6.4 \times 10^6)^2}}$$

$$\therefore (86{,}400)^2 = (2)^2 \times (3.14)^2 \times \frac{r^3}{9.8 \times (6.4 \times 10^6)^2}$$

$$r^3 = \frac{(86{,}400)^2 \times 9.8 \times (6.4 \times 10^6)^2}{(2 \times 3.14)^2} \qquad r = 4.283 \times 10^7 \text{ m.}$$

$\therefore$ Height of the satellite above the earth's surface.

$h = 4.283 \times 10^7 - 6.4 \times 10^6.$ $\quad = 4.233 \times 10^7 - 0.64 \times 10^7$

$\therefore$ h 3.593 $\times 10^7$ m $\quad$ h = 35930 km.

Q. 11. Discuss about Motion of satellites ?

Ans : We know that planets revolve around the sun. Similarly there are certain heavenly bodies which revolve around the planets. These bodies are called satellites. For example moon revolves around the earth and hence moon is a satellite of the earth. Thus any relatively small body moving round another relatively massive body is primarily called as satellite and its closed relative path is called by orbit.

Moon is a natural satellite of the earth. Now-a-days artificial satellites are also put into orbits round the earth. The satellites move round the earth under the action of gravitation attraction 'exerted by the planet on the satellite' is under the action of a central force. The launching of an artificial satellite is done by means of multi stage booster rockets.

1. The orbital velocity of the satellite te revolve around the earth must be $V = R\sqrt{\frac{g}{r}}$

   Where his the height of the satellite from earth's surface g is acceleration due to gravity.

2. The period of revolution of the satellite is $T = \frac{2\pi r}{r}$

SOLVED PROBLEMS

12. If earth is at one half of its present distance from sun, what will the number of days in a year.

Solution : From Kepler's law $\quad T^2 \alpha\, a^3$

$$\therefore \frac{T_2^2}{T_1^2} = \frac{a_2^3}{a_1^3} \qquad \text{Here } T_1 = 365 \text{ days and } a_2 = \frac{a_1}{2}$$

$$\therefore T_2 = \sqrt{T_1^2 \frac{(a_1/2)^3}{a_1^3}} = \sqrt{\frac{T_1^2}{8}} = \sqrt{\frac{(365)^2}{8}} = 129 \text{ days}$$

13. Estimate the mass of the sun assuming the orbit of earth around the sun is a circle. The distance between the sun and the earth is 1.49 x 10^11 m.

Solution : $T = 2\pi\sqrt{\frac{R^3}{GM}} \qquad M = 4\pi^2 \frac{R^3}{T^2 G}$

Radius of the orbit $\quad R = 1.49 \times 10^{11} m. \quad G = 6.67 \times 10{-}11\ Nm^2\ kg^{-2}.$

Time period of earth around Sun

$T = (365.25 \times 24 \times 60 \times 60)$ Sec

Mass of the Sun $M = \frac{4 \times (3.14)^2 \times (1.49)^3 \times 10^{33}}{(31557600)^2 \times 6.67 \times 10^{-11}} = 1.964 \times 10^{30}$ kg.

14. Show that the force $F = (y^2 - x^2)\, i + 2xyj$ is conservative.

Solution : $\nabla \times F = 0$ where $F = (y^2 - x^2)\,\hat{i} + 2xy\hat{j}$

$$\therefore \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ y^2 - x^2 & 2xy & 0 \end{vmatrix} = \hat{i}\left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(2xy)\right] + \hat{j}\left[\frac{\partial}{\partial y}(y - x^2) - \frac{\partial}{\partial x}(0)\right]$$

$$+\hat{k}\left[\frac{\partial}{\partial x}(2xy)-\frac{\partial}{\partial y}\left(y^2-x^2\right)\right]$$

$$=\hat{i}[0-0]+\hat{j}[0-0]+\hat{k}[2y-2y]=\hat{i}[0]+\hat{j}[0]+\hat{k}[0]=0$$

15. If the radius of the earth suddenly changes to half the present value without any change in mass. What would be the change in the duration of the day?

Solution : Applying the law of conservation of momentum $I_1\omega_1=I_2\omega_2$

where $I_1=MR_1^2$, $\omega_1=\dfrac{2\pi}{T_1}$ where $T_1=24$ hours $I_2=MR_2^2$, $\omega_2=\dfrac{2\pi}{T_2}$ But $R_2=\dfrac{R_1}{2}$

$$\therefore \frac{2}{5}MR_2^2\left(\frac{2\pi}{T_1}\right)=\frac{2}{5}M\left(\frac{R_1}{2}\right)\frac{2\pi}{T_2} \quad \text{or} \quad T_2=\frac{T_1}{4}=\frac{24}{4}=6 \text{ hours.}$$

16. The maximum and minimum distances of a comet from the sun are 1.6×10^{10} m and 8×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 m/sec. Calculate the speed at the farthest point.

Solution : Here the angular momentum is conserved. $L = mvr$ = constant

$$m\,v_1 r_1 = m\,v_2 r_2 \qquad v_1 r_1 = v_2 r_2$$

(or) $$v_1=\frac{v_2 r_2}{r_1}=\frac{6\times10^4\times8\times10^{10}}{1.6\times10^{10}}=3\times10^4 \text{ m/sec.}$$

& & &

UNIT - III

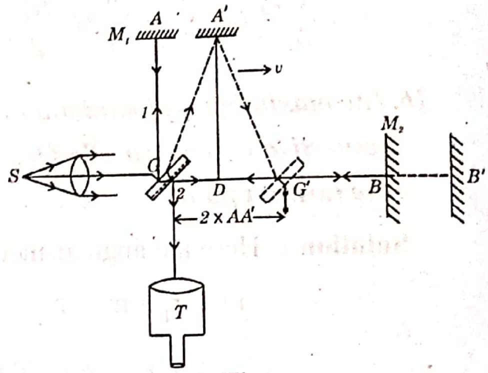
4. RELATIVISTIC MECHANICS

LONG ANSWER QUESTIONS

Q. 1. Describe Michelson - Morley experiment and explain the importance of its result?

Ans : *A.A. Michelson* and *E.W.Morley* carried out an experiment in the year 1887 to determine the absolute velocity of earth in the stationary ether basing on the principle of Interference.

Description : S is a monochromatic source of light of wave length λ. The light rays are made to fall on a convex lens L. The emergent light is made to fall on a semi silvered glass plate G kept at 45^0 to the beam. The beam is splitted into two parts. The reflected beam travels at right angle towards a plane mirror M_1 at A. The mirror again reflects the beam to G. The transmitted beam travels along the direction of initial beam and falls on the plane mirror M_2 at B. It is again reflected to G. The two reflected beams combine at G producing interference pattern, which can be observed with the help of Telescope T. To make the optical paths as equal another glass plate, identical to G^1 is introduced in the path of transmitted ray. The plate is called *compensating plate.*



Fig

Working : Let the two mirrors M_1 and M_2 are at a distance 'l' from G. If this arrangement is at rest in ether, the two rays (reflected) will take same time to reach G. But the whole apparatus is moving along with the earth. Let us suppose that the direction of motion of the earth is in the direction of initial beam. The optical paths travelled by both the beams are not the same.

Theory : Let c = Velocity of light in air

v = Velocity of earth.

Due to motion of earth the motion of light rays are as shown in the diagram. From diagram $GA^1 = ct$ and $AA^1 = vt$.

$GG^1 = 2\,AA^1$

$(GA^1)^2 = (AA^1)^2 + (A^1D)^2 \qquad (\because GD = AA^1)$ ....(1)

If t be the time taken by the ray to move from G to A, then from equation (1), we have

$$(ct)^2 = (vt)^2 + (l)^2 \quad \text{or} \quad t^2(c^2 - v^2) = l^2 \quad \text{or} \quad t = \frac{l}{(c^2 - v^2)^{1/2}}$$

Let t_1 be the time taken to travel the path GA^1G^1.

$$\therefore t_1 = 2t = \frac{2l}{(c^2 - v^2)^{1/2}} = \frac{2l}{c\left(1 - \frac{v^2}{c^2}\right)^{1/2}} = \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{\frac{-1}{2}} = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2}\right] \qquad ....(2)$$

$$\therefore t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2}\right]$$

Let t_2 be the time taken by the transmitted beam to travel from G to A and back from A^1 to G^1.

$$t_2 = \frac{l}{(c-v)} + \frac{l}{(c+v)} = t^2 = \frac{l(c+v) + l(c-v)}{(c^2 - v^2)} \quad \frac{2lc}{(c^2 - v^2)} = \frac{2lc}{c^2\left[1 - \frac{v^2}{c^2}\right]} = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1}$$

$$t_2 = \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right) \qquad ....(3)$$

$$\therefore \text{The time difference } \Delta t = t_2 - t_1 = \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = \frac{2l}{c} \times \frac{V^2}{2c^2} = \frac{lv^2}{c^3}$$

$\therefore$ Optical path difference $= velocity \times \Delta t = c \times \Delta t = c \times \dfrac{lv^2}{c^3} = \dfrac{lv^2}{c^2}$

$\therefore$ The path difference in terms of wave length $= n = \dfrac{lv^2}{c^2\lambda}$

When the apparatus is turned through 90^0, the positions two mirrors are changed.

$\therefore$ The path difference $= \left(\dfrac{lv^2}{c^2\lambda}\right)$

$\therefore$ The resultant path difference $= \left(\dfrac{lV^2}{\lambda c^2}\right) - \left(-\dfrac{lV^2}{\lambda c^2}\right) = \dfrac{2lV^2}{c^2\lambda}$

$\therefore$ Change in fringe shift $= \; n = \dfrac{2lv^2}{c^2\lambda}$

Substituting the various values

$l = 10 \times 10^3\ cm$; $\lambda = 5.0 \times 10^{-5}\ cm$; $v = 3 \times 10^6\ cm/sec$;

$c = 3 \times 10^{10}\ cm/sec$ or $n = \dfrac{2 \times 1.0 \times 1.0^3 \times (3 \times 10^6)^2}{5.0 \times 1.0^{-5} \times (3 \times 10^{10})^2} = 0.4\ fringe$

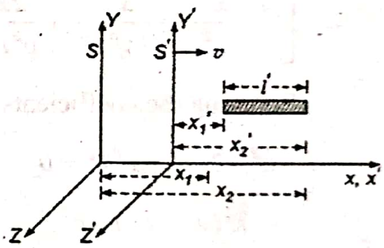
They repeated the experiment at different places and at different seasons of the year. But they could not detect any measurable shift. Hence it is a –ve result. Hence with this –ve result it was concluded that it is impossible to measure the speed of the earth relative to ether or the concept of a fixed frame of reference cannot be checked by experiment.

Significance : This experiment suggested that the speed of light in vacuum is the same in all frames of reference which are in uniform relative motion.

Q. 2. State Postulates of special theory of Relativity ? Derive Lorentz transformations.

Ans : Postulates of special theory of Relativity :

1. All Physical laws are the same in all inertical frames of reference which are moving with constant velocity relative to each other.
2. The speed of light in vacaum is the same in every inertial frame.



Fig

Consider two frames of references S and S^1. Let S^1 frame is moving relative to S with a velocity V along the +ve X–direction. Any event has co-ordinates $(x, y, z$ and $t)$ for an observer in S frame and (x^1, y^1, z^1, t^1) for an observer in S^1 frame.

$\therefore$ *Velocity of light* $= \dfrac{distance}{time}$ or $c = \dfrac{(x^2 + y^2 + z^2)^{1/2}}{t}$ $\qquad c^2t^2 = (x^2 + y^2 + z^2)$

$\therefore x^2 + y^2 + z^2 - c^2t^2 = 0$ ....(1)

Since the velocity of light is same in all reference frames

$x^{1^2} + y^{1^2} + z^{1^2} - c^2t^{1^2} = 0$ (c is constant) ....(2)

Since the frame S^1 is moving relative to S along X- direction

$y^1 = y$; $z^1 = z$ .....(3)

From equations (1) and (2), using eqn.(3), we have

$\therefore\ x^2 - c^2t^2 = x^{1^2} - c^2t^{1^2}$ ....(4)

The transformation between x and x^1 is can be represented by the simple relation

$x' = \lambda (x - Vt)$ ....(5)

If the S frame move with a velocity $-V$ along

where λ being independent of X and t +X direction $x = \lambda^{1} (x' + Vt')$ ....(6)

Substituting the value of x' from (5) in (6)

we have $x = \lambda'\left[\lambda(x - vt) + vt'\right]$

$\therefore \frac{x}{\lambda^1} = \lambda(x - vt) + vt^1$ or $Vt^1 = \frac{x}{\lambda^1} - \lambda(x - Vt)$ $t' = \lambda\left[t - \frac{x}{v}\left(1 - \frac{1}{\lambda\lambda^1}\right)\right]$ ....(7)

Substituting the values of x' value from (5) and t' value from (7) in equation (4) and on simplification.

$$x^2 - c^2t^2 = \lambda^2(x - vt)^2 - c^2\lambda^2\left[t - \frac{x}{v}\left(1 - \frac{1}{\lambda\lambda^1}\right)\right]^2$$

consider R.H.S term

$$\lambda^2(x^2 + v^2 + t^2 - 2xvt) - c^2\lambda^2\left[t^2 - \frac{x^2}{v^2}\left(1 - \frac{1}{\lambda\lambda^1}\right)^2 - \frac{2tx}{v}\left(1 - \frac{1}{\lambda\lambda^1}\right)\right]$$

$$\lambda^2x^2 + \lambda^2v^2t^2 - 2\lambda^2xvt - c^2\lambda^2\left[t^2 - \frac{x^2}{v^2}\left(1 - \frac{1}{\lambda^2\lambda^{12}} - \frac{2}{\lambda\lambda^1}\right)^2 - \frac{2tx}{v} + \frac{2tx}{v\lambda\lambda^1}\right]$$

$$\lambda^2x^2 + \lambda^2v^2t^2 - 2\lambda^2xvt - c^2\lambda^2\left[t^2 - \frac{x^2}{v^2} - \frac{x^2}{v^2\lambda^2\lambda^{12}} + \frac{2x^2}{v^2\lambda\lambda^1} - \frac{2tx}{v} + \frac{2tx}{v\lambda\lambda^1}\right] = 0$$

$$\therefore x^2 - c^2t^2 - \lambda^2x^2 - \lambda^2v^2t^2 + 2\lambda^2xvt + c^2\lambda^2$$

$$\left[t^2 - \frac{x^2}{v^2} - \frac{x^2}{v^2\lambda^2\lambda^{12}} + \frac{2x^2}{v^2\lambda\lambda^1} - \frac{2tx}{v} + \frac{2tx}{v\lambda\lambda^1}\right] = 0$$

Equating the coefficients of t^2 equal to zero, we get

$-c^2 - \lambda^2v^2 + c^2\lambda^2 = 0$ $\qquad -c^2 - \lambda^2(v^2 - c^2) = 0$

$-\lambda^2(v^2 - c^2) = c^2$ $\qquad \lambda^2(c^2 - v^2) = c^2$

$$\lambda^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \quad \therefore \lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{Similarly } \lambda^1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad .....(10)$$

Substituting the values of λ in eqn. (5) $x^1 = \dfrac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$ ...... (A)

Substituting the values of λ and λ^1 in eqn. (7) or $t^1 = \dfrac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$ ...... (B)

$z^1 = z; y^1 = y$

These are called Lorentz transformations. The inverse Lorentz transformations are or $x = \dfrac{x^1 + vt^1}{\sqrt{1-\dfrac{v^2}{c}}}$; $y = y^1$; $z = z^1$ and $t = \dfrac{t^1 + \dfrac{vx^1}{c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}}$ .... (C)

These are known as inverse Lorentz transformation equation.

Q. 3. Derive Einstein's mass-energy relation. ***(Or)***

Derive Equivalence of mass and energy.

Ans : Consider a particle of mass 'm' moving with a velocity v. Let a force F acts on it for a time dt. The particle moves through a distance dx. Let dk be the increase in kinetic energy. According to Newtons second law $F = \dfrac{dp}{dt} = \dfrac{d}{dt}(mv)$

.......(1)

According to theory of relativity, the mass as well as velocity are variables

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad ....(2)$$

When a particle is displaced through a distance dx by the application of a force, F, then the increase in kinetic energy dk is

work done = $dK = F \times dx$ ....(3)

Substituting the value of F, from eqn. (2) in eqn. (3), we get,

$$dK = \left[m\frac{dv}{dt} + v\frac{dm}{dt}\right]dx \qquad dK = mdv\left[\frac{dx}{dt}\right] + vdm\left[\frac{dx}{dt}\right]$$

$$\therefore dK = mvdv + v^2dm \qquad \left(\because \frac{dx}{dt} = v\right) \quad ....(4)$$

The variation of mass with velocity is given by $m = \dfrac{m_0}{\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}}$

where m_o = Rest mass of the particle.

m = mass of the particle when moving with velocity v.

Squaring both sides, we have $m^2 = \dfrac{m_0^2}{\left(\dfrac{c^2 - v^2}{c^2}\right)}$

$m^2(c^2 - v^2) = m_o^2 c^2$; $m^2c^2 = m_0^2 c^2 + m^2v^2$

Differentiating equation $c^2\, 2mdm - v^2 2m\, dm - m^2\, 2v\, dv = 0$

or $c^2\, dm - v^2\, dm - mv\, dv = 0$

or $c^2\, dm = mvdv + v^2\, dm$ ....(5)

From equations (1) and (3) $dk = c^2dm$ ....(6)

Integrating equation (4)

$$K = \int dK = c^2\int_{m_0}^{m} dm = c^2(m - m_0) \quad K = mc^2 - m_0c^2 \quad ....(7)$$

This is the relitivistic expression for $K.E.$ This is the increase in $K.E.$ The total kinetic energy of the body $E = K + m_0c^2$

or $E = c^2(m - m_0) + m_0c^2$. or, $E = mc^2$

$\therefore E = mc^2$.

This is Einsteins mass energy relation.

Q. 4. Explain variation of mass with velocity ?

Ans : Consider two systems of coordinates S and S', the latter moving with a velocity V. In order to consider the variation of mass with velocity, we shall consider a collission of two bodies in system S' and view it from system S.

Let the two bodies of equal masses m_1 and m_2 be travelling with velocities u' and $-u'$ parallel to x-axis in the system S'.

The velocities u_1 and u_2 are given as

$$u_1 = \frac{u^1+v}{1+\frac{u^1v}{c^2}} \text{ and } u_2 = \frac{u^1+v}{1-\frac{u^1v}{c^2}} \qquad \text{.........(1)}$$

The mass of the body travelling with velocity u_1, be m_1 and that of the body moving with velocity u_2 be m_2. Applying law of conservation of momentum.

forth is collission, wehave

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,u \qquad \text{.........(2)}$$

Substituting u_1 and u_2 values from eq 1 in eq 2

we get

$$m_1\left(\frac{u^1+v}{1+\frac{u^1v}{2}}\right) + m_2\left(\frac{-u^1+v}{1-\frac{u^1v}{2}}\right) = (m_1+m_2)u \qquad m_1\left(\frac{u^1+v}{1+\frac{u^1v}{2}}\right) - m_1u = m_2u - m_2\left(\frac{-u^1+v}{1-\frac{u^1v}{2}}\right)$$

on rearranging these equations, we get $\dfrac{m_1}{m_2} = \left(\dfrac{1+\frac{u^1v}{c^2}}{1-\frac{u^1v}{c^2}}\right)$ ....(3)

From eq (1) $u_1^{\,2} = \left(\dfrac{u^1+v}{1+\frac{u^1v}{2}}\right)^2$ or $1-\dfrac{u_1^{\,2}}{c^2} = 1-\dfrac{1}{c^2}\left(\dfrac{u^1+v}{1+\frac{u^1v}{2}}\right)$

solving weget

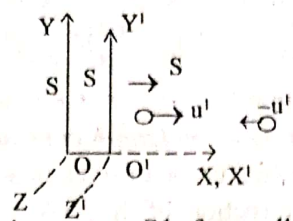
$$\therefore 1+\frac{u^1v}{c^2} = \left[\frac{\left(1-\frac{u_1^{\,2}}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_1^{\,2}}{c^2}\right)}\right] \qquad \text{.............(4A)}$$

Similarly we obtain

$$1-\frac{u_1v}{c^2} = \left[\frac{\left(1-\frac{u_1^{\,2}}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_2^{\,2}}{c^2}\right)}\right]^{\frac{1}{2}} \quad \frac{\left(1-\frac{u_1^{\,2}}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{u_2^{\,2}}{c^2}\right)} \qquad \text{..........(4B)}$$

Substituting above equation 4A and 4B in eq 3

We get $$\frac{m_1}{m_2}=\frac{\left(1-\frac{u_2^2}{c^2}\right)^{1/2}}{\left(1-\frac{u_1^2}{c^2}\right)^{1/2}}$$



Let the body of mass m_2 be moving with zero, velocity system S before collission is $u_2 = 0$

$$then\ \frac{m_1}{m_2}=\frac{1}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

In commonly used notation $m_1 = m\ m_2 = m_0$ and $u_1 = V$

$$\therefore \frac{m}{m_0}=\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$(or)\ m=\frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}} \qquad ....(5)$$

This is the relativistic formula for the variation of mass with velocity.

SHORT ANSWER QUESTIONS

Q. 5. Prove $x^2 + y^2 + z^2 = c^2t^2$ is invariant under Lorentz transformations.

Ans : From inverse Lorentz transformation

$$x=\frac{x^1+vt^1}{\sqrt{1-\frac{v^2}{c^2}}};\ y=y';\ z=z' \text{ and } t=\frac{t^1+\frac{vx^1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Substituting these values in the given expression

we get $$t=\frac{(x^1+vt^1)^2}{\left(1-\frac{v^2}{c^2}\right)}+y^{12}+z^{12}=c^2\frac{\left(t^1+\frac{vx^1}{c^2}\right)^2}{\left(1-\frac{v^2}{c^2}\right)}$$

$$y^{12}+z^{12}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(c^2t^{12}+\frac{v^2x^{12}}{c^2}+2rvx^1t^1-x^{12}-v^2t^{12}-2vx^1t^1\right)$$

$$y^{12}+z^{12}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(c^2t^{12}+\frac{v^2x^{12}}{c^2}-x^{12}-v^2t^{12}\right)$$

$$y^{12}+z^{12}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left((c^2t^{12}-x^{12})(1-\frac{v^2}{c^2})\right)$$

$y'^2 + z'^2 = (c^2t'^2 - x'^2)$ $\qquad$ $x'^2 + y'^2 + z'^2 = c^2t'^2$

Thus $x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 = c^2t'^2$

So the equation $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation.

Q. 6. Write short notes on length contraction.

Ans : Consider two frame of references S and S'. Let S' frame is moving relative to S with a velocity V along the +ve direction of X - axis. Let a rod of length l' is present in S' frame. The end coordinates be x_1' and x_2'

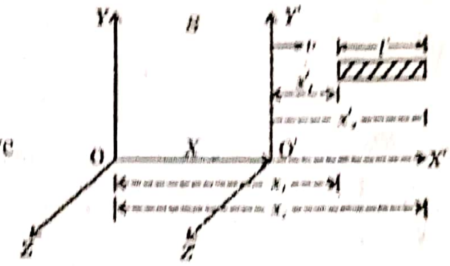
$$\therefore x_2' - x_1' = l' \qquad \text{.....(1)}$$

Let the end coordinates of the rod as given by S frame be x_1 and x_2,

where $x_2 - x_1 = l$

According to Lorentz transformations equation, we have

$$x_2' = \frac{(x_2 - vt)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \text{and} \quad x_1' = \frac{(x_1 - vt)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$



Fig

Substituting these values in eqn.(1), we

$$l' = \frac{(x_2 - vt)}{\sqrt{\left(1 - \frac{v^2}{c}\right)}} - \frac{(x_1 - vt)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\therefore \; x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\therefore \; l' = \frac{l}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\therefore \; l = l'\left(\sqrt{\left(1 - \frac{v^2}{c^2}\right)}\right)$$

1. If $v = 0$; $l = l'$. The length of the rod is same for the observers in S and S' frame.

2. If $v = c$, $l = 0$ the length of the rod is zero. Hence it was concluded that no material body can travel with velocity of light.

3. If v is comparable to c, $\sqrt{1 - \frac{v^2}{c^2}}$ is less than unity. Hence l is less than l'. The length of the rod appears to be small.

This is called length contraction.

Q. 7. Write short notes on Time Dilation.

Ans : Dilation means to lengthen. Consider two frames of references S and S'. Let S' frame be moving relative to S with a velocity v along the +ve direction of X–axis. Let a clock is present in S' frame. The initial and final time co-ordinates of an event with reference to S' frame be t_1' and t_2'.

$\Delta t^{l} = t_2^{\,l} - t_1^{\,l}$

The same event has different time coordinates with reference to S frame. They are t_1 and t_2.

$\Delta t = t_2 - t_1$

From inverse Lorentz transformations

$$t_1 = \frac{t_1^l + \frac{vx^l}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } t_2 = \frac{t_2^{\,l} + \frac{vx^l}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2 - t_1 = \frac{t_2^l - t_1^l}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Delta t = \frac{\Delta t^l}{\sqrt{1-\frac{v^2}{c^2}}}$$

1. If $v = 0$, $\Delta t = \Delta t^l$. The time interval is same for both the observers in S and S^l frames.

2. If $v = c$, $\Delta t = \alpha$. The event that has occured is S^l frame has taken infinite time for an observer in S frame. This is time dilation.

3. If v is comparable to c then $\sqrt{1-\frac{v^2}{c^2}}$ is less than unity. Hence $\Delta t > \Delta t^l$

SOLVED PROBLEMS

8. If rod travels with a speed 0.6c along its length claculate the percentage contraction.

Solution :

$$l_1 = l\sqrt{1-\frac{v^2}{c^2}} \quad \text{where } v = 0.6c$$

$$l_1 = l\sqrt{1-\frac{(0.6c)^2}{c^2}} \qquad l_1 = l\sqrt{1-0.36} \qquad l_1 = 0.8\,l$$

$$\text{Percentage contraction} = \frac{l-l_1}{l}\times 100 = \frac{l-0.8l}{l}\times 100 = 20\%$$

$\therefore$ % contraction = 20%

9. A rocket ship is 100 metre long on the ground. When it is in flight, its length is 99 metres to an observer on the ground. What is its speed?

Solution :

We know that

$$l = l'\sqrt{\left[1-\frac{v^2}{c^2}\right]}$$

Here $l' = 100 \times 100 = 10000\ cm$

and $l = 90 \times 100 = 9900\ cm$

$$\therefore 9900 = 10000\sqrt{\left[1-\frac{v^2}{c^2}\right]} \quad \text{or} \quad \frac{99}{100} = \sqrt{\left[1-\frac{v^2}{c^2}\right]}$$

or $$\frac{99\times 99}{100\times 100} = 1-\frac{v^2}{c^2}$$

or $$\frac{v^2}{c^2} = 1-\frac{99\times 99}{100\times 100} = \frac{199}{100\times 100} \text{ or}$$

$$v = c\frac{\sqrt{199}}{100} = 3\times 10^{10}\frac{\sqrt{199}}{100} = 4.23 \times 10^{9}\ cm/sec.$$

10. A clock showing correct time when at rest and loses 2 hours in a day when it is moving. What is its velocity?

Solution :

$$\Delta t = \frac{\Delta t^{1}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Delta t^{1} = 22 \text{ hours} \qquad \Delta t = 24 \text{ hours}$$

$$24 = \frac{22}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(1-\frac{v^2}{c^2}\right) = \left(\frac{22}{24}\right)^2 = \frac{121}{144}$$

$$\frac{v^2}{c^2} = 1-\frac{121}{144} = \frac{23}{144} \qquad \frac{v}{c} = \sqrt{\frac{23}{12}} = 0.4$$

$$\Rightarrow v = 0.4c \qquad \Rightarrow v = 0.4\times 3\times 10^{8} \qquad \Rightarrow v = 1.2\times 10^{8}\ m/sec.$$

The velocity is $1.2\times 10^{8}\ m/sec.$

11. At what speed the mass of an object will be double of its value at rest.

Solution : We know that

$$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \text{Here } m = 2m_0 \quad \therefore 2m_0 = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \text{or}$$

$$\sqrt{\left(1-\frac{v^2}{c^2}\right)} = \frac{1}{2} \text{ or } 1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\therefore \frac{v^2}{c^2} = 1-\frac{1}{4} = \frac{3}{4} \text{ or } v = \sqrt{\left(\frac{3}{4}\right)}\times 3\times 10^{10}\ cm/sec. = 2.6\times 10^{10}\ cm/sec.$$

12. If the total energy of a particle is exactly thrice its rest energy, what is the velocity of the particle?

Solution : We know that $E = mc^2$

Here $E = m_0c^2$ $\qquad \therefore 3\,m_0c^2 = mc^2$ $\qquad$ or $\qquad m = 3\,m_0$

The variation of mass with velocity is expressed as $m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$

$$\therefore \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = 3m_0 \quad \text{or } \sqrt{\left(1-\frac{v^2}{c^2}\right)}\frac{1}{3} \text{ or } \frac{v^2}{c^2} = \frac{8}{9} \quad \text{or}$$

$$v = \sqrt{\left(\frac{8}{9}\right)}\times c = \frac{2.828}{3}\times 3\times 10^{10}.$$

13. *A particle is moving with 90% of the velocity of light. Compare its relativistic mass with rest mass.*

Solution : We know that

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{m}{m_0} = \frac{1}{\sqrt{1-\left(\frac{0.9C}{C}\right)^2}} = \frac{1}{\sqrt{1-0.81}} = \frac{1}{\sqrt{0.19}} = \frac{1}{0.4359} = 2.294.$$

14. *With what speed shoult it be moved relative to an observer so that it may appear to lose 4 minutes in 24 hours.*

Solution : Given $\Delta t = 24 \times 60 = 1440$ min,

$$\therefore \Delta t^1 = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \quad \therefore 1444 = \frac{1440}{\sqrt{1-\frac{v^2}{c^2}}} = \text{if } v/c << 1 \text{ then}$$

$$\Delta t^1 = 1440 + 4 = 1444 \text{ min}$$

$$1444 - 1440\left(1+\frac{v^2}{2c^2}\right)$$

$$\frac{v^2}{2c^2} = \frac{1444}{1440} - 1 \quad V^2 = \frac{4\times 2}{1440} \times c^2 = v = 0.0745 \times c \times 2 = 2.23 \times 10^7 \text{ m/sec.}$$

& & &

5. UNDAMPED, DAMPED AND FORCED OSCILLATIONS

LONG ANSWER QUESTIONS

Q. 1. *What is simple oscillator? Give the equation of motion of a simple oscillator and its solution?*

Ans : *Simple oscillator* : The particle or body executing simple harmonic motion is called a simple oscillator.

Equation of motion of a simple oscillator : Consider a particle p of mass m executing S.H.M about equilibrium position 'o' along x-axis.

Let 'x' be the displacement of 'p' from 'o' at any instant. The force 'F' acting upon 'p' is given by

$$F \propto -x \; (or) \; F = -Kx \qquad \text{.......(1)}$$

Where K is proportionality factor.

According to Newton's second law of motion, the restoring force on mass m, produes an acceleration is

$$F = m\frac{d^2x}{dt^2} \qquad \text{......(2)}$$

From equation (1) and (2)

$$m\frac{d^2x}{dt^2} = -Kx \; (or) \; \frac{d^2x}{dt^2} = -\frac{K}{m} \times x$$

Put $\quad \dfrac{K}{m} = \omega^2 \quad$ then $\quad \dfrac{d^2x}{dt^2} + \omega^2 x = 0 \qquad$ ......(3)

This is known as the differential equation of simple harmonic oscillator.

Solution of the equation :

Let the solution be $x = c\, e^{\alpha t}$

Where c and α are oribitrary constants. Differentiating above equation, we get

$$\frac{dx}{dt} = c \propto e^{\alpha t} \; and \; \frac{d^2x}{dt^2} = c \propto e^{\alpha t}$$

Substituting these values in equation (3), we get

$$c\,\alpha^2 e^{\alpha t} + \omega^2 c\, e^{\alpha t} = 0 \qquad (or) \qquad c\, e^{\alpha t}(\alpha^2 + \omega^2) = 0$$

$$(\alpha^2 + \omega^2) = 0$$

$$\therefore \; \alpha = \pm\sqrt{-\omega^2} = \pm j\omega \quad \text{where } j = \sqrt{(-1)}$$

Now $x = Ce^{\pm j\omega t} \qquad$ (or) $\qquad x = Ce^{+j\omega t}$ and $x = Ce^{-j\omega t}$

So, the general solution can be written as $x = C_1\, e^{+j\omega t} + C_2\, e^{-j\omega t}$

Where C_1 and C_2 orbitrary constants.

Further $x = C_1(\cos\omega t + j\sin\omega t) + C_2(\cos\omega t - j\sin\omega t)$

(or) $x = (C_1 + C_2)\cos\omega t + j(C_1 - C_2)\sin\omega t$

Let us put $C_1 + C_2 = a\sin\phi$ and $j(C_1 - C_2) = a\cos\phi$

$$\therefore x = a\sin\phi\cos\omega t + a\cos\phi\sin\omega t \qquad (or) \qquad x = a\sin(\omega t + \phi) \qquad \text{.......(4)}$$

This is the solution of the equation of simple oscillator.

Q. 2. *Define damped harmonic oscillator. Derive the equation of Motion of damped harmonic oscillator. Discuss different cases.*

Ans : When a body vibrates in some medium, the medium offers some resistance to the motion of it. As a result the amplitude gradually decreases and the body finally comes to rest. Such type of motion of the body is called damped harmonic motion. Such oscillator is called *damped harmonic oscillator.*

Example : If a simple pendulum is displaced from its mean position and left free it oscillate with decreasing amplitude and finally comes to rest.

Equation : When a system is making damped oscillations there are two forces acting

1. Restoring force which is proportional to the displacement $(R.F.\ \alpha - y)$.

2. Frictional force which is proportional to the velocity $\left(F.F.\alpha - \frac{dy}{dt}\right)$

$$\therefore R.F \propto -y. \qquad R.F = -sy$$

where $s = R.F$ per unit displacement.

$$F.F\ \alpha - \frac{dy}{dt} \qquad \therefore F.F = \frac{rdy}{dt} \quad (r = F.F \text{ per unit velocity})$$

According to Newton's IInd law Force $(F) = m \times \frac{d^2y}{dt^2}$

$$\therefore m \times \frac{d^2y}{dt^2} = -sy - r\frac{dy}{dt} \quad \text{or} \quad \frac{d^2y}{dt^2} = -\frac{s}{m}y - \frac{r}{m}\frac{dy}{dt}$$

Put $\frac{r}{m} = 2k$ and $\frac{s}{m} = \omega^2$

$$\therefore \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = 0 \qquad \text{........(1)}$$

This is called differential equation of damped harmonic oscillator.

The solution for equation (1) will be of the form $y = Ae^{\alpha t}$ ....(2)

where A and α are arbitrary constants.

Differentiating equation (2)

$$\frac{dy}{dt} = A\alpha e^{\alpha t} \quad \text{and} \quad \frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$$

Substituting these values in equation (1)

$$A\alpha^2 e^{\alpha t} + 2kA\alpha e^{\alpha t} + \omega^2 A e^{\alpha t} = 0 \quad \text{or} \quad Ae^{\alpha t}(\alpha^2 + 2K\alpha + \omega^2) = 0 \quad \text{As } Ae^{\alpha t} \neq 0$$

$$\therefore \alpha^2 + 2k\alpha + \omega^2 = 0 \qquad \text{........(3)}$$

The solution is $\alpha = -k \pm \sqrt{k^2 - \omega^2}$

From equation (2)

$$y = A_1 e^{\left(-k+\sqrt{k^2-\omega^2}\right)t} + A_2 e^{\left(-k-\sqrt{k^2-\omega^2}\right)t} \qquad \text{........(4)}$$

where A_1 and A_2 are arbitrary constants.

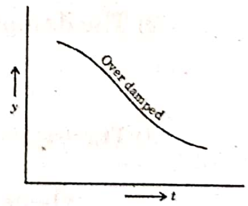


Fig.

Case (i) : Over damped motion : If $k^2 > \omega^2$; $\sqrt{k^2 - \omega^2}$ is real and less than k then both the powers in equation (4) are –ve. Hence the displacements consists of two terms both decreasing exponentially to zero with out making oscillations. This type of motion is called *dead beat* or *over damped*. This type of motion can be seen by a pendulum moving in a thick oil.

Case (ii) : Critical damping : If $k^2 \simeq \omega^2$ is substituted in equation (4) the solution does not satisfy the equation (1). Hence consider that $\sqrt{k^2 - \omega^2} = h$ (where h is a small quantity)

$\therefore$ Equation (4) reduces to

$$y = A_1 e^{(-k+h)t} + A_2 e^{(-k-h)t} = e^{-kt}\left[A_1 e^{ht} + A_2 e^{-ht}\right] = e^{-kt}\left[A_1(1+ht+....) + A_2(1-ht+....)\right]$$

$$= e^{-kt}\left[(A_1 + A_2) + ht(A_1 - A_2) + ...\right] = e^{-kt}\left[p + qt\right] \qquad \text{........(5)}$$

where $p = A_1 + A_2$ $\qquad q = h(A_1 - A_2)$

From equation (5) as t increases $(p + qt)$ increases. But the factor e^{kt} decreases. As a result the displacement approaches to zero as t increases. Such a motion is called critical damed motion.

Such a motion can be exhibited by ammeter and voltmeters.

Case (iii) : Under damped motion : If $k^2 < \omega^2$,

then $\sqrt{k^2-\omega^2}$ is imaginary.

$$\therefore \sqrt{k^2-\omega^2} = i\sqrt{\omega^2-k^2} = i\beta$$

where $i = \sqrt{\beta}$ and $\beta = \sqrt{\omega^2-k^2}$

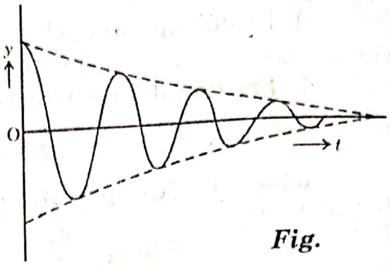


Fig.

$\therefore$ equation (4) becomes

$$y = A_1 e^{(-k+i\beta)t} + A_2 e^{(-k-i\beta)t} = e^{-kt}\left[A_1 e^{i\beta t} + A_2 e^{-i\beta t}\right]$$

$$= e^{-kt}\left[A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin\beta t)\right]$$

$$= e^{-kt}\left[(A_1+A_2)\cos\beta t + i(A_1-A_2)\sin\beta t\right]$$

Put $A_1 + A_2 = a\sin\phi$ $\qquad$ $i(A_1 - A_2) = a\cos\phi$

$$\therefore y = e^{-kt}\left[a\cos\beta t\sin\phi + a\sin\beta t\cos\phi\right] \qquad y = e^{-kt}\left[a\sin(\beta t+\phi)\right]$$

$$y = e^{-kt}\left[a\sin\left\{\sqrt{(\omega^2-k^2)}\,t+\phi\right\}\right] \qquad \text{........(6)}$$

This equation (6) represents the damped simple harmonic motion with amplitude ae^{-kt}

The time period of vibrating particle

$$T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{(\omega^2-k^2)}}$$

This type of motion is the motion of a pendulum in air.

Q. 3. *Define forced harmonic oscillators. Derive the differential equation and give its solution. Discuss different cases.*

Ans : The free vibrations produced in a body die away after some time due to dissipation of energy. But if some external periodic force is continuously applied on the body, the body continues to oscillate under the influence of that force. The vibrations of the body are called forced oscillations or driven oscillations.

Equation of forced vibration : When ever a body is making forced oscillations there are three forces acting on it.

1) Restoring force, which is directly proportional to displacement and oppositely directed

i.e., $R.F\ \alpha - y;\ R.F = -sy$

where $s = R.F$ per unit displacement

2) The damping force which is proportional to the velocity and opposite to the displacement.

$$\therefore D.F\ \alpha - \frac{dy}{dt};\ D.F = -r\frac{dy}{dt} \quad (r = D.F \text{ per unit velocity})$$

3) The external periodic force $F\sin pt$.

where $F =$ max.force $\frac{p}{2\pi}$ is its frequency.

So the total force acting on the particle is given by $-sy - r\frac{dy}{dt} + F\sin pt$

$\therefore$ According to Newtons IInd Law the force $F = \frac{md^2y}{dt^2}$

$$\therefore \frac{md^2y}{dt^2} = -sy - r\frac{dy}{dt} + F\sin pt \quad \text{or} \quad \frac{md^2y}{dt^2} + sy + r\frac{dy}{dt} = F\sin pt$$

$$\text{or} \quad \frac{d^2y}{dt^2} + \frac{s}{m}y + \frac{r}{m}\frac{dy}{dt} = \frac{F}{m}\sin pt \qquad \text{Put} \quad \frac{s}{m} = \omega^2 \text{ and } \frac{r}{m} = 2k \text{ and } \frac{F}{m} = f$$

$$\therefore \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = f\sin pt \qquad \text{........(1)}$$

This is the differential equation of motion of the particle.

The solution for this equation will be of the form $y = A\sin(pt-\alpha)$ ....(2)

Differentiating eqn.(2) we have

$$\frac{dy}{dt} = A\cos(pt-\alpha)p = Ap\cos(pt-\alpha) \text{ and } \frac{d^2y}{dt^2} = -Ap^2\sin(pt-\alpha)$$

Substituting these values in equation (1)

$$-Ap^2\sin(pt-\alpha) + 2kAp\cos(pt-\alpha) + \omega^2 A\sin(pt-\alpha) = f\sin pt = f[\sin\{(pt-\alpha)+\alpha\}]$$

R.H.S. = $f\sin(pt-\alpha)\cos\alpha + f\cos(pt-\alpha)\sin\alpha$

Comparing the coefficients of $\sin(pt-\alpha)$, we get

$$-Ap^2 + \omega^2 A = f\cos\alpha \qquad A(\omega^2 - p^2) = f\cos\alpha \qquad ....(3)$$

$$\therefore f\cos\alpha = A(\omega^2 - p^2)$$

Comparing the coefficients of $\sin(pt-\alpha)$ $\qquad 2kAp = f\sin\alpha$ ....(4)

Squaring and adding equations (3) and (4)

$$f^2\cos^2\alpha = A^2(\omega^2-p^2)^2 \qquad f^2\sin^2\alpha = 4k^2A^2p^2$$

$$\therefore f^2 = A^2(\omega^2-p^2)^2 + 4k^2A^2p^2 \qquad \therefore A^2 = \frac{f^2}{(\omega^2-p^2)^2+4k^2p^2}$$

$$\therefore A = \frac{f}{\sqrt{(\omega^2-p^2)^2+4k^2p^2}} \qquad ....(5)$$

This is the expression for the resultant amplitude.

While on dividing eqn. (4) by eqn. (3), we have

$$\tan\alpha = \frac{2kAp}{A(\omega^2-p^2)} = \frac{2kp}{(\omega^2-p^2)} \quad \text{or} \quad \alpha = \tan^{-1}\left(\frac{2kp}{(\omega^2-p^2)}\right) \qquad ....(6)$$

Substituting the value of A from eqn. (5) in equation (2)

$$y = \frac{f}{\sqrt{(\omega^2-p^2)^2+4k^2p^2}}\sin(pt-\alpha) \qquad ....(7)$$

Different cases of amplitude and phase :

Case 1 : When driving frequency is low $p << w$. In this case the amplitude of vibration is given by

$$A = \frac{f}{\sqrt{(w^2-p^2)^2+4k^2p^2}} \cong \frac{f}{w^2} = cant$$

and $$\theta = \tan^{-1}\frac{2kp}{(w^2-p^2)} \cong \tan^{-1}(0) \cong 0$$

Case 2 : Whe $p = w$ i.e., frequency of the force is equal to the frequencey of the body. In this cae, the amplitude of vibration is given by

$$A = \frac{f}{2kp} = \frac{F}{rw} \qquad \left[\because f = \frac{F}{m}, 2k = \frac{r}{m} \text{ and } p = w\right]$$

also $$\theta = \tan^{-1}\left(\frac{Kp}{0}\right) = \tan^{-1}(w) = \frac{\pi}{2}$$

Case 3 : When $p >> w$ i.e., the frequency of force is greater than the naturla frequency w of the body.

In this $$A = \frac{f}{\sqrt{p^4+4k^2p^2}} \cong \frac{F}{p^2} \cong \frac{F}{mp^2}$$

and $$\theta = \tan^{-1}\left(\frac{2Kp}{w^2-p^2}\right) = \tan^{-1}\left(\frac{-2K}{P}\right) \approx \tan^{-1}(-0) = \pi$$

SHORT ANSWER QUESTIONS

Q. 4. *What is simple-harmonic motion? What are its physical characteristics?*

Ans : *Simple harmonic motion* : When a particle (or) a body moves such that its acceleration is always directed towards a fixed point and varies directly as its distance from that point, the particle is said to execute S.H.M.

Characterstics of S.H.M :

1) Displacement : The displacement of any particle at any instant executing S.H.M is given by

$$x = a \sin(\omega t + \phi) \qquad \text{.........(1)}$$

The maximum displacement from the mean position is called amplitude.

2) Velocity : The velocity V of the oscillating particle can be obtained by differentiate equation (1), thus.

$$V = \frac{dx}{dt} = a\omega \cos(\omega t + \phi) = \omega\sqrt{a^2 - x^2}$$

at $x = 0$, the velocity is maximum $= a\omega$. The velocity is zero at extreme position.

3) Periodic Time : Time taken for a complete oscillation is called as periodic time.

$$T = 2\pi\sqrt{\frac{displacement}{acceleration}}$$

4) Frequency : The no. of oscillations made in one second is called frequency.

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

5) Phase : The angle $(\omega t + \phi)$ is called the phase of vibrator.

Q. 5. *Explain the amplitude and sharpness resonance.*

Ans : Sharpness of Resonance : The rate of fall in amplitude, with the change of forcing frequency on each side of resonance frequency is called *sharpness of resonance*. Smaller is damping, sharper is resonance.

In the case of forced vibration the amplitude

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4k^2p^2}} \qquad \text{........(1)}$$

and $\alpha = tan^{-1}\left(\dfrac{2kp}{(\omega^2 - p^2)}\right)$ ....(2)

The expression (1) shows that amplitude varies with the frequency of the force p. For a particular value of p the amplitude becomes maximum. The phenomenon is known as amplitude resonance. The amplitude is maximum when.

if $\sqrt{(\omega^2 - p^2)^2 + 4k^2p^2}$ is minimum, A is maximum.

or, $\dfrac{d}{dp}\sqrt{(\omega^2 - p^2)^2 + 4k^2p^2} = 0$ or, $2(\omega^2 - p^2)(-2p) = +4k^2(2p) = 0$

or, $\omega^2 - p^2 = 2k^2$ or, $p^2 = \omega^2 - 2k^2$

or, $p = \sqrt{\omega^2 - 2k^2}$ ....(3)

The amplitude is maximum when the frequency $\dfrac{p}{2\pi}$ is

$\dfrac{\sqrt{\omega^2 - 2k^2}}{2\pi}$. This is the Resonance condition.

This gives frequency of the system both in presence of damping i.e. $\sqrt{\frac{\omega^2 - 2k^2}{2\pi}}$ and in the absence of damping $\frac{\omega}{2\pi}$. If the damping in small then it can be neglected and the condition of maximum amplitude reduced of putting condition (3) in eqn. (1), we get

$$A_{max} = \frac{f}{\sqrt{\left((\omega^2 - \omega^2 + 2k^2)^2 + 4K^2(\omega^2 - 2k^2)\right)}} = \frac{f}{\sqrt{\left(4b^2\omega^2 - 4k^4\right)}} = \frac{f}{2k\sqrt{(\omega^2 - k^2)}}$$

$$= \frac{f}{2k\sqrt{(p^2 + k^2)}} \qquad (\; p^2 = \omega^2 - 2k^2)$$

and for low damping it reduces to

$$A_{max} \simeq \frac{f}{2kp}$$

Showing that $A_{max} \to \propto$ as $k \to 0$

The variation of amplitude with forcing frequency at different amounts of damping are as shown in the figure curve (1) shows when there is no damping $(k = 0)$. In this case amplitude becomes infinity at $p = \omega$. This is only an ideal case. The curves (2) and (3) shows the effect of damping on the amplitude.

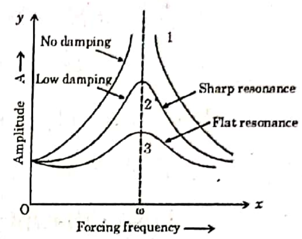


Fig.

Q. 6. *Define quality factor. Explain.*

Ans : The quality factor is defined as 2π times the ratio of the energy stored in the system to the energy lost per period.

$$\text{Quality factor} = Q = 2\pi.\frac{\text{Energy stored in system}}{\text{Energy lost per period}}$$

If P is the power dissipated and T is time period $Q = 2\pi.\frac{E}{PT}$

If $t =$ relaxation time $\quad P = \frac{E}{\tau}$

$$\therefore Q = 2\pi\frac{E}{\frac{E}{\tau}\times T} = \frac{2\pi\tau}{T} = \omega\tau \qquad \text{whose } \omega = \frac{2\pi}{T} \qquad \therefore Q = \omega\tau$$

If Q is high, damping is low   If Q is low, damping is high.

Q. 7. *Write short notes on logarthmic decrement of the oscillator.*

Ans : When an oscillator makes damped harmonic motion, the displacement decreases exponentially with increase of time.

The decrease of displacement of a damped oscillator in one second is indicated by the *logarthmic decrement.*

The amplitude of damped simple harmonic motion is represented as a_0^{-kt}

When $t = 0$, the amplitude is max (a_0) When $t = \frac{T}{2}$ the amplitude is max a_1

When $t = \frac{2T}{2}$ the amplitude is max a_2 (where T is the time period)

$$\therefore a_1 = A_0 e^{-k\left(\frac{T}{2}\right)}; \quad \therefore a_2 = A_0 e^{-k\left(\frac{2T}{2}\right)}$$

From the above equations $\quad \frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \ldots\ldots\ldots\ldots = e^{-k\left(\frac{T}{2}\right)} = d$

where d is called decrement.

The natural logarithm of this decrement is called *logarithmic decrement* (λ) of the oscillator

$\lambda = \log_e^{d} \Rightarrow e^{\lambda} = d$ i.e $e = \frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = e^{\lambda}$

$\therefore \lambda = \log_e\left(\frac{a_0}{a_1}\right) = \log_e\left(\frac{a_1}{a_2}\right) = \log_e\left(\frac{a_2}{a_3}\right).$

Q. 8. *Define relaxation time? Derive expression for it?*

Ans : The time taken for the total mechanical energy to decay to $\frac{1}{e}$ of its original value is called *relaxation time.* The expression for total énergy of damped harmonic oscillator is

$E = \frac{1}{2} sa^2 e^{-2kt}$ When $t = 0$; $E = E_0$

$\therefore E_0 = \frac{1}{2} sa^2$ $\therefore E = E_0 e^{-2kt}$ .... (1)

When $t = \tau$; $E = \frac{E_0}{e}$

Moving this substitution eqn. (1), we get

$\therefore \frac{E_0}{e} = E_0 e^{-2k\tau}$ or $\frac{1}{e} = e^{-2k\tau}$ or $e^{-1} = e^{-2k\tau}$

$\therefore 1 = 2k\tau$ $\therefore \tau = \frac{1}{2k}$ .... (2)

From eqn. (1) and (2), we get

$\therefore E = E_0 e$ $E = E_0 e^{-1} = \frac{E_0}{e}$

$\therefore$ Power dissipation $= P = 2Ek$ $P = \frac{E}{\tau}$

SOLVED PROBLEMS

9. *The displacement of a particle making S.H.M is given by $x = 0.5 \cos\left(10\pi t + \frac{\pi}{3}\right)$ calculate (1) amplitude, (2) Frequency, (3) Phase, (4) Displacement after 1 sec.*

Solution :

$x = 0.5 \cos\left(10\pi t + \frac{\pi}{3}\right) m$

Comparing the given equation with the standard equation

$x = a\cos(\omega t + \phi)$

(1) Amplitude $a = 0.5\ m$ (2) Frequency $n = \frac{10\pi}{2\pi} = 5 Hz$

(3) When $t = 0$, initial phase $\phi = \frac{\pi}{3} = 60^0$

Phase angle $= \left(10\pi t + \frac{\pi}{3}\right) = 10\pi \times 1 + \frac{\pi}{3}$

when $t = 1 sec$ $= 10\pi + \frac{\pi}{3} = 60^0$

(4) displacement x, when $t = 1$

$x = 0.5 \cos\left(10\pi \times 1 + \frac{\pi}{3}\right)$ $x = 0.5 \cos\left(31\frac{\pi}{3}\right)$

$x = 0.5 \cos 60^0$ $x = 0.5 \times 0.5 = 0.25 m$

10. *A particle vibrates simple harmonically with a period of 2 sec. Find the amplitude if its max. velocity is 10 cm/s.*

Solution : $v_{max} = a\omega = a\left(\frac{2\pi}{T}\right)$

$\therefore a = \frac{v_{max} T}{2\pi}$; $T = 2 sec$; $V_{max} = 10\ cm/s$ $a = \frac{v_{max} T}{2\pi} = \frac{10 \times 2 \times 7}{2 \times 22} = 3.18\ cm$

11. *The amplitude of seconds pendulum falls to half of its initial value in 150 sec. Calculate the Q – factor.*

Solution : For damped oscillator the amplitude after time t.

$a = a_0 e^{-\alpha t}$

But when $t = 150s$ $a = \frac{a_0}{2}$ $\therefore$ $\frac{a_0}{2} = a_0\, e^{-\alpha \times 150}$

$$e^{\alpha \times 150} = 2$$

$$150\alpha = \log_e 2 = 2.303 \log_{10} 2$$

$$\alpha = \frac{2.303 \log_{10} 2}{150} = 0.00462$$

Relaxation time $\tau = \frac{1}{\alpha} = \frac{1}{0.00462} = 216.4\ s$

$$Q = \frac{\omega\tau}{2} = \frac{2\pi n\tau}{2} = \frac{2 \times 3.14 \times 0.15 \times 216.4}{2}$$

$$Q = 339.8.$$

12. ***The Q value of a spring loaded with 0.3 kg is 60. If it vibrates with a frequency of 2 Hz. Calculate the force constant and the mechanical resistance.***

Solution : Load attached to the spring $(m) = 0.3\ kg.$

$Q = 60$

Frequency of vibration $n = 2\ Hz.$

From $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $\quad \omega = 2\pi n = 2 \times 3.14 \times 2 = 12.56\ RS^{-1}$

Force constant, $k = 4\pi^2 n^2 m = 4\,(3.14)^2\,(2)^2(0.3) = 47.33\ Nm^{-1}$

Mechanical resistance $r = \frac{m\omega}{Q} = \frac{0.3 \times 12.56}{60} = 0.0628\ kgs^{-1}.$

13. ***The quality factor Q of a sonometer wire is 2×10^2. On plucking, the wire emits a note of frequency 120 Hz. Calculate the time in which the amplitude falls to $(1/e^2)$ of the initial value.***

Solution :

The quality factor Q is given by

$Q = \omega\tau = 2\pi n\tau = 2\pi \times 120\tau = 240\pi\,\tau\ sec^{-1}$

Given that $Q = 2 \times 10^2$

$\therefore\ 2 \times 10^2 = 240\,\pi\,\tau$ or $\tau = (2 \times 10^2)\,/\,240\,\pi$ or

$\tau = 2.652\ sec.$

The instantaneous amplitude of a damped oscillator is given by $a = a_0 e^{-bt}$.

At $t = 0,\ a = a_0$

Let after a time $t = x$, the amplitude falls to $(1/e^2)$ of initial value, then

$\left(\frac{1}{e^2}\right) a_0 = a_0\, e^{-bx}$ or $e^{-2} = e^{-bx}$ or

$2 = bx$ or $x = \frac{2}{b}$

Further $\tau = \frac{1}{2}b$ or $b = \frac{1}{2}\tau$

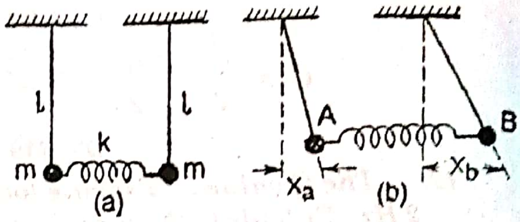
$\therefore\ x = 4\tau = 4 \times 2.652 = 10.608\ sec.$

6. COUPLED OSCILLATORS

LONG ANSWER QUESTIONS

Q. 1. ***Obtain the normal mode and normal coordinates of two identical pendulums with their bobs connected by means of elastic massless spring?***

Ans : Consider a system of two identical simple pendulums A and B, each of mass m and length l coupled by a spring of force constank k as shown in fig.(2 a). Here the separation between the two bobs is such that the spring is relaxed in equilibrium position.



Consider the situation in which the system slightly disturbed from the equilibrium position as shown in fig.(2 b). The two pendulums begin to oscillate. Led x_a and x_b be the diplacements of the bobs at an instant of time t. The spring will be stretched when $x_b > x_a$ and compressed when $x_a > x_b$. The magnitude of the tension in the spring is $k(x_b - x_a)$. If $x_b > x_a$ the tension will act against the acceleration of pendulum B but in favour of the acceleration of A.

Considering bob B, there are two forces acting on it :

(i) restoring force or return force due to gravity. This is equal to $-m\,g\,\sin\,\theta = -mg\,(x_b/l)$. Negative sign is used to show that restoring force is opposite to displacement x_b.

(ii) return force due to stretching of spring. This is equal to $-k\,(x_b - x_a)$, where k is force constant of spring. This is also opposite to displacement x_b.

The equations of motion of pendulum A and B, for small oscillations in a plane, are

$$m\frac{d^2x_a}{dt^2} = -\frac{mg}{l}x_a + k(x_b - x_a) \qquad \text{and} \qquad m\frac{d^2x_b}{dt^2} = -\frac{mg}{l}x_b - k(x_b - x_a)$$

These equations are not of simple harmonic motions because the acceleration of the pendulum is not proportional to its own displacement. When k = 0 (the spring were absent), the two pendulums will execute harmonic oscillations whose angular frequency is given by

$$\omega_0 = \sqrt{(g/l)}$$

In terms of ω_0, the two coupled equations are

$$\frac{d^2x_a}{dt^2} = -\omega_0^{\,2}x_a + (k/m)\,(x_b - x_a) \qquad .....(1)$$

$$\frac{d^2x_b}{dt^2} = -\omega_0^{\,2}x_b - (k/m)\,(x_b - x_a) \qquad .....(2)$$

In order to find out the effect of coupling on each pendulum, these equations must be solved for x_a and x_b.

Adding eqs. (1) and (2), we get

$$\frac{d^2}{dt^2}(x_a + x_b) = -\omega_0^{\,2}(x_a + x_b)$$

$$\frac{d^2}{dt^2}(x_a + x_b) + \omega_0^{\,2}(x_a + x_b) = 0 \qquad .....(3)$$

$$\frac{d^2X}{dt^2} + \omega_0^{\,2}\,X = 0 \quad \text{where } X = x_a + x_b \qquad .....(a)$$

Subtracting eq.(1) from eq.(2), we get

$$\frac{d^2}{dt^2}(x_b - x_a) = -\,\omega_0^{\,2}\,(x_b - x_a) - 2\,(k/m)\,(x_b - x_a)$$

$$\frac{d^2}{dt^2}(x_b - x_a) + \left(\omega_0^{\,2} + 2\frac{k}{m}\right)(x_b - x_a) = 0 \qquad .....(4)$$

$$\frac{d^2X'}{dt^2}+\left(\omega_0^2+2\frac{k}{m}\right)X'=0 \quad where\ X'=(x_b-x_a) \qquad .....(4)$$

$$\frac{d^2X'}{dt^2}+\omega_2^2X'=0 \quad where\ \omega_2^2=\left(\omega_0^2+\frac{2k}{m}\right)$$

Eqs. (3) and (4) are familiar equations of simple harmonic oscillations. In eq. (3), the variable is (x_a+x_b) while the variable in eq. (4) is $(x_b - x_a)$.

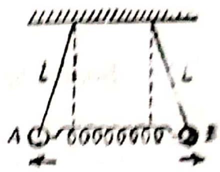
If $x_a = x_b$ at all times, the motion is completely described by eq.(3) since eq.(4) vanishes. The angular frequency of oscillations is given by

$$\omega_1=\omega_0=\sqrt{\left(\frac{g}{l}\right)} \qquad .....(5)$$

Both the pendulums are always in phase and spring has the natural length throughout the motion. (The spring is neither compressed nor extending while swinging). This is the first normal mode (in phase mode.). If $x_a = -x_b$ at all lines, the motion is completely described by eg (4)

Since of (3) vanishes. In this case, the angular frequency is given by

$$\omega_2=\left(\omega_0^2+\frac{2k}{m}\right)^{1/2} \qquad .... (6)$$



So $\omega_2 > \omega_1$ *i.e.*, the frequency of oscillation of the coupled system is greater than the natural frequency of the pendulum when they are separate. The out of phase mode is shown in fig.(3). This is second normal mode.

Normal modes solution : The normal modes are represented by oscillations of variables $(x_a + x_b)$ and $(x_b - x_a)$. These are called *Normal coordinates*. The changes in the values of $(x_a + x_b)$ occur independently of $(x_b - x_a)$ and vice versa. The frequencies of individual normal modes are called Normal frequencies. The characteristic of normal frequency is that both x_a and x_b can oscillate with that frequency. The *in phase mode* and *out phase mode* are called the normal modes of coupled system.

The equations governing the motion of the pendulums are

$$\frac{d^2x_a}{dt^2}=-\omega_0^2x_a+\frac{k}{m}(x_b-x_a) \quad \text{and} \quad \frac{d^2x_b}{dt^2}=-\omega_0^2x_b-\frac{k}{m}(x_b-x_a)$$

To find out the normal modes, let us solve these differential equations. Consider that a normal mode exists at an angular frequency ω and phase constant ϕ. This implies that both the pendulums move with a simple harmonic motion at same angular frequency ω and same phase constant ϕ. Then

$$x_a = C\cos(\omega t+\phi) \qquad ......(7)$$

and

$$x_b = C'\cos(\omega t+\phi) \qquad ......(8)$$

where the amplitudes C and C' (in general) may be different.

From eqs. (7) and (8), we have

$$\frac{d^2x_a}{dt^2}=-\omega^2C\cos(\omega t+\phi) \quad \text{and} \quad \frac{d^2x_b}{dt^2}=-\omega^2x_b$$

Hence,

$$-\omega^2x_a=-\omega_0^2x_a+\frac{k}{m}(x_b-x_a)$$

or

$$\left(\omega_0^2-\omega^2+\frac{k}{m}\right)x_a=\frac{k}{m}x_b \qquad .....(9)$$

and $$\left(\omega_0^2 - \omega^2 + \frac{k}{m}\right) x_b = \frac{k}{m} x_a \qquad .....(10)$$

From eq. (9) $$\frac{x_b}{x_a} = \frac{\{\omega_0^2 - \omega^2 + (k/m)\}}{(k/m)} \qquad .....(11)$$

Similarly, from eq. (10), we have

$$\frac{x_b}{x_a} = \frac{(k/m)}{\{\omega_0^2 - \omega^2 + (k/m)\}} \qquad ......(12)$$

From eqs. (11) and (12)

$$\frac{\{\omega_0^2 - \omega^2 + (k/m)\}}{(k/m)} = \frac{(k/m)}{\{\omega_0^2 - \omega^2 + (k/m)\}}$$

or $$\{\omega_0^2 - \omega^2 + (k/m)\}^2 = (k/m)^2$$

Hence $$\omega_0^2 - \omega^2 + (k/m) = \pm (k/m)$$

or $$\omega^2 = \omega_0^2 + (k/m) \mp (k/m)$$

So, we have two solutions for ω, let us call them as ω' and ω''. Then

$$\omega'^2 = \omega_0^2 \qquad .....(13)$$

and $$\omega''^2 = \omega_0^2 + 2(k/m) \qquad .....(14)$$

Thus the positive square roots of these expressions are the two normal frequencies of the system *i.e.,* two modes. The angular frequency of mode 1 is ω' while that of mode 2 is ω''.

The configuration of mode 1 can be found by substituting $\omega_0^2 = \omega'^2$ in eq. (11) or eq. (12). Thus

$$\left(\frac{x_a}{x_b}\right)_{\text{mode 1}} = \left(\frac{C'}{C}\right)_{\text{mode 1}} = +1 \qquad .....(15)$$

The displacement of oscillators in mode 1 are given by

$$(x_a)_1 = C\cos(\omega' t + \phi_1) \qquad .....(16)$$

and $$(x_b)_1 = C\cos(\omega' t + \phi_1) \qquad .....(17)$$

The configuration of mode 2 can be found by substituting $\omega^2 = \omega''^2$ in either eq. (11) or in eq. (12) Hence

$$\left(\frac{x_a}{x_b}\right)_{\text{mode 2}} = \left(\frac{C'}{C}\right)_{\text{mode 2}} = -1 \qquad .....(18)$$

For mode 2, the displacements are

$$(x_a)_2 = D\cos(\omega'' t + \phi_2) \qquad .....(19)$$

$$(x_b)_2 = -D\cos(\omega'' t + \phi_2) \qquad .....(20)$$

The most general solution is given by the superposition of two normal modes, i.e.,

$$x_a = (x_a)_1 + (x_a)_2 = C\cos(\omega' t + \phi_1) + D\cos(\omega'' t + \phi_2)$$

and $$x_b = (x_b)_1 + (x_b)_2 = C\cos(\omega' t + \phi_1) - D\cos(\omega'' t + \phi_2)$$

Q. 2. ***Obtain the equation of motion, considering the case of N-coupled oscillators and derive the equation for the frequency of the system ?***

Ans : Consider a flexible elastic string of negligible mass. Suppose N indentical particles, each of mass m and equally spaced at a distance l are attached to the string as shown in fig.(6a). Let the string is fixed at two points, one at a distance l to the left of first particle and the other at a distance l to the right of Nth particle. The particles are labeled from 1 to N or from 0 to $N + 1$ if two fixed ends are considered. The two particles at fixed ends are considered as if they were particles of zero displacement. Fig. (6b) shows a configuration of p, $(p - 1)$ and $(p + 1)$th particles at some instant of time during their transverse oscillation. Here it is assumed that the amplitude of these oscillations is small and initial tension T in the string does not change as the particles oscillate.

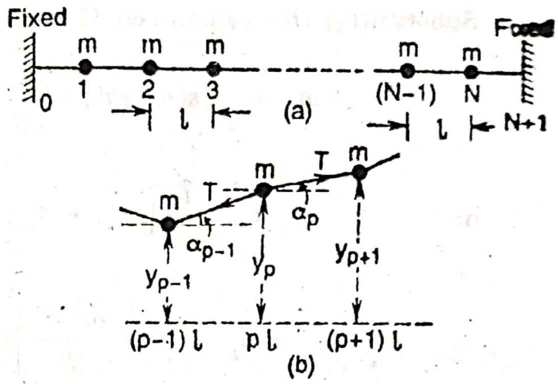


Fig.

Equation of motion : Here we focus our attention on pth particle together with two immediate neighbours $(p - 1)$th and $(p + 1)$th particles. Let the displacements of these particles from the equilibrium state be y_p, y_{p-1} and y_{p+1} respectively, where $p = 1, 2, 3, ... (N - 1), N$.

Referring to fig. (6b), the resultant y component of force on pth particle is

$$F_p = -T\sin\alpha_{p-1} + T\sin\alpha_p \quad \text{or} \quad Fp = -T\tan\alpha_{p-1} + T\tan\alpha_p$$

$$(\because \tan\alpha_p \approx \sin\alpha_p) \qquad .....(1)$$

From figure (6b), $\tan\alpha_p = \dfrac{y_{p+1} - y_p}{l}$ and $\tan\alpha_{p-1} = \dfrac{y_p - y_{p-1}}{l}$

Therefore,
$$F_p = \frac{T}{l}(y_p - y_{p-1}) + \frac{T}{l}(y_{p+1} - y_p)$$

or
$$F_p = \frac{T}{l}[y_{p+1} + y_{p-1} - 2y_p]$$

This force must be equal to mass m times the transverse acceleration of pth particle. Thus equation of motion of pth particle can be written as

$$m\frac{d^2y_p}{dt^2} = \frac{T}{l}[y_{p+1} + y_{p-1} - 2y_p]$$

or
$$\frac{d^2y_p}{dt^2} = \frac{T}{ml}[y_{p+1} + y_{p-1} - 2y_p] \qquad .....(1)$$

This is the differential equation for pth particle. By putting p = 1,2,3.....N, we can construct set of N differential equations. Here we have the following two boundary conditions.

$$x = 0, \qquad y_0 = 0$$

and
$$x = (N+1)l \qquad y_{N+1} = 0 \qquad .....(2)$$

Normal modes : For normal modes, let there exists a mode with angular frequency ω and phase constant ϕ. In normal mode, all particles execute harmonic oscillations with same frequency and constant phase ϕ. Hence for pth particle, we have

$$y_p = A_p\cos(\omega t + \phi) \qquad .....(3)$$

where A_p is the amplitude of harmonic oscillations of pth particle. Similarly,

$$y_{p-1} = A_{p-1}\cos(\omega t + \phi) \quad \text{and} \quad y_{p+1} = A_{p+1} + 1\cos(\omega t + \phi)$$

From eq. (3) $\frac{d^2 y_p}{dt^2} = -\omega^2 A_p \cos(\omega t + \phi)$

Substituting this value in eq. (1) we get

$$-\omega^2 A_p \cos(\omega t + \phi) = \frac{T}{ml}\left[A_{p+1} + A_{p-1} - 2Ap\right]\cos(\omega t + \phi)$$

or $$-\omega^2 A_p = \frac{T}{ml}\left[A_{p+1} + A_{p-1} - 2A_p\right] \quad \text{or} \quad -\frac{\omega^2 ml}{T} A_p = A_{p+1} + A_{p-1} - 2A_p$$

or $$A_{p+1} + A_{p-1} = \left(2 - \frac{\omega^2 ml}{T}\right) A_p \qquad .....(4)$$

According to boundary conditions $A_0 = 0$ and $A_{N+1} = 0$. Eq. (4) represents a set of N equations which have to be simultaneously solved to give the possible mode of frequencies.

General solution :

$$\frac{A_{p-1} + A_{p+1}}{A_p} = 2 - \frac{\omega^2}{\omega_0^2} \qquad \text{where } \omega_0^2 = \frac{T}{ml}$$

$$\frac{A_{p-1} + A_{p+1}}{A_p} = \frac{2\omega_0^2 - \omega^2}{\omega_0^2} \qquad (p = 1, 2 \ldots N) \qquad .....(5)$$

For any particular value of ω, the right hand side of eq. (5) is constant and is independent of p. Therefore the ratio of left hand side must be a constant and independent of p. Let us assume that the amplitude of particle p can be expressed as

$$Ap = C \sin P\theta \qquad .....(6)$$

So that $$A_{p-1} = C \sin (p-1)\,\theta$$

and $$A_{p+1} = C \sin (p+1)\,\theta$$

$$A_{p-1} + A_{p+1} = C\,[\sin (p-1)\,\theta + \sin (p+1)\,\theta]$$
$$= 2\,C \sin p\theta \cos \theta$$

Hence $$\frac{A_{p-1} + A_{p+1}}{A_p} = \frac{2\,C \sin p\theta \cos\theta}{C \sin p\theta} = 2\cos\theta \qquad .....(7)$$

The right hand side is independent of p. So the proposed solution [eq. (6)] is successful and will satisfy all N equations. The value of θ can be obtaied by applying the boundary conditions i.e., $Ap = 0$ for $p = 0$ and $P = N + 1$. This condition will be satisfied if $(N+1)$ (is an integral multiple of θ i.e.,

$$(N+1)\theta = n\pi \qquad (n = 1, 2, 3........)$$

$$\theta = \frac{n\pi}{(N+1)} \qquad .....(8)$$

Substituting the value of (from eq. (8) in eq. (6), we get

$$A_p = C \sin\left(\frac{p\,n\,\pi}{N+1}\right) \qquad ......(9)$$

Now the allowed frequencies of the normal modes can be determined from eqs. (5) to (9), we have

$$\frac{A_{p-1}+A_{p+1}}{A_n}=\frac{2\omega_0^2-\omega^2}{\omega_0^2}=2\cos\left(\frac{n\pi}{N+1}\right)$$

or $$2\omega_0^2-\omega^2=2\omega_0^2\cos\left(\frac{n\pi}{N+1}\right)$$ or $$\omega^2=2\omega_0^2\left[1-\cos\left(\frac{n\pi}{N+1}\right)\right]$$

or $$\omega^2=\left[4\omega_0^2\sin^2\left(\frac{n\pi}{2(N+1)}\right)\right] \qquad .....(10)$$

or $$\omega=2\,\omega_0\sin\left\{\frac{n\pi}{2(N+1)}\right\} \qquad .....(11)$$

In eq. (9), different valuse of n define different normal mode frequencies. So, in general, eq. (9) can be expressed as

$$\omega_n=2\,\omega_0\sin\left\{\frac{n\pi}{2(N+1)}\right\} \qquad .....(12)$$

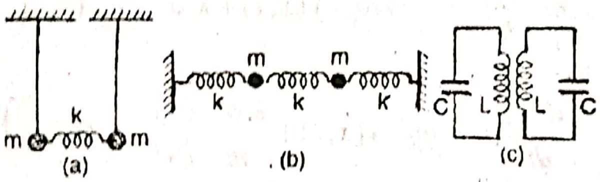
At each frequency x_n, the pth particle has amplitude

$$A_p=C\sin\left\{\frac{pn\pi}{(N+1)}\right\} \qquad .....(13)$$

SHORT ANSWER QUESTIONS

Q. 3. What are coupled oscillators and give examples ?

Ans : When two or more oscillating systems are connected in such a manner as to allow motion energy to be exchanged between them. Such systems oscillations are called couple oscillations and such oscillators are called coupled oscillators. Coupled oscillators occur in nature or can be found in man made devices.



Some examples of two coupled oscillating systems are shown in fig. (1) fig (1a), the bobs of two simple pendulums are connected to each other by means of a spring. Fig. (1b) shows two masses attached to each other by three springs. Here the middle spring provides the coupling. Fig (1c) shows to to coupled LC circuit.

Q. 4. Derive the wave equation of N - coupled oscillators ?

Ans : Consider n coupled pendulums. In fig. (8) only (n – 1)th, nth and (n + 1)th pendulums are shown.

Let y_{n-1}, y_n and y_{n+1} be the displacements of $(n-1)^{th}$, n^{th} and $(n+1)^{th}$ masses respectively. Now we shall consider the resultant force on n^{th} mass. The following forces are acting on the n^{th} mass :

(i) Restoring force due to gravity. This is given by

$$-mg\sin=-mg\,(y_n/l)=-m\,(g/l)\,yn$$

$$=-m\,\omega_0^2\,y_n \quad where\ \omega_0^2=(g/l)$$

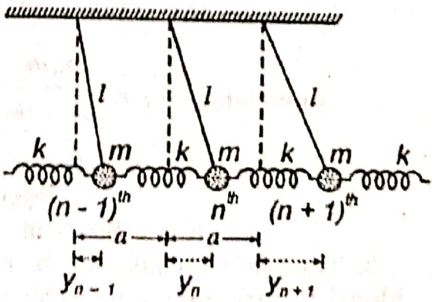


Fig.

(ii) The right hand spring increases the displacement of n^{th} mass. This can be derived as :

stretching of the spring $= y_{n+1} - y_n$

$\therefore$ Force that tends to increase the displacement $= k\,(y_{n+1} - y_n)$

where k is force constant of the spring.

(iii) The left hand spring decreases the displacement of n^{th} mass. This can be derived as :

stretching of the spring $= y_n - y_{n-1}$

$\therefore$ Force that tends to increase the displacement $= -k\,(y_n - y_{n-1})$

Thus the resultant force on n^{th} mass is given by $= -m\,\omega_0^2\,x_n + k\,(y_{n+1} - y_n) - k\,(y_n - y_{n-1})$

Now the equation of motion of n^{th} mass is given by

$$m\frac{d^2y_n}{dt^2} = -m\,\omega_0^2\,y_n + k\,(y_{n+1} - y_n) - k\,(y_n - y_{n-1}) \quad .....(1)$$

According to Taylor's series.

$$y_{n+1}(t) = y\,(x + a,\,t) = y\,(x,\,t) + a\frac{\partial y}{\partial x}(x,t) + \frac{1}{2}a^2\frac{\partial^2 y}{\partial x^2}(x,t) + ...$$

Similarly, $$y_{n-1}(t) = y\,(x - a,\,t) = y\,(x,\,t) - a\frac{\partial y}{\partial x}(x,t) + \frac{1}{2}a^2\frac{\partial^2 y}{\partial x^2}(x,t) + ...$$

So we have

$$y_{n+1} - y_n = a\frac{\partial y}{\partial x} + \frac{1}{2}a^2\frac{\partial^2 y}{\partial x^2} \quad .....(2)$$

and $$y_n - y_{n-1} = a\frac{\partial y}{\partial x} - \frac{1}{2}a^2\frac{\partial^2 y}{\partial x^2} \quad .....(3)$$

From eqs. (1) (2) and (3), we get

$$m\frac{d^2y_n}{dt^2} = -m\,\omega_0^2\,y(x,t) + k\,a^2\frac{\partial^2 y}{\partial x^2}(x,t)$$

or $$\frac{d^2y_n}{dt^2} = -\omega_0^2\,y(x,t) + \frac{k\,a^2}{m}\frac{\partial^2 y}{\partial x^2}(x,t) \quad .....(4)$$

This is wave equation of n - coupled oscillator.

SOLVED PROBLEMS

Q. 5. Sodium chloride molecule has a natural vibrational frequency = 1.14×10^{13} Hz. Calculate the interatomic force constant. Mass of sodium atom = 23 a.m.u. Mass of cl atom = 35 a.m.u. (1 a.m.u. = 1.67×10^{-27}kg).

Ans: Given $\upsilon = 1.14 \times 10^{13}$ Hz.

$m_1 = 23 \times 1.67 \times 10^{-27}$ kg.

$m_2 = 35 \times 1.67 \times 10^{-27}$ kg.

$$\text{reducedmass } \mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{23x35x(1.67x10^{-27})^2}{(23+35)+1.67x10^{-27}} = 23.18 \times 10^{-27}\text{ kg}$$

$K = 4x^2\,\upsilon^2\mu = (4 \times (3.14)^2 \times (1.14 \times 10^{13})^2 \times 23.18 \times 10^{-27}$

$= 1188.07 \times 10^{-1}$

$K = 118.8$ N/m

2. The CO_2 molecule may be represented by a system consisting of a central mass m_2 connected by identical springs of spring constant k to two masses m_1 and m_3 (with $m_1 = m_3$) as shown in fig. Write down the equations of motion of each mass and slove them for the two normal modes in which the masses oscillate along the line joinging the centres.

Solution : Equation of motions are :

$$m_1 = \frac{d^2x_1}{dt^2} = k\,(x_2 - x_1)$$

$$m_2 = \frac{d^2x_2}{dt^2} = k\,(x_3 - x_2) - k\,(x_2 - x_1)$$

and $$m_3 = \frac{d^2x_3}{dt^2} = -\,k\,(x_3 - x_2)$$

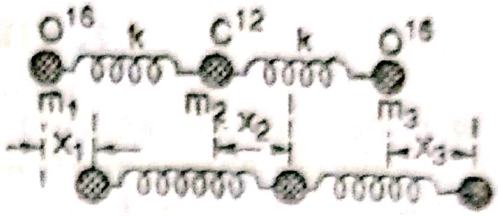


Fig.

from these equations $$m_1 = \left(\frac{d^2x_3}{dt^2} - \frac{d^2x_1}{dt^2}\right) = -\,k\,(x_3 - x_1)\ \text{(setting } m_1 = m_3\text{)}$$

or $$\frac{d^2x}{dt^2} = -\frac{k}{m_1}x \qquad \text{where } x = (x_3 - x_1)$$

$$\omega_1^2 = k/m_1$$

Assume $x_1 = A\cos(\omega t + \phi)$ $\quad x_2 = B\cos(\omega t + \phi)$ $\quad x_3 = C\cos(\omega t + \phi)$

Substituting these values, we get

$$m_1\omega^2 A = k\,(A - B) \qquad m_2\omega^2 B = k\,(2B - A - C) \text{ and } \qquad m_1\omega^2 C = k\,(B - C)$$

eliminating A, B and C from these equations, we get

$$\omega_1^2 = k\left(\frac{2m_1 + m_2}{m_1 m_2}\right)$$

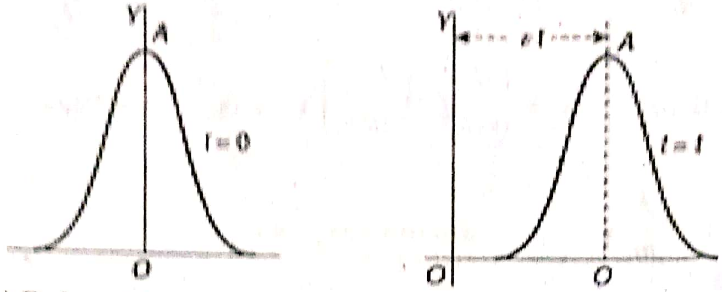
UNIT – V

7. VIBRATING STRINGS

LONG ANSWER QUESTIONS

Q. 1. *Derive wave equation of Transverse wave propagation along a stretched string and give it's general solution*

Ans : Consider a string strethed in the x-direction along which a transverse wave is travelling with the shape of the wave at $t=0$ is shown in fig (a). The pulse is described by $y = f(x)$ when time $t=0$.



(a) Pulse at time t = 0 *(b) Pulse at time t = t*

Let the pulse is travelling to the right with a velocity v changed. After a time t, the pulse reaches a distance vt along x-axis shown in fig (b).

The wave form now can be represented as $y = f(x - vt)$. If the pulse is traveling along the negative x-direction i.e., $y = f(x+vt)$.

Thus the wave travelling with a constant speed v along the x-axis can be represented by

$y = f(x \pm vt)$ ....(1)

We now consider a special case in which vibration is a sinusoidal (or) hormonic function then

$$y(x, t) = A \sin K(x - vt)$$

Let us replace x by $x + \left(\frac{2\pi}{K}\right)$ then

$$y(x, t) = A_0 \sin\left(K\left(x + \frac{2\pi}{K} - Vt\right)\right) = A_0 \sin(K(x - vt) + 2\pi) = A_0 \sin(K(x - vt))$$

So, the replacement of x by $x + \left(\frac{2\pi}{K}\right)$ give the same value of y. In other words.

$$\lambda = \frac{2\pi}{K} \ (or)\ K = \frac{2\pi}{\lambda}$$

k is known as wave number

Eq (1) is expressed by $y = f(vt \pm x)$ ...... (2)

Differentiating eq (2) partially with respect to x twice, we get

$$\frac{\partial y}{\partial x} = \pm f^{1}(vt \pm x) \ \& \ \frac{\partial^2 y}{\partial x^2} = \pm f^{11}(vt \pm x) \quad ......(3)$$

Simiarly, differentiating (2) partially with respect to 't' we get

$$\frac{\partial y}{\partial t} = V f^{1}(vt \pm x) \ \& \ \frac{\partial^2 y}{\partial t^2} = V^2 f^{11}(vt \pm x) \quad ......(4)$$

From eq. (3) and (4) we get $\dfrac{\partial^2 y}{\partial t^2} = V^2 \dfrac{\partial^2 y}{\partial x^2}$ ......(5)

This is called the differential form of the wave equation.

Simple harmonic solution of wave equation : The simple harmonic function can be expressed by

$y(x, t) = a \sin (wt \pm kx)$ (or) $y(x, t) = a \cos (wt \pm kx)$ .....(6)

$\therefore$ $y(x, t) = a \sin (wt - kx)$ is the function taking the wave advancing in positive x-direction.

$$\therefore y(x,t) = a\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

Where $\omega = 2\pi n = \frac{2\pi}{T}$ where n is the frequency and T is time period and $K = 2\pi\lambda$ where λ is wavelength.

$$\therefore y(x,t) = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$$y(x,t) = a\sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda t}{T} - x\right)\right] \quad \text{.......(7)}$$

But, $\upsilon = n\lambda = \frac{\lambda}{T}$ $\quad y(x,t) = a\sin\left[\frac{2\pi}{\lambda}(\upsilon t - x)\right]$ ....(8)

Equations (7) & (8) are the alternative forms of S.H wave equation.

Q. 2. *Describe the modes of vibrations of a stretched strings clamped at both ends. What are overtones?*

Ans : Modes of vibrations of stretched clamped at both ends :

A uniform string of length l having mass per unit length m and stretched by a tension T. The general solution of the wave equation is given by

$y = a_1 \sin(wt - kx) + a_2 \sin(wt + kx) + b_1\cos(wt - kx) + b_2 \cos t\,(wt + kx)$ ....(1)

Where a_1, a_2, b_1 and b_2 are orbitrary constants.

As the string is rigidly supported at the two ends, we have the following boundary conditions.

1. $y = 0$ at $x = 0$ at all time t, ...(2)
2. $y = 0$ at $x = \lambda$ at all time t

Applying first conditions (2) in eq. (1) we get

$O = a_1 \sin wt + a_2 \sin wt + b_1 \cos wt + b_2 \cos wt$

$O = (a_1 + a_2) \sin wt + (b_1 + b_2) \cos wt$

As $\sin wt \neq 0$ and $\cos t\, wt \neq 0$ Hence $(a_1 + a_2) = 0$ and $(b_1 + b_2) = 0$

Thus we have $a_1 = -a_2$ and $b_1 = -b_2$ Now eq (1) becomes

$y = a_1\,[\sin(wt - kx) - \sin(wt + kx)] + b_1[\cos(wt - kx) - \cos(wt + kx)$

$= a_1[\sin wt \cos kx - \cos wt \sin kx] - [\sin wt \cos kx + \cos wt \sin kx]\}$

$+ b_1[\cos wt \cos kx + \sin wt \sin kx] - [\cos wt \cos kx - \sin wt \sin kx]$

$= -2a_1 \cos wt \sin kx + 2b_1 \sin wt \sin kx.$

$= (-2a_1 \cos wt + 2b_1 \sin wt) \sin kx$ ....(3)

The solution now consists of two terms one depending on t and second on x. Thus the first boundary condition reduces the opposite travelling waves to stationary wave.

Now applying second boundary condition of eq (2) in eq (3) we have, $\sin wt \neq 0$ $\cos wt \neq 0$ hence $\sin kl = 0$.

Which gives the general solution for angle kl to be $kl = n\pi$ where $n = 1, 2, 3,$ ....

As l is constant K is limited to discreate set of values known as

$$Kn = \frac{n\pi}{l} \quad \text{where } n = 1, 2, 3 \quad \text{.....(4)}$$

$$\therefore \upsilon_n = n\left(\frac{V}{2l}\right) \quad \text{where } n = 1, 2, 3 \quad \text{.....(5)}$$

From eq (5) that the string can have a set of eigen-frequencies or proper frequencies only.

The fundamental frequency corresponding to $n = 1$ is given by;

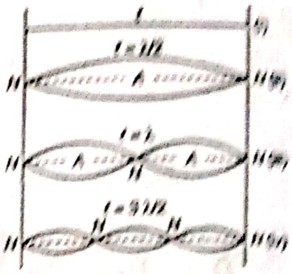
$$\upsilon_1 = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{M}} \quad \text{..... (6)}$$

where $V = \sqrt{\frac{T}{M}}$

The nth harmonic mode of frequency is given by $\upsilon_n = \frac{n}{2l}\sqrt{\frac{T}{M}}$ ....(7)

Overtones and Harmonics : Let us consider the case of string fixed at the two ends and plucked at the middle it vibrates with nodes at the ends and antinode at the middle. The tone emitted under this condition known as fundamental (or) first harmonic. The frequency υ is given by,

$$\upsilon_1 = \frac{1}{2l}\sqrt{\frac{T}{M}}$$



If string is plucked at 1/4th of its length. The string vibrates in two segments. The frequency of vibration of string, υ_2 is given by,

$$\upsilon_2 = \frac{2}{2l}\sqrt{\frac{T}{M}} = 2\upsilon_1$$

This is called first over tone or second harmonic.

When the string vibrates in three segments, the frequency of vibrations given by

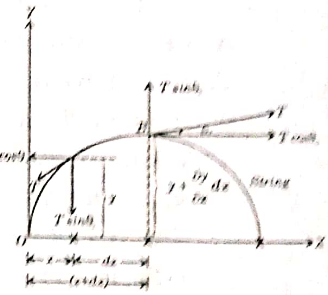
$$\upsilon_3 = \frac{3}{2l}\sqrt{\frac{T}{M}} = 3\upsilon_1$$

This is called second over tone or third harmonic.

Q. 3. *Derive the expression for velocity of a transverse wave along a stretched string?*

Ans : Ideal string : A perfectly elastic, uniform and flexible cord having very large in length compared to its diameter is called *Ideal string.*

Expression : Consider a string fixed between two rigid supports under a tension 'T' along X-axis. When the string is plucked perpendicular to its length, transverse vibrations are set up. Let at any instant the string is as shown in the diagram. Consider a small element AB of length dx. Let the coordinates of A and B be x and $(x + dx)$. Let y be the displacement of any point in between A and B. Let θ_1 and θ_2 be the angles made by the tension at A and B with X-axis. The tension at A and B can be resolved into components.



At A : The horizontal component of Tension = $T\cos\theta_1$

The vertical downward component = $T\sin\theta_1$

At B : The horizontal component of Tension = $T\cos\theta_2$

The vertical upward component = $T\sin\theta_2$

The horizontal components are nearly balanced.

$\therefore$ The vertical upward component = $T\sin\theta_2 - T\sin\theta_1$

$\therefore$ The net upward force = $F = T\sin\theta_2 - T\sin\theta_1$ .....(1)

If θ_1 and θ_2 values are small $\sin\theta_1 = \tan\theta_1$: $\sin\theta_2 = \tan\theta_2$

$\therefore F = T\tan\theta_2 - T\tan\theta_1$ $F = T(\tan\theta_2 - \tan\theta_1)$

$$\tan\theta_1 = \left(\frac{\partial y}{\partial x}\right)_x \quad ; \quad \tan\theta_2 = \left(\frac{\partial y}{\partial x}\right)_{x+dx}$$

$$\therefore F = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right] \quad .....(2)$$

Consider the term $\left(\frac{\partial y}{\partial x}\right)_{x+dx}$ using Taylor's expansion

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx + \frac{\partial^3 y}{\partial x^3}(dx)^2 + \ldots\ldots\ldots$$

Neglecting high powers $\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx$ .....(3)

Substituting the value of $\left(\frac{\partial y}{\partial x}\right)_x$ from eqn.(3) in eqn.(2), we get,

$$F = T\left[\left\{\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx\right\} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$\therefore \quad F = T.\frac{\partial^2 y}{\partial x^2}dx \qquad \text{.......(4)}$$

If m is the mass per unit length of the string, the mass of the element $AB = mdx$.

The acceleration of the element $AB = \frac{\partial^2 y}{\partial t^2}$.

$\therefore$ The force in the upward direction = mass x acceleration

$$\therefore F = mdx\frac{\partial^2 y}{\partial t^2} \qquad \text{.......(5)}$$

$\therefore$ From (1) and (2)

$$T\left(\frac{\partial^2 y}{\partial x^2}\right)dx = mdx.\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\therefore \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}.\frac{\partial^2 y}{\partial x^2} \qquad \text{.......(6)}$$

The standard form of differential equation of wave motion is

$$\therefore \frac{\partial^2 y}{\partial t^2} = v^2.\frac{\partial^2 y}{\partial x^2} \qquad \text{.......(7)}$$

$\therefore$ From equations (6) and (7) $v^2 = \frac{T}{m}$

$$\therefore v = \sqrt{\frac{T}{m}} \qquad \text{.......(8)}$$

$\therefore$ This is the expression for the velocity of transverse wave in a stretched string.

SOLVED PROBLEMS

4. ***A string of length 8 m fixed at both ends has a tension of 49 N and mass of 0.4 kg. Find the speed of transverse wave.***

Solution : $l = 8\,m$ $\quad T = 49\,N$

mass of string = 0.04 kg.

$$\text{Linear density} = \frac{\text{Mass of string}}{\text{length of string}} = \frac{0.04}{8} = \frac{4}{8 \times 100}$$

$$m = \frac{1}{200} = 0.005\ kg./m^3$$

$$\therefore \text{Velocity of Transverse wave } v = \sqrt{\frac{T}{m}} = \sqrt{\frac{49}{0.005}} = v = \sqrt{9800} = 98.99\ m/s$$

5. ***A steel wire of diameter 1 c.m. is kept under a tension of 5 KN. The density of steel is 7.8 g/c.c. Calculate the velocity of the transverse wave.***

Solution : $v = \sqrt{\frac{T}{m}}$

Tension $T = 5\ KN = 5 \times 10^3 \times 10^5$ dyne

Linear density $m = \pi r^2 e$

Radius of the wire $r = \frac{1}{2}cm = 0.5\ cm$

Density of wire $e = 7.8\ gm/cm^3$

Linear density $m = \frac{22}{7} \times 0.5 \times 0.5 \times 7.8\ gm/cm$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{5 \times 10^3 \times 10^5 \times 7}{22 \times 0.5 \times 0.5 \times 7.8}}$$

Velocity of transverse wave $(v) = 9033\ cm/sec.$

6. ***A steel wire 50 cm long has mass of 5 gms. It is stretched with a tension of 400 N. Find the frequency of the wire in fundamental mode of vibration.***

Solution : $\upsilon = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$

Here, $l = 50\ cm = 0.5\ m, m = 5 \times 10^{-3}\ kg/0.5 \times 10^{-2}\ kg$ and $T = 400$

$$\therefore \upsilon = \frac{1}{2\times 0.5} \times \sqrt{\frac{400}{10^{-2}}} = \frac{1}{1.0}\sqrt{4000 \times 10^2}$$

7. ***Two similar wires are under same tension. When tension in one wire increased by 6.09% and the two wires vibrate simultaneously, 6 feats are heard per second. Find the original frequency of the two strings.***

Solution : Let υ_1 be the original frequency and after loading further, 1st υ_2 be its frequency. In this case $T_2 = T_1 + \frac{6.09}{100}T_1 = \frac{106.09}{100}T_1$

Hence $\upsilon_1 = \frac{1}{2l}\sqrt{\left(\frac{T_1}{m}\right)}$ ....(1)

2nd $\upsilon_2 = \frac{1}{2l}\sqrt{\left(\frac{106.09T_1}{100m}\right)}$ ....(2)

Dividing eq. (2) by eq. (1), we get

$$\frac{\upsilon_2}{\upsilon_1} = \sqrt{\left(\frac{106.09}{100}\right)} = \frac{10.3}{10} = \frac{103}{100} \qquad \upsilon_2 = \frac{103}{100}\upsilon_1 \qquad \text{........(3)}$$

Given $\upsilon_2 - \upsilon_1 = 6$ $\therefore = \frac{103}{100}\upsilon_1 - \upsilon_1 = 6$

$3\upsilon_1 = 600$ $\upsilon_1 = 200\ Hz$

8. ***The fundamental frequency of vibration of a stretched string of length 1m is 256 Hz. Find the frequency of the same string of half the original length under identical condition.***

Solution : Fundamental frequency $n_1 = 256\ Hz$

Length of the string $l_1 = 1\ m$

If the length changes to $l_2 = 0.5\ m$

Let the frequency be n_2

From the laws of transverse vibrations

$n_1 l_1 = n_2 l_2$

$$n_2 = \frac{n_1 l_1}{l_2} = \frac{256 \times 1}{5} = 512\ Hz.$$

8. ULTRASONICS

LONG ANSWER QUESTIONS

Q.1. Write an essay on the production of ultrasonics.

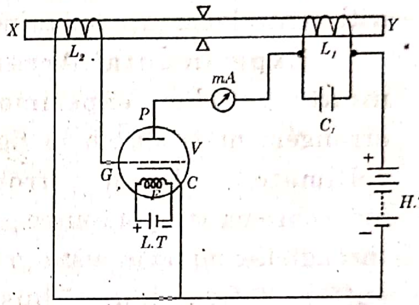
Ans : There are two methods by which ultrasonics can be produced 1. Magnetostriction method and 2. Piezo electric genetor method.

1. Magnetostriction Method : In this method magnetostriction principle is used .

Principle : When a ferromagnetic material in the form of a bar is subjected to alternating magnetic field, the bar expands and contracts in length alternately. This means the rod make vibrations.

The alternating magnetic field is produced with the help of an oscillatory circuit as shown in the diagram. By this method ultrasonics of frequency 300 *kHz* can be produced.

The experimental arrangement is shown in the diagram. *XY* is a Ferromagnetic bar, clamped at its centre. L_1 and L_2 are two coils surrounding the bar. The parallel L_1, C_1 combination is connected to the plate circuit. Let *V* be the thermionic valve. *G* is grid. *C* is cathod. *F* is filament. *P* is plate. The values of L_1 and C_1 will determine the frequency of the oscillatory circuit. The frequency of the oscillatory circuit is $\dfrac{1}{2\pi\sqrt{LC}}$



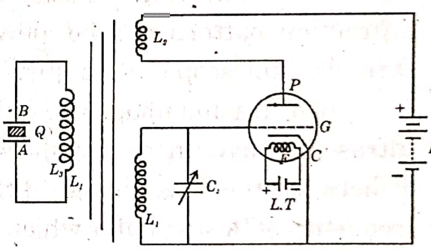
Initially the bar is magnetised by passing direct current. The value of C_1 is adjusted untill the frequency of L_1 C_1 circuit is made equal to the natural frequency of longitudinal vibrations of the bar. If the frequency of the bar is more than 20 *kHz* it emits Ultrasonics.

2. Piezo electric generator : In this method converse of Piezo electric effect is used.

Piezo electric effect : Crystals like quartz, tourmaline, rochelle salt exhibit this effect.

If one pair of opposite faces of a crystal are subjected to pressure, opposite electric charges are developed on the other pair of opposite faces. The polarities of the charge can be reversed when the crystal is subjected to tension.

The converse of Piezo electric effect is, if alternating voltages are applied to one pair of faces, there is corresponding changes in the dimensions of the other pair of faces of the crystal are produced.



Fig

The experimental arrangement is as shown in the diagram. *Q* is a thin slice of quartz crystal, cut with its opposite faces perpendicular to the optic axis. The crystal is placed between the plates *A* and *B*. The inductance L_1, capacitor C_1 are connected in parallel in the plate circuit. The variable condenser capacity is adjusted untill the natural frequency of the crystal is made equal to the frequency of the oscillatory circuit. By this method ultrasonics of frequency 500 *kHz* can be produced.

SHORT QUESTION ANSWERS

Q.2. What are ultrasonics ? What are their properties ?

Ans : Ultrasonics are the sound waves having frequency more than *20,000 Hz* or *20 kHz*

Properties :

1. They travel with velocity of sound waves.
2. Their wave length is very small. Hence their energy is more.
3. They are less absorbed by the medium through which they pass.
4. They can propogate as a fine beam over longer distances. Hence used for communication purpose in war field.

Q.3. What are the methods for the detection of ultrasonics?

Ans : 1. Kundt's tube method, 2. Sensitive flame method, 3. Piezo - electric detector and 4. Thermal detector.

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1. Kundt's tube method : Lycopodium powder is sprinkled in the Kundt's tube method. When ultrasonics are sent into the tube stationary ultrasonics are formed. Hence the powder collects as small heaps at nodes. The distance between two nodes is half the wave length of ultrasonics .

2. Sensitive flame method : When a narrow sensitive flame is moved through the medium where the ultrasonics are present, the flame will be steady at antinodes and flickers at nodes.

3. Piezo electric detector : When one pair of opposite faces of a quartz crystal is exposed to the ultrasonics waves opposite charges are developed on the other pair of opposite faces.

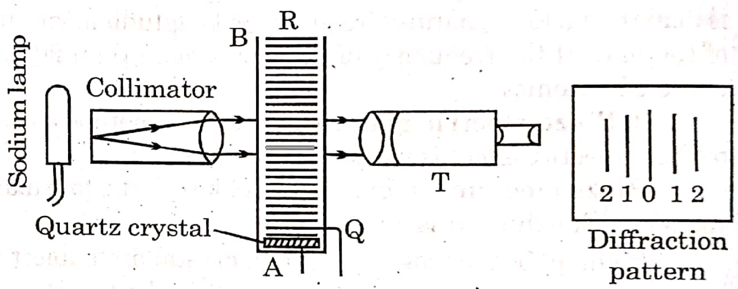
4. Thermal detector : When a platinum wire is moved through the medium where ultrasonics are present, at nodes the resistance of the wire changes with time. At antinodes the resistance of the wire remains constant.

Q.4. Write short notes on Acoustic grating.

Ans : A quartz crystal *'Q'* is placed between two metal plates. These two plates are connected to audio frequency oscillator. Due to the vibrations of the crystal ultrasonics are produced. They are reflected by the reflector *R*. Hence stationary ultrasonics are formed. Hence density variations are created. This arrangement is called *acoustic grating*. By this arrangement the wave length and velocity of ultrasonics can be determined.

When monochromatic light is passed through such grating, diffraction pattern is observed, with central maxima and principal maxima on either side of it.

Experimental Arrangement : The experimental arrangement is shown in fig. A collimated beam from a monochromatic light source passes through the liquid acoustic grating suffers diffraction. Thus the diffraction pattern can be viewed through a telescope. A number of diffracted images on either side of the central image are observed. As the frequency of the ultrasonic waves increases, the seperation of the diffraction lines increases. The angular separation 'θ' between the direct image of the slit and the diffracted image of the nth order is measured. The frequency of the oscillator which drives the crystal gives the frequency of the vibrating crystal. Thus the frequency of the ultrasonic wave υ is known. On substituting these in the above equation we can calculate velocity of ultrasonic wave.



Q.5. Write a note on applications of ultrasonics.

Ans : 1. Depth of Sea : To measure depth of sea. If t is the time interval between the transmission of the ultrasonic wave and receipt of the echo and υ is the velocity of sound waves in sea water, then depth of the sea $a = \frac{\upsilon \times t}{2}$

2. Direction Signaling : Since ultrasonic have high frequency they are used in directional signalling.

3. Detection of flaws in metals : Ultrasonic waves can be used to detect flaws in metal. We know that flaw in the metal produces a change in the medium due to which reflection of ultrasonic waves takes place.

4. Detection of submarines, Iceberg and other objects in ocean : A sharp ultrasonic beam is directed in various directions into the sea. The reflection of waves from any direction shows the presence of some reflecting body in the sea.

5. Soldering and metal cutting : Ultrasonic waves can be used for drilling and cutting processes in metals. These waves can also be used for soldering, for example, aluminium cannot be soldered by normal methods.

6. Formation of alloys : The constituents of alloys, having widely different densities, can be kept mixed uniformly by a beam of ultrasonic. Thus it is easy to get alloy of uniform composition.

7. Ultrasonic mixing : A colloid solution or emulsion of two non - miscible liquids like oil and water can be formed by simultaneously subjecting to ultrasonic radiations. Now-a-days most of the emulsion like polishes, paints, food products and pharma-ceutical preparations are prepared by using ultrasonic mixing.

8. Coagulation and crystallisation : The particles of a suspended liquid, by ultrasonics, can be brought quite close to each other so that coagulation may tale place. The crystallisation rate is also affected by ultrasonics. The size of crystals, when molten metal is put to crystallisation, can be made smaller and more uniform by the use of ultrasonics.

9. Ultrasonics in metallurgy : To irradiate molten metals which are in the process of cooling so as to refine the gain size and to prevent the formation of cores and to release trapped gases, the ultrasonic waves are used.

10. Destruction of lower life : The animals like rats, frogs, fishes, etc. can be killed or injured by high intensity ultrasonics.

11. Treatment of neuralgic pain : The body parts caffected due to neuralgic or rheumatic pains on being exposed to ultrasonics get great relief from pain.

12. Detection of abnormal growth : Abnormal growth in the brain, certain tumours which can not be detected by X-rays can be detected by ultrasonic waves.

Q.6. Explain about SONAR.

Ans : Sonar (Sound navigation and ranging) is a technique that uses sound propagation (in under water) to navigate, communicate with or detect objects on or under the surface of the water, such as other vessels.

Mainly there are two types of sonar. They are passive sonar and active sonar. Passive sonar is essentially listening for the sound made by vessels (objects). Active sonar is emitting pulses of sounds and listening for echoes - Sonar has a wide spectrum of under water applications, including communication between submarines and navigation of submarines. Mainly Sonar device uses the ultrasonic waves to measure the distence, direction and speed of under water objects.

Sonar consists of a transmitter and a detector and is installed at the bottom of boats and ships. The transmitter produces and transmits ultrasonic waves. These wave travel through water and after striking the object on the seabed, get reflected back and are sensed by the detector. The detector converts the ultrasonic waves into electrical signals which are approximately interpreted. The distence of the object that reflected the sound wave can be calculated by knowing the speed of sound in water and the time travel between transmission and reception of the ultra sound.

Let the time travel between transmission and reception of ultra sound signal be 't' and the speed of sound through sea water be $2d = v \times t$. This method is called echo - ranging. Sonar technique is used to determine the depth of sea, and to locate the underwater hills, valleys, submarine, Ice bergs etc.

SOLVED PROBLEMS

7. ***A magnetostriction oscillator has frequency 20 kHz. If it produces sound wave of velocity 6.2 x 10³ m/s, find the length of ferrite rod.***

Solution : $\nu = \frac{v}{2l}$

$$\therefore 20 \times 10^3 = \frac{6.2 \times 10^3}{2l}$$

$$l = \frac{6.2 \times 10^3}{2 \times 20 \times 10^3} = \frac{6.2}{40} = \frac{31}{200} = 15.5 \times 10^{-2} = 0.155\ m$$

8. ***Calculate the frequency of fundamental note emitted by a piezo electrtic crystal. Use the following data.***

Solution :

$Y = 8 \times 10^{11}\ N/m^2$; Vibrating length $= 3\ mm$

$e = 2.5 \times 10^3\ kg./m^3$

$$n = \frac{1}{2t}\sqrt{\frac{Y}{P}}$$

$Y = 8 \times 10^{11}\ N/m^2$

$p = 2.5 \times 10^{-3} kg. m^{-3}$

$t = 3 \ mm = 3 \times 10^{-3} \ m$

$$n = \frac{1}{2 \times 3 \times 10^{-3}} \sqrt{\frac{8 \times 10^{11}}{2.5 \times 10^{-3}}} = 0.943 \ M \ Hz.$$

9. *A quartz crystal thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency. Given Y for quartz = 7.9×10^{10} newton/m² and ρ for quartz = 2650 kg/m³.*

Solution : We know $v = \sqrt{Y/\rho}$

Substituting the given values, we get $v = \sqrt{\left(\frac{7.9 \times 10^{10}}{2650}\right)} = 5461$ m/sec.

For the fundamental mode of vibration the thickness should be equal to $\lambda/2$. Hence

$\lambda = 2t = 2 \times 0.001 = 0.002$ metre

Now $v = \nu\lambda$ or

$$\nu = \frac{v}{\lambda}$$

$$\therefore \ \nu = \frac{5461}{0.002} = 2.730 \times 10^6 \ Hz$$

10. *A piezo electric crystal has a thickness 0.002 m. If the velocity of sound wave in crystal is 5750m/s, calculate the fundamental frequency of crystal.*

Solution : $\nu = \frac{v}{\lambda} = \frac{v}{2t}$

$$\therefore \ \nu = \frac{5750}{2 \times 0.002} = \frac{5750}{0.004}$$

$$= 1.4375 \times 10^6 \ Hz = 1.4375 \ MHz$$

11. *Calculate the capacitance to produce ultrosonic waves of 10^6 Hz with an inductance of 1 henry.*

Solution : The frequency of LC circuit is given by

$$\upsilon = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)}$$

$$\therefore \ C = \frac{1}{4\pi^2\upsilon^2 L} = \frac{1}{4 \times (3.14)^2 \times (10^6)^2 \times 1} = 0.025 \times 10^{-12} \ Fard = 0.025 \ \mu F.$$

PHYSICS PRACTICALS

MECHANICS, WAVES AND OSCILLATIONS

1. YOUNG'S MODULUS OF THE MATERIAL OF BAR BY UNIFORM BENDING

Experiment No

Date

Aim : To determine the Young's modulus of the material of a given beam by uniform bending method.

Apparatus : A metal beam of iron or steel having a uniform rectangular cross section, two knife edge supporters, weight hangers, metre scale, travelling microscope, vernier calipers, screw gauge.

Formula :

In the uniform bending method, the Young's modulus of a beam of rectangular cross section is given by

$$Y = \frac{3gal^2}{2bd^3}\left(\frac{M}{e}\right) \text{ dynes/cm}^2 \text{ (or) N/m}^2.$$

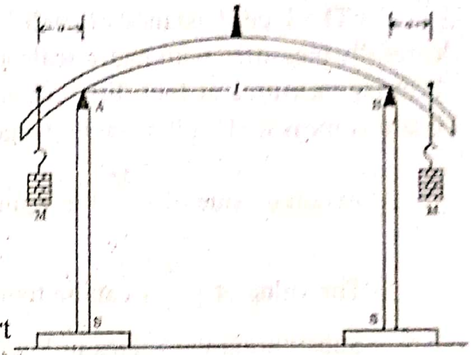


Fig 1

Here, the arrangement is as shown in Fig-1 and

g = acceleration due to gravity (cm/s²)

l = length of the beam between the two knife edge supports = CD (as in Fig-1 (cm)

a = distance on each side, between the knife edge support and place from where weight hanger weight hanger is hung (cm) EC = FD

b = breadth of the beam (cm)

d = thickness of the beam (cm)

M = mass hung from each weight hanger (gm)

e = elevation of the mid point due to a mass M (on either side) (cm)

Description :

The experimental arrangement will be as shown in Fig-1. Here AB is a uniform rectangular beam of length 100 cm made of iron or steel. C and D are two knife edge supports that can be placed in fixed positions at a distance (l) a part. The beam is placed on these two knife edges in a symmetric manner. That is the length AC and length DB will be exactly the same. A pin is fixed with wax at the midpoint N of the beam. This will be midpoint of the length 'l' of the beam between C and D due to the symmetric arrangement. Through the eye piece of travelling microscope the pin appears inverted.

Near the two ends A and B, at equal distance on either side from A and B, at E two grooves are marked with a file and the weight travelling hangers are placed on these grooves. By symmetry AE = FB and EC = DF. Weights can be added to these weight hangers. The E and F should be between A and C and D and B and not between C and D. By this symmetric arrangements, the beam will be subjected to a uniform bending.

Theory of Experiment

Let the weight hangers (of equal weights) alone are suspended from E and F and the horizontal cross wire in the eye piece of travelling microscope is made to coincide with the tip of the (image of the) pin. Let the reading on the vertical scale of travelling microscope is (Z_0). Let the mass of each weight hanger is M_0.

Now let us add additional masses each of value M on either weight hanger. Let the reading of the travelling microscope be now (z). That is, due to a mass M, the elevation of the pin (mid point of the beam) e = ($Z - Z_0$). Now, the Youngs modulus of the material of the beam can be calculated from

$$Y = \frac{3gal^2}{2bd^3}\left(\frac{M}{e}\right) \text{ (dynes/cm}^2\text{)}$$

Experimental Procedure :

First of all the arrangements are made as shown Fig-1. The length l of the beam between C and D is adjusted to be 60 cm. The weight hangers are suspended at E and F at a distance of 2 cm from C and D. The pin is fixed vertically with wax at the mid point N of AB = l of the beam. The travelling microscope is brought before N and is focussed such that the horizontal cross wire coincides tangentially with the tip of the pin. The reading on the vertical scale is noted as (Z_0). Let each weight hanger has a mass M_0.

Now a mass (additional) M of 500 gm is added on either weight hanger and the reading of travelling microscope is noted as z. Now $e = z - z_0$ is the elevation of the midpoint for a mass M (on either side) [In the case of wooden beam, the mass M should be only 50 gm].Next, the masses are increased each time by 500 gm (50 gm for wooden beam) and the corresponding reading z is noted and e is calculated. This process is continued upto a mass of 3000gm. Now, the weights are gradually decreased, each time by 500 gm and the reading (with mass decreased) are taken once again and entered into the tabular form-1A and form Table-1 into Table-2B.

Now, the distance CD between the knife.edges is measured on the beam with a metre scale as 'l'. CD =1 cm Distance of each knife edge from the corresponding knife edge - that is EC = DF is carefully measured with metre scale as 'a'. EC = DF = a cm. With a vernier calipers, the breadth 'b' of the beam is measured at four different places and readings are entered in Table-3D. The thickness 'd' of the beam is measured with a screw gauge at six different places. Readings are entered in Table-4E.

Average value of $\left(\frac{M}{e}\right)$ is found from Table-2.

The value of $\left(\frac{M}{e}\right)$ can be found from a graph drawn between M and e also.

Substituting the values in F and finally in Equation-1 we calculated Y.

We can change the length l - distance between the knife edges and determine y - two or three times as we wish.

Observations : Readings are noted first in table - 1 and then transferred to table-2.

A Readings on the vertical scale of the travelling microscope

1 Main scale division S = 0.05 cm. Total number of vernier divisions n = 50.

Least count of vernier $L.C = \frac{S}{n} = 0.05 \text{ cm} = 0.001 \text{ cm}$.

Table - 1

Mass added (M) (gm)	Main scale a cm.	Vernier coincidence 'n'	Vernier measurement $b = n \times l.c = n \times 0.001\ cm$	Total reading (a+b) cm = z
Weight hanger only M_0				$Z_0 =$
500 gm				
1000 gm				
1500 gm				
2000 gm				
2500 gm				
3000 gm				
3000 gm				
2500 gm				
2000 gm				
1500 gm				
1000 gm				
500 gm				

Above M - Z values are transferred into Table-2.

B) Readings of M and e.

Table - 2

Mass added M gm	Readings on the vertical scale of Microscope(3)			Elevation of N for travelling $e = Z - Z_0$	$\frac{M}{e}$
	Load increasing	Load decreasing	Mean (3)		
Weight hanger only M_0			Z_0		
500 gm 1000 gm 1500 gm 2000 gm 2500 gm 3000 gm					

Average value of $\left(\frac{M}{e}\right)$ = gm/cm

C) M – e graph :

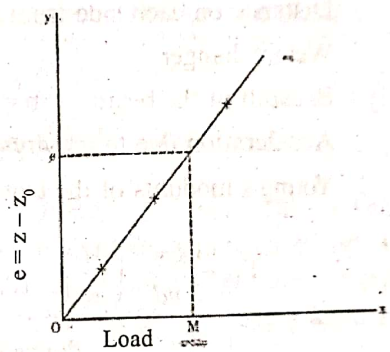
M on x axis e : Scale :

e on y axis e : Scale :

M

e

value of $\frac{M}{e}$ from graph



D) Breadth (b) of the beam with a vernier calipers

1 Main scale division s = 0.1 cm

Total number of vernier divisions n = 10

Least count of vernier L.C. = $\frac{S}{n} = \frac{0.1\ cm}{10}$ = 0.01 cm

Table - 3

S.No.	Main scale reading (a) cm	Vernier coincidence 'n'	Vernier measurement (b) = n × l.c= n × 0.001 cm	Total reading (a+b) cm = Breadth(b)
(1) (2) (3) (4)				

Average breadth of the beam b = cm

E) Thickness (d) of the beam with a screw guage

Pitch of the screw = 0.1 cm

Number of Head scale divisions = 100

Least count of screw gauge

$$\frac{Pitch\ of\ the\ screw}{No.of\ Head\ scale\ divisions} = \frac{0.1cm}{1n} = 0.001cm$$

Zero error correction = ________ Head scale divisions.

Sl.No.	Pinch scale reading a cm	Head scale Reading		Head scale measurement bcm = n × lc n × 0.001 cm	Total reading (a+b) cm Tickness (d) of the beam
		Observed	Corrected (n)		
(1)					
(2)					
(3)					
(4)					
(5)					
(6)					

Average thickness of the beam (d) = cm

F) Calculations - Data : (1) Average value of $\left(\frac{M}{e}\right)$ from table = gm/cm

Distance between knife edges l = cm

Distance on each side from knife edge to

Weight hanger a = cm

Breadth of the beam b = cm

Acceleration due to gravity g = cm/s²

Young's modulus of the material of the beam

$$Y = \frac{3\,gal^2}{2\,bd^3}\left(\frac{M}{e}\right)$$

= dynes/cm² --------- (2)

= N/m² -------- (3)

The values of Y in dynes/cm² is to be divided by 10 to get the numerical values of Y in N/m².

(2) $\frac{M}{e}$ from graph =

$$Y = \frac{3\,gal^2}{2\,bd^3}\left(\frac{M}{e}\text{ from graph}\right) = \text{\_\_\_\_\_\_\_\_ dynes/cm}^2$$

= ________ N/m²

Precautions : The following precautions are to be carefully observed during the experiment.

1. The beam should be placed symmetrically on the knife edge. That is AC = BD and CN = ND also, where N is the mid point of AB.
2. The weights are to be suspended symmetrically.

   That is AE = BF and EC = FD also.

   Unless the conditions (1) and (2) are perfectly satisfied, we do not get a uniform bending of the beam.
3. The adjustment screw on the vertical scale of the travelling microscope should be always rotated in the same direction. Otherwise there will arise 'Back lash error'. The adjustment to get all the readings by moving the screw only in one direction should be attended to in the beginning itself.
4. The weights should be added in the weight langer in a regular fashion and in an orderly manner and in a smooth way. While adding or removing the weights, care should be taken to see that the symmetric arrangements are not disturbed.

2. YOUNG'S MODULUS OF THE MATERIAL OF BAR BY NON-UNIFORM BENDING

Experiment No

Date

Aim To determine the Young's modulus of the material of a given beam by non-uniform bending (double cantilever) method.

Apparatus A metal beam of iron or steel of uniform rectangular cross section, two knife edge supporters, pin, one weight hanger, metre scale, travelling microscope, vernier calipers, screw gauge.

Formula In the uniform bending method (double cantilever) the Young's modulus of a beam of rectangular cross section is given by

$$\frac{3gl^3}{4bd^3}\left(\frac{M}{e}\right) \text{ dynes/cm}^2 \quad \text{——— (1)}$$

$$(\text{or N/m}^2) \quad \text{——— 1(a)}$$

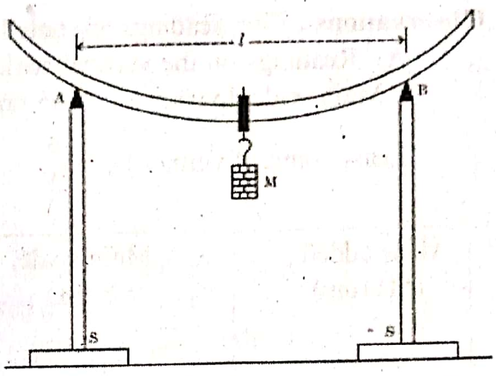


Fig - 1

Here, the arrangement is as shown in Fig-1 and

g = acceleration due to gravity (cm/s²)

l = length of the beam between the two knife edge supports (cm) [before adding weights] = AB

b = breadth of the beam (cm)

d = thickness of the beam (cm)

M = mass suspended at the middle of the beam (gm)

e = Depression of the mid point. Due to the mass M suspended at the middle (cm)

Description

The experimental arrangement will be as shown in Fig-1. PQ is a metal beam of iron steel of uniform rectangular cross section This beam is supported on two knife edge supporters A and B in a symmetric manner. That is PA = QB. At the mid point of AB - that is, at the mid point N of PQ, we suspend a weight hanger from which weights can be hung. Let the mass of the weight hanger alone is M_0. Due to this mass suspended, the mid point N gets depressed. The two half portions of the beam act as two cantilevers and hence the arrangement is called a double cantilever. A pin is fixed vertically at N (to the weight hanger itself) with wax. The horizontal cross wire in the travelling microscope is adjusted to coincide tangentially with the tip of the pin and the reading on the vertical scale is noted down. It should be kept in mini here that, the pin appears inverted through the eye piece.

Theory of Experiment

If M is the mass suspended at the mid point N of the beam, the weight W = Mg. But here we have a double cantilever and hence the weight for each cantilever will be $W = \frac{Mg}{2}$ and length of each cantilever is $\frac{l}{2}$ In the case of a beam of rectangular cross section with $\frac{Mg}{2}$ load for a length $\frac{l}{2}$ we have

$$Y = \frac{gl^3}{4bd^3}\left(\frac{M}{e}\right)$$ Here e is the depression due to a load Mg.

Experimental Procedure

The arrangements are setup as shown Fig-1. The beam is so adjusted to have PA = QB. Now, the weight hanger alone is suspended from N. Let the mass of weight hanger is M_0. The horizontal cross wire in the eye piece of travelling microscope is adjusted to coincide tangentially with the tip of the pin and the reading on the vertical scale is noted as Z_0 The value of M_0 need not be known to us.

Next an additional 500gm mass is added to the weight hanger and the corresponding reading (z) is noted. The process continued in steps of adding 500 gm. [In the case of a wooden beam the masses should be 50gm and final mass 300gm only] Next, the masses are reduced gradually each time by 500gm and again the microscope readings (z) are noted. The readings are first entered into Table-1. (A) From this table, the values of M and Z are entered into Table-2 (B) The length of the beam 'l' between A and B is measured with a metre scale. The breadth (b) of the beam is measured at four different places with a vernier calipers. Reading are entered in Table-3. (D) The thickness 'd' of the beam is measured at six different places with a screw gauge. Readings are entered in Table-4 (E).

Average value of $\frac{M}{e}$ is found from Table-2.

A graph is dawn with 'M' on x axis and correspondly 'e' on y axis and $\left(\frac{M}{e}\right)$ is found from the graph also. Data is entered in order as in F.

Using these value of $\frac{M}{e}$ we can immediately calculate y.

The experiment may be repeated 2 or 3 times by changing the length $l = AB$ of the beam between A and B knife edges.

Observations : First readings are noted in table - 1 and then transferred to table-2.

A) Readings on the vertical scale of the travelling microscope

1 Main scale division S = 0.05 cm, Total number of vernier divisions N = 50

Least count of vernier l.c = $\frac{S}{N} = \frac{0.05\ cm}{50}$ = 0.001 cm.

Table - 1

Mass added (M) (gm)	Main scale a cm.	Vernier coincidence 'n'	Vernier measurement b = n × l.c= n × 0.001 cm	Total reading (a+b) cm = z
Weight hanger only M_0			Z_0 =	
500 gm				
1000 gm				
1500 gm				
2000 gm				
2500 gm				
3000 gm				
3000 gm				
2500 gm				
2000 gm				
1500 gm				
1000 gm				
500 gm				

Above M - Z values are transferred into Table-2.

B) Readings of M and e.

Table - 2

Mass added M gm	Readings on the vertical scale of Microscope(3)			Elevation of N for travelling $e = Z - Z_0$	$\frac{M}{e}$
	Load increasing	Load decreasing	Mean (3)		
Weight hanger only M_0			Z_0		
500 gm 1000 gm 1500 gm 2000 gm 2500 gm 3000 gm					

Average value of $\left(\frac{M}{e}\right) =$ gm/cm

C) M – e graph :

M on x axis e : Scale :

e on y axis e : Scale :

M

e

From graph $\left(\frac{M}{e}\right) =$

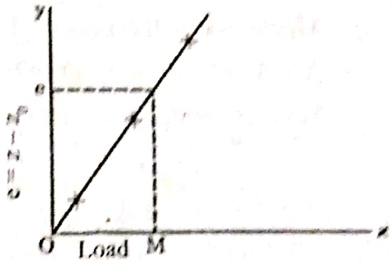


Fig - 2

D) Breadth (b) of the beam with a vernier calipers

1 Main scale division S = 0.1 cm

Total number of vernier divisions n = 10

Least count of vernier L.C. = $\frac{S}{n} = \frac{0.1\ cm}{10} = 0.01$ cm

Table - 3

S.No.	Main scale reading (a) cm	Vernier coincidence 'n'	Vernier measurement (b) = n× l.c= n × 0.001 cm	Total reading (a+b) cm = Breadth(b)
(1)				
(2)				
(3)				
(4)				

Average breadth of the beam b = cm

E) Thickness (d) of the beam with a screw guage

Pitch of the screw = 0.1 cm

Number of Head scale divisions = 100

Least count of screw gauge

$$\frac{Pitch\ of\ the\ screw}{No.of\ \ Head\, scale\, divisions} = \frac{0.1cm}{1n} = 0.001cm$$

Zero error correction = ————— Head scale divisions.

Sl.No.	Pinch scale reading a cm	Head scale Reading		Head scale measurement bcm = n × lc n × 0.001 cm	Total reading (a+b) cm Tickness (d) of the beam
		Observed	Corrected (n)		
(1)					
(2)					
(3)					
(4)					
(5)					
(6)					

Average thickness of the beam (d) = cm

F) Calculations - Data

(1) Average value of $\left(\frac{M}{e}\right)$ from table - 2 = gm/cm

Distance between knife edges $l =$ cm

Breadth of the beam $b =$ cm

Thickness of the beam $d =$ cm/s²

Acceleration due to gravity $g =$ cm/s²

Young's modulus of the material of the beam

$$Y = \frac{3\,gal^2}{2\,bd^3}\left(\frac{M}{e}\right) = \frac{dynes}{cm^2} \quad \text{--------(2)}$$

$$= \text{N/m}^2 \quad \text{-------(3)}$$

The values of Y in dynes/cm² is to be divided by 10 to get the numerical values of Y in N/m².

(2) $\frac{M}{e}$ from graph =

$$Y = \frac{3\,gal^2}{2\,bd^3}\left(\frac{M}{e}\ from\ graph\right) = \text{---------- dynes/cm}^2$$

$= \text{---------- N/m}^2$

Precautions : The following precautions are to be carefully observed during the experiment.

1. The beam should be placed symmetrically on the knife edges. That is PA = QB.
2. Weigh hanger should be suspended exactly at the mid point N of the beam.
3. The adjustment screw on the vertical scale of the travelling microscope should be always rotated in the same direction. Otherwise there will arise 'Back lash error'. The adjustment to get all the readings by moving the screw only in one direction should be attended to in the beginning itself.
4. The weights should be added in the weight langer in a regular fashion and in an orderly manner and in a smooth way. While adding or removing the weights, care should be taken to see that the symmetric arrangements are not disturbed.

Viva – Voce

1. What is a beam? What are the different types of beams?
2. What is bending moment?
3. When is the bending said to be uniform and when is it said to be non-uniform?
4. When is the reason for calling the non-uniform bending as double cantilever ? What are the two cantilevers formed in this experiment?
5. In the case of non-uniform bending experiment, what are the points on the beam that are symmetrically arranged with respect to the C.G. of the beam?
6. In the case of uniform bending experiment, what are the points on the beam that are symmetrically arranged with respect to the C.G. of the beam?
7. In the case of uniform bending, which physical quantity to be measured more accurately and why?
8. In the case of non-uniform bending, which physical quantity is to be measured more accurately and why?
9. What is 'backlash error' with a travelling microscope? How can it be eliminated?
10. While using the Vertical cross wire in the eye piece, on which scale you should observe readings?
11. What is neutral axis of a beam?
12. What is meant by eleasticity? What is plasticity?
13. What is Hooke's law?
14. How many kinds of elasticities are there? What is the relation between them?
15. Define Young's modulus?
16. When is a beem called a cantilever?

Viva - Voce Answers

1. A beam is structural member that is designed to resist forces acting transverse to its axis. Usually a beam is of uniform cross section and of length large in comparison to its breadth and thickness. Beams are of three types (1) Simple supported beam, (2) Cantilever beam and (3) A beam with over hang (Protrusion).
2. The moment of the bending (elastic couple) couple is called the bending moment. It will be in a direction perpendicular to the plane of paper.
3. If the value of R, the radius of curvature is the same for all parts of neutral axis of the beam, the bending is said to be uniform. If the value of R is different for different parts of the neutral axis of the beam, then the bending is said to be non-uniform.
4. Here, the beam is supported on two knife edges at two points. Each half of the beam, between one support (on knife edge) and the point where the beam is loaded (mid point) behaves like a cantilever. As the two halves of the beam acting as two cantilevers, the arrangement is referred to as Double Cantilever.

   AO - one cantilever, OB - second cantilever.
5. The two points at which the beam rests on the knife edges should be symmetric about the centre of gravity (C.G.) of the beam. That is, on either side of C.G., the points of support should be at equal distances from C.G.
6. Here, the two points at which the beam is supported on the knife edges, and the two points where the beam is loaded should by symmetric about the centre of Gravity-that is they should be (each pair) at equal distances from C.G.
7. The thickness 'd' of the beam should be more accurately determined as it occurs in the third power (d^3) in the formula.
8. The thickness 'd' of the beam as well as the distance (l) between the points of support on knife edges should be more accurately determined as both of them occur in third powers (d^3 and l^3) in the formula.
9. While taking readings with a travelling microscope - either on horizontal scale or vertical scale, we have to rotate the tangential screw (T.S.) for finer adjustments. During the experiment, if we rotate the screw once forward, then backward and again in forward direction an error called backlash error arises in readings.
10. While using horizontal cross wire - we take readings on vertical scale.

    While using Vertical cross wire - we take readings on horizontal scale.
11. When a beam is subjected to bending, there will be a portion of the beam which is neither elongated or compressed. This portion is called the neutral surface. The line along which the neutral surface intersects the plane of bending is called neutral axis.
12. The property of a body by virtue of which its deformation is resisted and the body regains its original size and shape after the deformation force is removed is called elasticity.

    If the body cannot regain its original shape and size when the deformation force is removed, the property is called plasticity.
13. Within the proportionality limit, $\frac{stress}{strain}$ = a constant. This is calle Hooke's Law. The constant of proportionality is called the coefficient of elasticity.
14. Coefficients of elasticity are of 3 types (1) Young's Moduluss Y, (2) Bulk Modulus k and (3) Rigidity Modulus η. The relation between them is $Y = \frac{9k\eta}{(3k+\eta)}$.
15. Youngs Modulus $Y = \frac{Stress}{Linear\ strain}$
16. A cantilever beam is a beam that is built in or rigidly fixed at one and free at the other end. Before addings weights at the free end, the beam will be horizontal. But when weights are added at the free end it gets bent.

3. SURFACE TENSION OF A LIQUID BY CAPILLARY RISE METHOD

Experiment No

Date

Aim To determine the surface tension of a given liquid by capillary rise method.

Apparatus A uniform capillary glass tube of length about 20 cm, travelling microscope, rubber bands, retort stand, beaker, $G\frac{|C}{F|}E$

Formula By the method of capillary rise, the surface tension (T) of a liquid is given by

$$T = \frac{\left(h+\frac{r}{3}\right)r\,dg}{2} \text{ dynes/cm}$$

Here, h = the height to which the liquid rises up in the capillary tube (cm)

r = radius of the capillary tube (cm)

d = density of the liquid (gm/cm³)

g = acceleration due to gravity (cm/s²)

Description

As shown in Fig-1, A uniform capillary tube AB is placed vertically immersed to a certain height in a beaker B filled with given liquid, by means of retort stand R. Due to capillary action, the liquid rises to a certain height 'h' inside the capillary tube. (liquids like water rise up, where as mercury goes down in the capillary tube). The rise 'h' can be accurately determined with a travelling microscope. The horizontal cross wire in the eye piece is adjusted to coincide tangentially with the lower (concave) meniscus of water inside the capillary tube. The reading on the vertical scale is noted as R_1. The thin wire $G\frac{|C}{F|}E$ bent twice at right angles is attached to the capillary tube, with rubber bands such that the lower end of EF - that is F just touches the surface of liquid (water) in the beaker. After removing the beaker without distrubing the wire, the horizontal cross wire in the eye piece is adjusted to coincide tangentially with the tip F of the wire. The reading on the vertical scale is noted as R_2. The difference between these two readings (R_1-R_2) gives the rise 'h'.

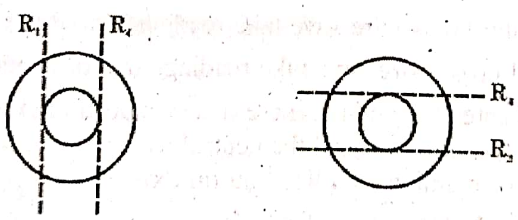


Fig - 1

Theory of Experiment

The angle of contact between glass and water is less than 90° and such liquids rise up in capillary tubes. The height 'h' to which the liquid rises, the radius 'r' of the capillary tube, density of the liquid 'd' are related to surface tension, according theory, as

$$T = \frac{\left(h+\frac{r}{3}\right)r\,dg}{2}$$

In our experiment, we should carefully measure the height h and radius r very accurately. For this, we make use of a travelling microscope.

Experimental Procedure

First of all the capillary tube is throughly cleaned with acidified potassium dichromate solution. This is to ensure that there are no grease and dust inside the capillary tube. Then the tube is cleaned with distilled water and dried. Next the glass beaker filled with water is placed at a certain height. The capillary tube is vertically dipped into the liquid inside the beaker and is rigidly fixed in position by means of a retort stand. The capillary's lower and should not touch the bottom of the beaker and should be slightly above. First the capillary is completely dipped vertically into the liquid and then raised up. This ensures that the liquid wets the tube.

Next, the twice bent thin wire is attached to the capillary tube with the help of rubber bands, such that the end F touches the surface of liquid in the beaker (water).

Next, the horizontal cross wire in the eye piece of the travelling microscope is adjusted to coincide tangentially with the upper surface (lower meniscus) of liquid raised inside the capillary tube, the reading on the vertical scale is noted as R_1.

Next, the beaker is gentaly removed without disturbing the thin wire and now the horizontal errors wire is adjusted to coincide tangentially with the tip of the bent wire (F). Reading on the vertical scale is noted as R_2 $(R_1 - R_2)$ gives the height 'h'.

Next, the capillary tube is taken out, cleaned and dried once again and then its radius 'r' is determined with the travelling microscope.

(B) Determination of the inner radius of the capillary tube with a travelling microscope

As shown in Fig-1(a) let us coincide the vertical cross wires as tangents to the inner circle of cross section and let the respective readings on the horizontal scale be x_1 and x_2 respectively. The diameter of the inner circle will be $d = x_2 - x_1$. Similarly, as shown Fig-1(b), if the readings on the vertical scale corresponding to the coincidences of horizontal cross wires as tangents are y_1 and y_2 respectively, then $d = y_2 - y_1$.

From these readings we can determine the radius $r = \frac{d}{2}$.

The readings are arranged in a tabular from.

Observations

(A) Determination of h

1 main scale division S = 0.05 cm

No. of divisions on the vernier n = 50

Least count $l.c. = \frac{S}{n} = \frac{0.05\ cm}{50} = 0.001\ cm$

S.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measurement (b) = n× l.c= n × 0.001 cm	Total reading (a+b) cm
Reading of upper surface of liquid risen in capillary R_1				R_1 =
Reading of the tip of F R_2				R_2=

(B) Determination of Inner radius (r) of capillary tube

Readings on the horizontal scale of the microscope

Least count of the vernier l.c. = 0.001 cm

S.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measurement (b) = n× l.c= n × 0.001 cm	Total reading (a+b) cm
1.				x_1 = ...
2.				x_2 = ...

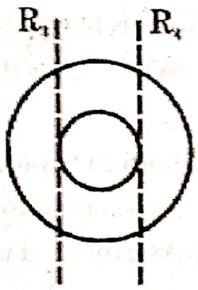


Fig - 2

Inner diameter of the capillary tube $d = x_2 - x_1 =$ cm

reading $r = \frac{d}{2} =$ cm ------- (i)

Readings on the vertical scale

S.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measurement (b) = n × l.c= n × 0.001 cm	Total reading (a+b) cm
1.				y_1 = ...
2.				y_2 = ...

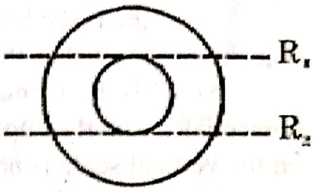


Fig - 4

Inner diameter of the capillary tube $d = y_2 - y_1 =$ cm

reading $r = \frac{d}{2} =$ cm ------- (ii)

Average radius $= \frac{r_1 + r_2}{2} =$ cm

(C) Height to which the liquid rised inside the capillary h = cm

Inner radius of the capillary tube r = cm

Density of liquid d = gm/cm^3

Acceleration due to gravity g = cm/s^2

Surface Tension of the liquid

Result

Surface tension of the given liquid by capillary rise method T = ... dynes/ cm ------- III

Precautions

The following precautions should be carefully observed during the experiment.

1. Initially the capillary should be cleaned to remove any grease, oil or dust.
2. The bore of the capillary should be uniform through out.
3. The capillary tube should be always vertical.

Viva - Voce

1. In determining the surface tension of a liquid which method is the easiest? and
2. How does the surface tension of a liquid change with temperature?
3. Where is the property of surface tension exhibited more - at the surface or inside the liquid ?
4. Mention certain uses and examples of surface tension?
5. When a capillary is dipped into a liquid, do all the liquids rise up in the capillary? On what factor does the behaviour of a liquid depend upon?
6. When a glass capillary tube is immersed in mercury, what will happen? And why?
7. What is the behaviour of (a) detergents and (b) water proofing agents? What is the reason for the respective behaviours?
8. Which method is more accurate in your opinion and why?

Viva - Voce Answers

1. Surface tension by method of drops is the easiest method.
2. As temperature increases, the surface tension of liquids gets decreased.
3. As expressed in the name itself, the properly of surface tension of exhibited at the surface only.
4. Artificial dentures get attached in the mouth due to surface tension.

   It is due to capillarity, a consequences of surface tension, that the following phenomena happen :

   (a) The rise of oil through a wick.

   (b) Rise of solutions through the roots to the plant.

   (c) Blotting paper absorbing ink.

(d) Round shape of (spherical shape of) rain drops.

(e) Lead pellets having spherical shape.

5. Not all the liquids rise up through a capillary. Mercury gets depressed in a glass capillary tube. If the angle of contact between the liquid and the material of the capillary tube is less than 90^0, the liquid rises up in the capillary, but if the angle of contact is more than 90^0, the liquid gets depressed in the capillary.
6. Mercury gets depressed in a glass capillary. This is because, the angle of contact between mercury and glass is ($\sim 135^0$) greater than 90^0.
7. Detergents make water get attached and imbued by clothes by reducing the angle of contact.
8. The more accurate method is the capillaryrise method. It has a sound theoretical back ground and measurements can be taken accurately.

4. VISCOSITY OF LIQUID BY THE FLOW METHOD (POISEUILLE'S METHOD)

Experiment No

Date

Aim To determine the coefficients of viscosity of different liquids by studying the flow of liquids through capillaries.

Apparatus An aspirator bottle, retort stand and travelling microscope or mercury, watch glass and a common balance, sensitive stop clock, measuring jar, metre scale, pinch cock clip, beaker, rubber tube, water and other liquids having low viscosity.

Formula Coefficient of viscosity η of a liquid through Poiseille's method is given by

$$\eta = \frac{\pi p r^4 t}{8 V L} \text{ dynes/square cm/unit velocity gradient or Poise} \qquad \text{----- (1)}$$

Here, p = The pressure difference between the two ends of the capillary tube of length 'l' (dynes/cm²) and p=hdg where

h = the height of the water level in the glass vessel from the axis of the capillary tube. (cm) (average height) If h_1 is the height before starting the experiment and h_2 is the height at the end of the experiment, then

$$h = \frac{h_1 + h_2}{2}$$

d = density of the liquid (gm/cm³)

g = acceleration due to gravity (cm/s²)

r = inner radius of the capillary tube (cm)

t = time through which the liquid of lows through the capillary tube (time through which water is collected in the beaker) (seconds)

V = the volume of the liquid flown through the capillary tube for a time (t) - that is the volume of the liquid collected in the beaker (cm³)

L = length of the capillary tube (cm)

(A) Determination of inner radius of the capillary tube by mercury pellet method

from m = $(\pi r^2 l)\rho$. ----- (2)

radius

$$r = \sqrt{\frac{m}{\pi l \rho}} \text{ cm} \qquad \text{----- (3)}$$

Here, 1 = length of the mercury pellet drawn into the capillary tube (cm)

m = mass of this mercury pellet (gm)

ρ = density of mercury (gm/cm³)

(B) Determination of the inner radius of the capillary tube with a travelling microscope.

As shown in fig-1(a) let us coincide the vertical cross wires as tangents to the inner circle of cross section and let the respective radings on the horizontal scale be x_1 and x_2 respectively. The diameter of the inner circle will be $d = x_2 - x_1$. Similarly, as shown in Fig-(1)b, if the readings on the vertical scale corresponding to the coincidences of horizontal cross wires as tangents are y_1 and y_2 respectively, then

From these readings we can determine the radius $r = \frac{d}{2}$.

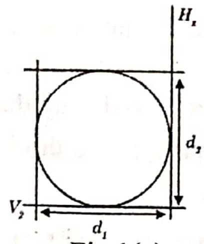


Fig 1(a)

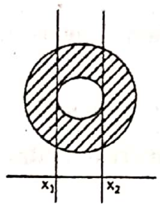


Fig 1(b)

Description

The arrangement of the apparatus consists of an aspirator bottle of about two litres capacity provided with an opening at the side near the bottom. The opening is closed by one-hold rubber stopper through which passes a short glass tube. The capillary tube is connected to the outer end of the glass tube with a short rubber tubing which is provided with a pinch-cock. When the aspirator is filled with water, it flows out through the capillary tube which is kept in a horizontal position. When the collection of water is over, the flow of water can be stopped by closing the pinch-cock.

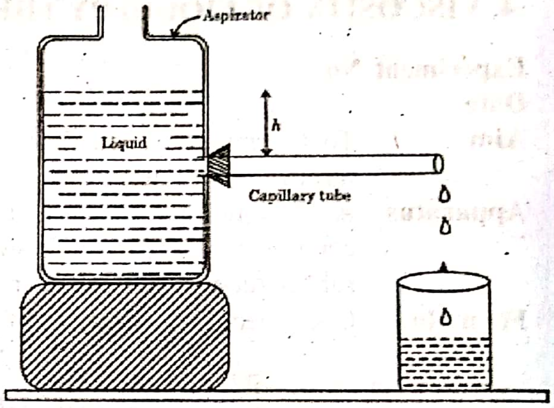


Fig 2

If h_1 and h_2 represent the heights of water level in the aspirator bottle above the axis of the capillary tube before and after the collection of water, the mean pressure exerted is given by P = hdg where h is the average height $\left(\frac{h_1 + h_2}{2}\right)$.$d$ the density of water and g the acceleration due to gravity.

Theory of Experiment

If a liquid having a coefficient of viscosity (flows through a capillary tube of inner radius 'r' and length 'l' under a pressure difference 'p'. for a time 't' then poisemble showed that the volume of liquid flown through time 't' is given by

$$V = \frac{\pi p r^4 t}{8 \eta L}$$ From this, we can determine η by measuring V.

Experimental Procedure

The capillary tube is cleaned well first with acidified potassium dichromate solution and then with tap water. It is then fixed to the upper tube and clamped with its axis horozontal. A short length of fine thread is tied to tube at free end of the capillary tube so that the water coming out of the capillary tube trickless down along the thread in drops.

A clean and dry breaker is taken and is placed under the free end the capillary tube. The height h_1 cm of water level in the aspirator bottle above the axis of the capillary tube is measured with a metre scale. By opening pinch-cock completely water is allowed to flow through the capillary tube into the beaker for a sufficient interval of time l seconds (about 15 minutes). The pinch-cock is closed and the height h_2 of the water level in the aspirator bottle above the axis of the tube is measured. The beaker is removed. The volume of water collected is given by a measuring jar in which the water collected is found and its volume is determined.

To determine the internal radius r of the capillary tube, the tube is almost completely filled with mercury and the length l of the mercury and the length l of the mercury thread is measured with a scale The mercury is then transferred into a weighed watch glass and its weight is determined correct to a milligram. From this, the mass m of mercury is found. The radius r of the capillary tube is then calculate from the relation $m = \pi r^2 l \rho$ where ρ is the density of mercury.

$$\text{That is } r = \sqrt{\frac{m}{\pi l \rho}}$$

The radius 'r' may also be found with a travelling microscope as explined already in (B) of Formula. Finally the length l of the capillary tube is measured with a scale.

The coefficient of viscosity of water is then calculated using the formula.

$$\eta = \frac{\pi P r^4 t}{8 l V} \text{ dynes/sq.cm./unit vel. gradient.}$$

Exactly the same procedure is followed with other liquids and the corresponding coefficient of viscosity η_L is calculated in each case.

Observations

(A) Inner radius of (r) of the capillary tube by mercury pellet method

Length of mercury pellel l = cm

Density of mercury ρ = gm/cm^3

Mass of empty watch glass W_1 = gm

Mass of watch glass + mercury W_2 = gm

Mass of mercury pellet m = $(W_2 - W_1)$ gm

Table – 1 for measuring W_1 and W_2 - Simple Balance

S.l. No.	Contents in the pans		Turning points		Mass in gm Average		Resting point
	Left Pan (object)	Right Pan (weight)	Left (3)	Right (2)	Left (a)	Right (b)	$R.P = \left(\frac{a+b}{l}\right)$
1.	0	0					Z.R.P. =

$$\text{Internal radius of capillary } r = \sqrt{\frac{m}{\pi l \rho}} = \text{ (cm)}$$

(B) Inner radius or capillary tube bya travelling microscope

Readings on the horizontal scale of the microscope

Least count of the vernier l.c. = 0.001 cm

Sl.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measure bcm = n × lc n × 0.001 cm	Total reading = (a+b) cm
1.				x_1 = ...
2.				x_2 = ...

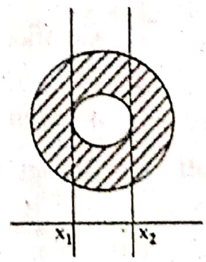


Fig 3

Inner diameter of the capillary tube $d = x_2 - x_1$ = cm

reading $r = \frac{d}{2}$ = cm (i)

Readings on the vertical scale

Sl.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measure· bcm = n × lc n × 0.001 cm	Total reading = (a+b) cm
1.				$y_1 = ...$
2.				$y_2 = ...$

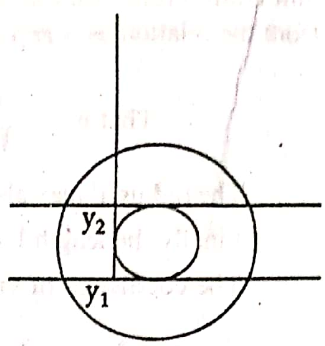


Fig : 4

Inner diameter of the capillary tube $d = y_1 - y_2 =$ cm

Radius $r = \frac{d}{2} =$ cm ..... (ii)

Average radius $\frac{r_1 + r_2}{2} =$ cm

Observations Now the data is entered as follows.

Sl.No.	Measured quantity	Value
1.	Length of the capillary tube	$l =$ cm
2.	Time of flow of water	$t =$ s
3.	Inner radius of capillary tube	$r =$ cm
4.	Acceleration due to gravity	$g = 980$ cm/s²
5.	Initial height of water column	$h_1 =$cm
6.	Final height of water column	$h_2 =$.... cm
7.	Average height of water column	$h = \frac{h_1 + h_2}{2} = ...cm$
8.	Pressure difference between the two ends of the capillary tube (density of water d = 1 gm/c.c)	$p = hdg =$ .... dynes/cm²
9.	Volume of water flow n through time 't' is	$V =$ .... cm³
10.	Coefficient of viscosity of water (η) at room tempertature	$\frac{\pi p r^4 t}{8 Vl}$ = dynes/cm²/unit velocity gradient

Result Coefficient of viscosity of water η

$$(\eta) = \frac{\pi p r^4 t}{8 V l} = ...$$ poise or dynes/com² / unit velocity gradient

Same procedure is followed for other liquids also.

Precautions Following precautions should be carefully observed during this experiment.

1. The capillary tube should be perfectly horizontal.
2. The capillary tube should be cleaned throughly with acidified potassium dichromate solution.
3. While determining the volume of liquid flow, the pinch cock should be completely open.

Viva – Voce

1. How should be the flow of liquid through the capillary tube ? How should be drops be falling from the end of the tube into the beaker?
2. If the viscosity increases, how does the liquid flow rate change ? (increase or decrease)

3. On what factors does the viscosity of a liquid depend upn?
4. How does the viscosity of a liquid change with increase of temperature?
5. What is critical velocity?
6. What is viscosity? Do gases also have viscosity or not?
7. How does the constant pressure head work.
8. What is critical velocity?
9. How are you eliminating the effect of gravity on the flow of liquid through the tube?
10. Can Poiseuille's method be used for highly viscous liquids?

Viva - Voce Answers

1. The liquid flow should be steady and the liquid drops should come out at a steady rate and in round shape.
2. When the viscosity increases, the rate of flow decreases as per $\frac{V}{t} \alpha \frac{1}{\eta}$.
3. The viscosity of a liquid depends upon
   1. Temperature (η decreases with increase of temperature)
   2. Pressure (η increases with increase of pressure)
4. As temperature is increased, the viscosity of liquids decreases.
5. The velocity of the fluid above which the steady flow becomes turbulent is called the critical velocity (v_4)
6. Viscosity is a property of a fluid by which the fluid resists any relative motion between its different layers. As gases are also fluids, they also do have discosity.
7. Here there will be a tube (usually at the middle) which allows the water raising above its height to flow down through it. Hence the liquid level always remains constant at the height of the tube.
8. The velocity of flow of a liquid above which the stream the flow becomes turbulent is called critical velocity (v_c)
9. By keeping the tube horizontal we can eliminate the effect of gravity.
10. Poiseullie's method is not applicable for highly viscous liquids.

5. BIFILAR SUSPENSION - MOMENT OF INTERIA OF REGULAR RECTANGULAR BODY

Experiment No

Date

Aim To study the oscillations under a bifilar suspension with the two filaments not being parallel and hence find out the moment of inertia of a given metal plate of rectangular cross section.

Apparatus As shown in Fig-1 the essential part of this experiment is a metal plate ABCDEFGH of rectangular cross section with a length (a) of 20 cm, breadth (b) of 10 cm and thickness (t) of 0.5cm. Besides this, a rigid support having two hooks P and Q two metal wires of equal lengths, a sensitive stop clock, beam compass, vernier calipers, pin and pointer, balance.

Formula

A regular object like a rectangular metal block (or plate) ABCDEFGH is hung by means two metal of equal lengths but not parallel from a rigid support IJ as shown in Fig-1. When the metal block (or plate) is drawn slightly aside and released, the (block) plate makes oscillations in a horizontal plane around a vertical axis passing through its centre of mass. These oscillations are S.H.M. in quality and the time period of these simple harmonic oscillations is given by

$$T = 2\pi\sqrt{\frac{I.h}{Mgd_1d_2}} \quad \text{----- (1)}$$

From this, the moment of inertia

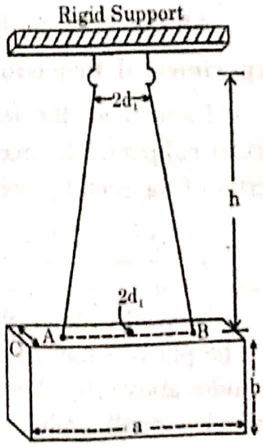


Fig -1

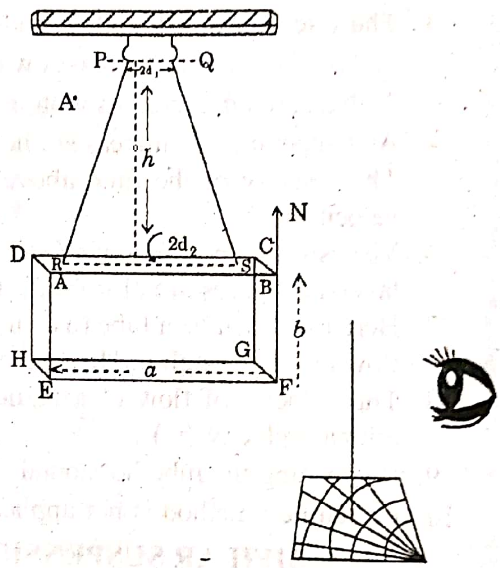
$$I = \frac{Mg}{4\pi^2}\frac{T^2 d_1 d_2}{h}\,(gm - cm^2) \qquad \text{------ (2)}$$

Here,

M = mass of the metal plate (or block) (gm)
g = acceleration due to gravity (cm/s^2)
T = time period of oscillations of the plate (or block) (s)
$2d_1$ = Distance between the two points of suspension P – Q on the support (cm)
$2d_2$ = Distance between the two points R – S where the wrires are attached to the plate (or block) (cm)
h = Vertical distance from the point of suspension (P or Q) to the body that is PM = QN (cm)
I = Moment of inertia of the plate (or block) about a vertical axis passing through the centre of mass of the plate (or block) (gm - cm^2)

Description

The apparatus consists of a rigid support with two hooks P and Q symmetrically situated. From these two hooks, two metal wires of equal lengths are suspended. A metal plate (or block) of regular shape as AB CD EF GH has two hooks at R and on the face ABCD of the metal plate. These hooks R and S are exactly situated on the straight line joining the mid points of AD and BC and are symmetrically situated. These hooks are attached to the lower ends of the metal wires. Thus, the metal plate (or block) is hung. In this arrangement PQ (=$2d_1$) and RS ($2d_2$) distance are not equal and hence the wires PR and QS (even though of equal lengths) will not be parallel to each other. On the face BCGF a pin is attached with wax such that a part of it protrudes out vertically above CD. A pointer is placed infront of the pin and parallel to it. Keeping our eye behind the pointer, we can count the oscillations of the metal plate (or block) as shown in Fig-2.



Theory of Experiment

When a regular body is suspended from a rigid support by means of two wires that are not parallel and when the body is slightly drawn aside and released, it makes simple harmonic (motion) oscillations in a horizontal plane. If 'T' is the time period of oscillation, then

$$T = 2\pi\sqrt{\frac{Ih}{Mgd_1 d_2}}$$

$$I = \frac{Mg}{4\pi^2}\frac{T^2 d_1 d_2}{h}$$

All the terms in this equation are already explained in the Equation - 2 Formula.

Experimental Procedure

First of all the length (a) breadth (b) and thickness (t) of the metal plate are determine with a vernier cälipers. The mass of the plate (M) can be determined with a rough balance. Now, the moment of inertia of the metal plate about a vertical axis passing through its centre of mass is given by

$$I = \frac{M(a^2 + l^2)}{12}$$

Next, the metal plate is hung from the rigid support by means of metal wires from hooks at P and Q. The pin is attached to the face BCGF with wax in such a manner that, a part of the pin vertically protudes above BC. The pointer is placed in front of the needle and parallel to it. The plate is drawn aside through a small angle (<5^0) and is released. Now, the plate starts making oscillations. When the pin once again crosses the pinter in the same direction (from left to right) it is counted as one (oscillation completed). Thus 20 oscillations are counted and time taken for these twenty oscillation is noted from the stop clock.

The same procedure is repeated once again and time taken for 20 oscillations is counted for a second time. The averages of these two values is counted for a second time. The average of these two values is divided by 20 to get the time period T. Readings are entered in the tabular form.

The distances PQ (= $2d_1$) and RS (=$2d_2$) are measured quite accurately with a beam compass. The value of h is determined with a metre scale.

Readings are also entered in the tabular form.

The experiment can be repeated five or six times by changing the values of h - that is by changing the lengths of the wires.

Observations

(i) To determine the length (l), breadth (b) and thickness (t) of the plate.

On the main scale of the vernier calipers, 1 M.S.D. s = 0.1 cm

Total no. of divisions on the vernier n = 10

Least count of the vernier $l.c. = \frac{S}{n} = \frac{0.1\ cm}{10} = 0.01\ cm$

S.No.	Main scale reading (a) cm	Vernier coincidence (n)	Vernier measurement $(b) = n \times l.c = n \times 0.01\ cm$	Total reading (a+b) cm *length (l)*
1.				
2.				
3.				
4.				

Average length (l) of the plate l = ——— cm

Reading for breadth (b) and thickness (t) can be had in a similar tabular form form each.

(ii) To determine the moment of inertia (I) of the plate

Mass of the plate M = ——— gm; g = ——— cm/s²

Sl. No.	h cm	$2d_1$ cm	$2d_2$ cm	d_1 cm	d_2 cm	Time taken for 20 (t) oscillations (s) 1st time	2nd time(t) s	(s) Average	T (s²)	T² gm - cm²	$I = \frac{Mg}{4\pi^2} \times \frac{T^2 d_1 d_2}{h}$
1											
2											
3											
4											

Average value of the moment of inertia I = ... gm - cm² = ... kg-m²

If I_1 is the volue of moment of inertia in gm - cm², then its value in Kg-m² is given by

$$I_2 = I_1 \times 10^{-7}.$$

Result

The moment of inertia of the rectangular plate about a vertical axis passing through its centre of mass is

I = gm - cm²

= kg - m²

Precautions

The follow precautions are to be taken very carefully during the experiment.

1. The wires should be equal lengths (PR = QS)
2. The face ABCD and EFGH of the plate should be horizontal.
3. The oscillations of the plate should be confined to a single horizontal plane.
4. Amplitude of oscillation should be small ($<5^0$)
5. There should be no kincks in the wires.

Viva - Voce

1. What is the moment of inertia of a body? On what factors does this depend upon ?
2. A rigid body is rotating about a fixed axis. The mass of the body remains constant. The, does the moment of inertia depend on the shape of the body?
3. In the Bifilar suspension experiment, what are the points that are symmetrically situated?
4. Which physical quantity in this experiment should be measured more accurately? And why?
5. On what factors does the time period of oscillation of abifilar pendulum depend upon ?

Viva - Voce Answers

1. It is a measure of the inertia of a body in rotatory motion. It depends upon the axis of rotation mass of the body and also on the distribution of the mass about the axis.
2. Changes. As the shape of the body changes, the distribution of mass around the axis changes and hence the moment of inertia $I = \Sigma mr^2$ also changes.
3. The following points (pairs) should be symmetric about the vertical axis passing through the C.G of the body. They are (a) The points of suspension at the support (b) The lower ends of the threads.
4. The time period 'T' should be more accurately measured as it occurs in the second power (T^2) in the formula .
5. The time period of a bifilar suspension is given by

$T = 2\pi\sqrt{\frac{I\,h}{Mg\,d_1\,d_2}}$ and hence, it depends on (1). Moment of inertia of the body. (2) Mass of the body. (3) The distance between the wires at the points of suspension and (4) The distance between the wires at their lower points. (5) The vertical distance between the ends of each wire.

6. FLYWHEEL – DETERMINATION OF MOMENT OF INERTIA

Experiment No

Date

Aim To determine the moment of inertia of a fly wheel about an axis passing through its centre of mass.

Apparatus Fly wheel, sensitive stop clock, weights of known masses, twine thread, metre scale, vernier calipers.

Formula The moment of inertia of the fly wheel about an axis passing through its centre of mass is determined in the laboratory, using the following formula.

$$I = \frac{n_2}{(n_1 + n_2)}.m\left(\frac{2gh}{\omega^2} - r^2\right) gm - cm^2$$

and $\omega = \frac{4\pi n_2}{t}$

Here,

m = the known mass we have used (gm)

g = acceleration due to gravity (cm/s²)

h = the heightof the mass above the ground (cm)

n_1 = the number of complete rotations made by the fly wheel during the time interval through which the mass 'm' comes down and meets the ground.

= the number of windings of the twine thread around the axle.

n_2 = the number of rotations made by the fly wheel during the time interval - from the instant the mass touches the ground and gets detached) and the wheel comes to rest.

ω = the angular velocity acquired by the wheel after the mass touches the ground (and gets detached).

Description

The fly wheel is as shown in Fig-1. It is a circular wheel (W) of very large mass and is made of metal. This heavy circular wheel consists of a long axle. A passing through its centre of mass as shown in Fig-1 The axle is horizontal and rotates on ball bearing which are fixed in a metal frame as shown in figure. Due to its support on ball-bearings, the firction due to rotation of the axle is minimized. As the axle 'A' rotates, the wheel 'W' also rotates along with it. Both of them rotate together.

There is a peg 'p' on the axle. The end of a long twine thread is made into a loop and is passed over the peg. The twine thread is wound over the axle without overlap in circular successive circles. The second end of the twine thread is attached to a known mass 'm'. As shown in the Fig-1, the mass 'm' will initially be at rest at a height 'h' above the ground. The height 'h' and the length of the thread are so adjusted such that, when the mass 'm' descends and finally touches the ground, the end of the thread around the peg should get detached from the peg and fall down. By this arrangement we ensure that the number of windings of the thread on the axle is exactly equal to the number of rotations made (n_1) by the wheel during the descent of mass through height 'h'.

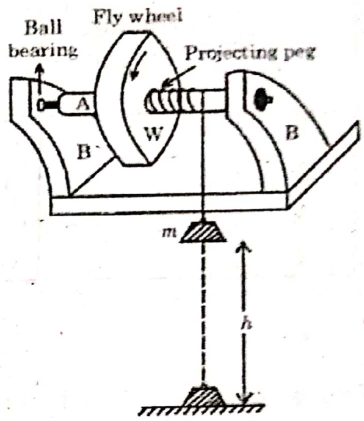


Fig - 1

To count the number of rotations made by the fly wheel, a pointer is fixed in ahorizontal position and infront of the fly wheel. (It is not shown in the figure). Just before the pointer, a horizontal line is drawn on the rim of the wheel. Every time, the mark (line) crosses the pointer, the wheel completes one revolution.

Let us wind the twine thread n_1 times without any overlap over the axle, so that the mass 'm' is stationary at a height 'h' above the ground as shown in the figure. If the wheel is now released to rotate, then the thread gets detached from the peg P and simultaneously the mass 'm' touches the ground after n_1 revolutions of the wheel. The number of revolutions made by the wheel during this time in interval will be n_1.

The number of revolution can also be counted from the horizontal mark on the rim and pointer arrangement also.

The stop clock is immediately started at the moment the mass touches the ground (and simultaneously the thread gets detached from the peg P) and the time taken (t) and number of revolutions (n_2) made by the wheel from this instant till the wheel getst stopped are noted down.

Experiment Procedure

First of all, the length of the twine thread and the height h are so adjusted such that just when the mass touches the ground, the thread gets detached from the peg P and falls down. A mass m = 200 gm is tied to the thread is wound without overlap n_1 times around the axle. The height 'h' of the mass 'm' above the ground is measured with a metre scale.

Now, the wheel is released to rotate and as the wheel rotates, the mass descends down. With the help of the horizontal line on the rim and the pointer, the number of revolutions (n_1) made by the wheel during the descent of the mass are counted.

Just as the mass touches the ground, the stop clock is started. The number of revolution made by the wheel from this instant upto complete stop page of revolutions (n_2) and the time taken (t) are noted.

$$\omega = \frac{4\pi n_2}{t} \text{ is calculated}$$

The entire procedure is repeated once again with the same 'h' and the time is noted and 'ω 'is found. Average value of ω is calculated.

The radius 'r' of the axle is determined with the vernier calipers. From (12) and (16) Equations I is calculated.

Now, the same experiment can be repeated by changing the mass in steps of 50 gm each time.

Observations

(i) Radius (r) of the axle with Vernier Calipers

1 Main scale division s = 0.01 cm

No. of Vernier divisions n = 10

Least count of the vernier $l.c. = \frac{s}{n} = \frac{0.1}{10} = 0.01 cm$,

S.No.	Main scale reading (a) cm	Vernier coincidence (n) n × 0.001 cm	Vernier measurement (b) = n× l.c= ter (d)	Total reading (a+b) cm = of axle diame
1.				
2.				
3.				
4.				

Average diameter of axle (d) = cm

Radius $r = \frac{d}{2} =$ cm

(ii)Rotations of the fly wheel

Height of the mass m above the ground = h cm = cm

radius of the axle = r cm cm

Result

The moment of inertia of the fly wheel about an axis passing through its centre of mass $I_2 = (I_1 \times 10^{-7})kg - m^2$

By noting down the mass of the wheel M and its radius R we can estimate its moment of inertia about an axis through its centre of mass as $I = \frac{MR^2}{2} =$ $gm - cm^2$

Precautions

The following precautions are to be carefully observed during the experiment.

1. The twine thread is to be wound around the axle without any overlap.
2. The moment the mass touches the ground, counting of n_2 and t should be simultaneously started.
3. The ball bearings should be oiled to avoid friction.

Viva-Voce

1. What is the use of a fly wheel ?
2. We mention that the thread is to be wound around the axle without overalp. What happens if there is an overlap?
3. Is the moment of inertia a scalar or a vector? or, is is something else?
4. On what factors does the moment of inertia depend on ?
5. What is the relation between moment of inertia and the radius of gyration?
6. Through which axis, the moment of inertia of a body will have the minimum value?
7. What is the equation in rotatory motion analogus $\vec{F} = m a$ in linear motion?
8. What is the relation between the moment of inertia (I) and angular momentum (J) ?
9. Where is the most part of the mass of fly wheel is concentrated - at its centre or at the rim ?
10. To increase the moment of inertia of a body about an axis passing through its centre, where should the mass be added the centre or at the rim ?

Viva - Voce Answers

1. The direction of motion can be specifically fixed.
2. If there is any overlap, the thread will not get detached from the and even after the mass touches the ground.
3. Moment of inertia is a Tensor.
4. The moment of inertia depends upon (1) The axis of rotation (Its position), (2) Mass of the body and (3) The distribution of the mass about the axis.
5. $I = MK^2$ with I = Moment of inertia, M = Mass of the body and k = Radius of gyration.
6. The moment of inertia of a body will be minimum about an axis passing through its centre of mass ($I_{C.M}$). About any parallel axis at a distance a from this, $I = I_{C.M.} + Ma^2$.
7. Torque $\vec{\tau} = I.\vec{\alpha}$; I = Moment of inertia and $\vec{\alpha}$ = Angular acceleration.
8. $\vec{J} = I\,\vec{\omega}$ ($\vec{\omega}$ = Angular Velocity).
9. Most part of the mass is concentrated at the rim of the wheel.
10. At the rim, as a increases Ma^2 and hence I increase.

7. RIGIDITY MODULUS OF MATERIAL OF A WIRE DYNAMIC METHOD (TORSIONAL PENDULUM)

Experiment No

Date

Aim **To determine the rigidty modulus (η) of the material of a given wire using torsional pendulum.**

Apparatus The wire in the arrangement of a torsional pendulum, pointer, sensitive stop clock, vernier calipers, screw gauge, metre scale.

Formula

Rigidity modulus (η) of the material of the wire made as a torsional pendulum is given by

$$\eta = \frac{4\pi MR^2}{a^4}\left(\frac{l}{T^2}\right)\frac{dynes}{cm^2} \quad \text{------ (1)}$$

Here M = Mass of the disc (gm)

R = Radius of the disc (cm)

l = length of the wire (cm)

a = radius of the wire (cm)

T = time period of oscillation of a torsional pendulum of length 'l' (seconds)

Units of $\eta \rightarrow \frac{dynes}{cm^2}$. By deviding n by 10 we get the value in $\left(\frac{n}{10}\right)\frac{newton}{m^2}$

Description

A torsional pendulum is as shown in Fig-1. It consists of a disc 'D' (usually of a circular shape) hung by means of a long wire of uniform cross section. The thickness of the disc 'D' will be very small compared toits diameter. The disc is suspended by the wire passing through its centre C and is tightly fixed by chucknut. The upper end of the wire is suspended from a rigid support by passing the wire through chuck nut as shown in the figure. The disc hangs in a horizontal position. The disc is usually made of a metal like brass. A vertical line mark is made on the side (thickness) of the disc and the pointer is placed vertically in front of this mark. We place our eye behind the pointer and count be oscillations of the disc by noting the crossing of the line mark. Keeping the wire in its position, if we draw the disc rotated through a small angle and release it, then due to the twist developed in the wire (torsion) the

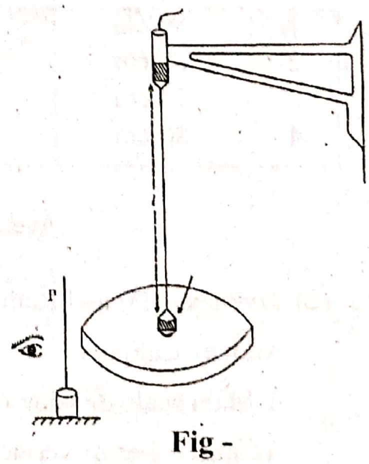


Fig -

disc will be making oscillations in a horizontal plane about the wire as its axis if ritation. These oscillations are torsional oscillations and hence the arrangement is called a torsional pendulum.

We leave out the first few oscillations. Then, as the line mark crosses the pointer from left to right we count it as zero (oscillation) and start the stop clock, When the line mark once again crosses the pointer once again in the same direction (from left to right) we count it as one (oscillation). In this manner we can count about 20 oscillations and find the time period (T).

Experimental Procedure

The disc D is supended by means of the given wire to form a torsional pendulum as shown in Fig-1. With the help of the ckuck nuts, we first keep the length of the wire $l = 50$ cm. A vertical line mark is drawn on the side (thickness) of the wire and we keep the pointer vertically infront of the line mark. Then we rotate the disc slightly so that the wire gets twisted (by $< 5^0$). Now we release the disc and allow it to make simple harmonic oscillations.

Observing the line mark from behind the pointer and with the help of the stop clock we note down the time taken for 20 oscillations.

Without changing the length, the experiment is once again repeated and the time taken for 20 oscillations is noted for a second time. The average value of these two is divided by 20 to get the time period of oscillation 'T'.

Next, the length of the wire is increased by 10 cm, that is $l = 60$ cm and the same procedure as above is repeated. In this way the experiment is repeated for 4 or 5 different lengths. The readings are entered in the tabular form.

Each time, the length l can be measured with a metre scale. The diameter of the disc is measured with a vernier calipers and the diameter of the wire is measured with a screw gauge. The mass M of the disc is found with a rough balance.

From the table, average value of $\frac{l}{T^2}$ is calculated.

A l - T^2 graph is drawn and $\frac{l}{T^2}$ value is found from the graph also

These value are substituted in formula (1) to get the rigidity modulus 'η' of the material of the given wire.

Observations

(1) l - T values and $\frac{l}{T^2}$ values.

Sl.No.	Lenght of the wire l cm	Time taken for 20 oscillations t (s)			Time period $T = \frac{t}{20}$ s	$\frac{l}{T^2}\left(\frac{cm}{s^2}\right)$
		1st time	2nd time	Average t(s)		
1.	50 cm					
2.	60 cm					
3.	70 cm					
4.	80 cm					

Average value of $\left(\frac{l}{T^2}\right) =$ cm/s²

(2) Diameter (D) and Radius (R) of the dis

Vernier Calipers

1 Main scale division on the calipers $S = 0.1$ cm

Total number of vernier divisions $n = 10$

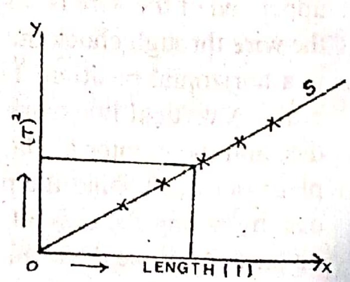


Fig.

∴ Least count of the vernier $l.c. = \frac{S}{n} = \frac{0.1cm}{10} = 0.01cm$

S.No.	Main scale reading (a) cm	Vernier coincidence 'n'	Vernier measurement (b) cm= n× l.c = n × 0.01 cm	Total reading (a+b) cm = Diameter of the disc
1.				
2.				
3.				
4.				

Average diameter of the disc D = cm

Radius $R = \frac{D}{2} =$ cm

(3) Diameter (d) and radius (a) of the wire-Screw Gauge

Pitch of the screw = 0.1 cm

No. of divisions on head scale = 100

Least count (l.c) of screw gauge= $\frac{Pitch\ of\ the\ screw}{No.of\ head\ scale\ divisions} = \frac{0.1cm}{100} = 0.001cm$

Correction for the zero error = ———————— Head scale divisions.

Sl.No.	Pinch scale reading a (cm)	Head scale Reading		Head scale measurement bcm = n × lc n × 0.001 cm	Total reading (a+b) cm = Diameter of wire
		Observed	Corrected (n)		
(1)					
(2)					
(3)					
(4)					
(5)					
(6)					

Average diameter of wire d = cm

Radius $a = \frac{d}{2} =$ cm

(4) Mass of the disc M = gm (with a rough balance)

Average value of $\frac{l}{T^2} =$ cm/s²

Mass of the disc M = gm

Average radius of the disc R = cm

Average radius of the wire a= cm

Result Rigidity modules of the material of the wire

$$\eta = \frac{4\pi MR^2}{a^4}\left(\frac{l}{T^2}\right) dynes/cm^2$$

$$\eta = \left(\frac{n}{10}\right) =$$ newton/m²

Precautions :

The following precautions are to be carefully observed during the experiment.

1. There should be no kinks any where is the wire.
2. The diameter of the wire is to be carefully measured with a screw gauge - atleast at six different places - and the average is to be calculated.
3. The diameter of the disc should also be carefully measured with vernier calipers atleast at four different places and the average is to be calculated.
4. The disc should oscillate in the same horizontal plane always. It should no wobble up and down. It should not oscillate to and for as a simple pendulum.
5. The angular amplitude of oscillation should be small ($<5^0$).

Viva -Voce

1. What is rigidity modulus ?
2. In this experiment, which physical quantity should be more accurately determined and why ?
3. What is the meaning in calling this a pendulum?
4. How does the time period change when the length of the wire in torsional pendulum is increased ?
5. How should be the amplitude of torsional oscillations ?
6. If the mass of the disc increases, how does the time period of torsional oscillations change ?
7. If the diameter of the disc increases, how does the time period of torsional oscillations change ?
8. If the thickness (diameter) of the wire increases, how does the time period of torsional oscillations change?
9. What is the difference between a simple pendulum and a torsional pendulum? Explain in terms of restoring forces and torques.

Vova-Voce Answers

1. When tangential surface forces and applied on a body, the successive layers of the material are moved or sheared. This type of strain is called shearing strain.

   The ratio of shearing stress to shearing strain in called Rigidity modulus.

2. $\eta = \frac{4\pi MR^2}{a^4}\left(\frac{l}{T^2}\right)$ has a in its fourth power (as a^4). Hence, the radius of the wire should be measured more accurately.

3. Here, the disc is also making oscillations - Torsional oscillations around a vertical axis passing through its centre of mass (about the wire) - and hence the arrangement is called a torsional pendulum.

4. If l is increased, T will also increasing as per $\frac{l}{T^2}$ = a constant.

5. Should be small, less than 5^0.

6. If mass (M) of the disc is increased, time period (T) also increases according to $\frac{M}{T^2}$ = a constant.

7. If diameter (D = 2R) of the disc is increased, time period (T) also increases to $\frac{R}{T}$ = a constant.

8. If the thickness (or diameter or radius 'a') of the wire is increased, time period decreases as per a^4T^2 = a constant.

9. In a simple pendulum the S.H.M is due to the restoring force which is the component of the weight of the job. In torsional pendulum the S.H.M. is due to the restoring couple arising out of torsion and shearing strain.

8. THE VOLUME RESONATOR

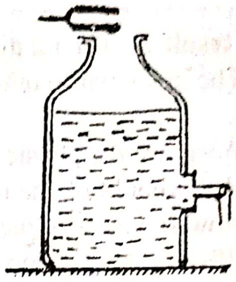
Experiment
Date

Aim :To verify the relation between the volume of an air cavity and the frequency of the note producing resonance in it.

Apparatus : Aspirator bottle, tuning forks of difference known frequencies and a measuring jar.

Description : The volume reisonator consists of an aspirator bottle of 1 to 2 litre - capacity, having opening in its side near bottom.

It is fitted with a one holed rubber into which a short glass tube is inserted. A rubber tube is connected to the glass tube and a pinch cock is attached to it. When the aspirator is filled with water, a required amount of water may be drawn out by opening the pinch cock.



Fig

Procedure : The natural frequency of the air in the resonator is given by

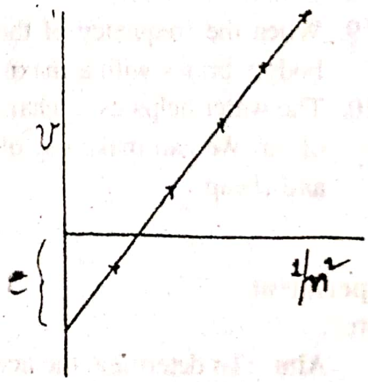
$$n = \frac{V}{2\pi}\sqrt{\frac{A}{vL}}$$

Where V is the velocity of sound in air, v is the volume of the vibrating air upto the neck of the bottle and L is the length of the neck. A is the cross sectional area of the neck. Since V, A and L are constants.

$$n \,\alpha\, \frac{1}{\sqrt{v}}$$

If 'e' is the end correction at the mouth of the bottle, $n^2 (v + e)$ = constant. This relation is verified experimentally.

The pinch cock is closed and the aspirator bottle is filled with water upto its neck. One of the tuning fork is taken and it is excited by the hammer. The tuning fork is held above the neck of the aspirator bottle without the prongs of the fork touching neck and water in the aspirator bottle is slowly let out and collected in a jar, by opening the pinch cock. When the volume of air inside reaches a particular value a sharp loud note or resonance is produced. Then the pinch cock is closed and the volume of air in the aspirator is found by measuring the water that flowed out with a measuring jar. The experiment is repeated twice with the same tuning fork and the mean volume (v) of the air in the bottle resonating with a tuning fork of known frequency is found.



The experimental is repeated with three tuning forks of different frequencies. The results are tabulated as shown in tabulatar form.

The experiment is repeated with there unknown tuning fork and volume of water is found.

A graph is plotted taking v on y axis and $1/n^2$ on x - axis. The negative intercept on y-axis gives the end correction.

From the graph the unknown frequency of a tuning fork can be determined by measuring volume with the fork.

Precautions : 1. The tuning fork should be held with its prongs horizontal.

2. The position of maximum sound should be noted carefully.

Observations

S.No.	Frequency of tuning fork (n)	Air resonating (v)			n^2v	n^2 (v+e)
		Trial I	Trial II	Mean		

Result : The relation between natural frequency and its volume is verified.
The unknown frequency of tuning fork = _______ Hz

Viva - Voce

1. What are the physical quantities that are in resonance in this experiment ?
2. How should be the neck of the vessel used in volume resonator experiment ?
3. How should be tuning fork (in vibration) should be kept on the resonator - horizontal or vertical ?
4. What is the end correction and why is it necessary ?
5. Is it possible to find the end correction with the observation of n and V without drawing the graph?
6. In the resonating air column experiment we get two resonating lengths (l_1 and l_2) with the same tuning fork. Is there any possibility in the volume resonator experiment also to get two resonating volumes for the same frequency ?
7. Why should we hold the tuning fork at the stem only, while it is vibrating ?
8. What kind of motion does the air in the neck of the volume resonator exhibit ?
9. What is resonance ?
10. What purpose does the water serve in this experiment ? Can we use any other liquid ?

Viva - Voce Answers

1. Volume of air in the bottle and the tuning fork.
2. The neck should be narrow and have a finite length.
3. The length of the prong of the fork should be horizontal.
4. As we keep the prong a little above the neck of the vessel.
5. $n_1^2 (V_1 + e) = n_2^2 (V_2 + e)$

   $n_1^2 V_1 - n_2^2 V_2 = (n_2^2 - n_1^2) e$

   and $e = \dfrac{n_1^2 V_1 - n_2^2 V_2}{(n_2^2 - n_1^2)}$ can be determined.
6. No.
7. If we hold at the prongs, immediately the vibrations of the two prongs come to rest.
8. Simple harmonic.
9. When the frequency of the external driving force equals the natural frequency of the body, the body vibrates with a maximum amplitude and this called the Resonance.
10. The water helps us in changing the volume of air in the vessel and also in determining the volume of air. We can make use of any other liquid instead of water. However water is available readily and cheap.

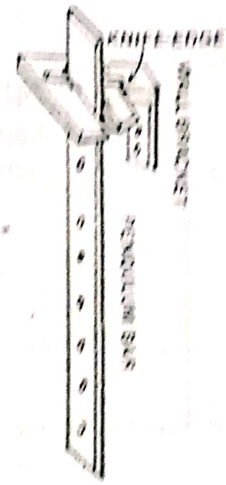
9. COMPOUND PENDULUM

Experiment
Date

Aim :To determine the acceleration due to gravity (g) using a compound pendulum.

Apparatus : The compound pendulum, stop watch knife edge, metre scale and a telescope.

Description : The compound pendulum consists of a uniform rectangular bar made up of iron or brass with a number of holes drilled along its length at equal distances, symmetrically on either side of C.G. The pendulum can be suspended vertically by means of a horizontal knife edge passing through one of the holes.



Procedure : The pendulum is suspended vertically by passing a horizontal knife edge through the hole near one end. A pin is fixed by wax vertically at the lower end of the pendulum. A telescope is arranged about $1\frac{1}{2}$ metre from the pendulum and the pin is focussed. The distance between one end of the pendulum and the knife edge is measured. Pin is taken as reference point for counting oscillations.

The pendulum is drawn to a side through a small distance and released so that it oscillates with small ampitude in the vertical plane without any wobbling. Looking through the telescope, with the help of the pin at the end of pendulum, time for 20 oscillation is found twice and the mean time period T is calculated.

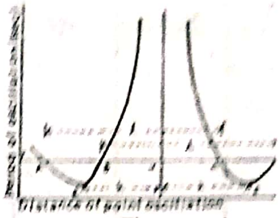


Fig.

This process is repeated by suspending the pendulum from successive holes and in each case the period T and the distance of the hole from the same end are measured. On approaching the C.G. the period becomes very large. The pendulum is then reversed and the experiment is repeated by suspending the pendulum from each hole, till the other end is reached.

It should be noted that even after eversal, the distance of the knife edge should be measured from the same end.

A graph is drawn taking the distance of the hole from one end on the x-axis and the corresponding time period (1) on the y-axis. The graph is as shown as fig. The graph consists of two symmetrical curves corresponding to the two halves of the bar.

A line is drawn parallel to the x -axis cutting the curves and four points A, B, C, D. The period is same for all these points. The points A, C and B, D are two sets of interchangeable points about which T is same. Hence AC or BD gives the length of an equivalent simple pendulum l. Actually the mean of AC and BD is taken as 'l'.

In the same way two or three more lines are drawn parallel to the x-axis cutting the graph. In each case the length of the equivalent simple pendulum (l) and the corresponding time period (T) are noted. The readings are tabulated in the tabular form (2). A second graph is drawn taking the value of 'l' on the x-axis and the corresponding values of (T) on the y-axis. The graph is a straight line passing through the origin as shown fig.

From the graph $\frac{l}{T^2}$, value is calculated.

The acceleration due to gravity is calculated by substituting the value of $\frac{l}{T^2}$ obtained either from the graph or from calculations of the relation.

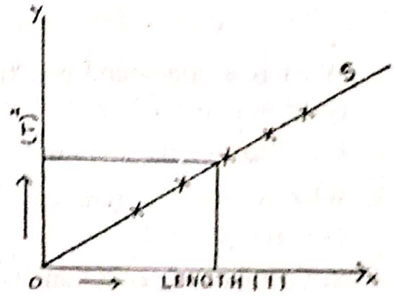


Fig.

$$g = 4\pi^2 \frac{l}{T^2}$$

Precautions : 1. The pendulum must be oscillated in the vertical plane with small amplitude and without any wobbling.

2. The knife edge should be horizontal.

Result : Acceleraton due to gravity (g) = ... cm/sec2

Observations

Table – 1

S.No.	Distance of the knife edge from one end	Time for 20 oscillations			Period (T)
		Trial I	Trial II	Mean	

Table – II

S.No.	AC	BD	Length of the equivalent simple pendulum $l = \frac{AC+BD}{2}$	T	T^2	$\frac{l}{T^2}$

Mean = $\frac{l}{T^2}$

Substituting mean value of $\frac{L}{T^2}$ in the equation $g = 4\pi^2 \frac{l}{T^2}$ g can be calculated.

Note : To determine radius of gyration (k) of the pendulum about an axis through its centre of gravity ⊥ er to the broad face of the bar.

Let E be the point where the A B C D cuts the ordinate through 'G', If AE = h_1 and EC = h_2, then

$k = \sqrt{h_1 h_2}$

Also if E and F are the points on the curves corresponding to minimum period, then $k = \frac{EF}{2}$. Thus K can be determine.

Viva - Voce

1. What is a compound pendulum ? What are the differences between a simple pendulum and a compound pendulum ?
2. How shall be the amplitude of the pendulum ?
3. What is radius of gyration ? Does it depend upon (a) The mass of the body (b) The position of the axis of rotation ?
4. In a compound pendulum, through how many points the axis of rotation can pass through such that the time period will be the same ?
5. What is meant by "Equivalent Simple Pendulum".
6. In a compound pendulum we draw a graph between '*d*' and '*T*'. Where from the values of '*d*' are measured ? When the pendulum is inverted, where from the values of '*d*' are measured ?
7. The time period of a compound bendulum becomes infinity when the axis of rotation passes through a certain point in the pendulum. What is that particular point ?

8. The two branches of the d - T will be symmetric about a line. What is that line ?
9. Explain the point of suspension and point of oscillation of a pendulum. What is the distance between these two points in (a) A simple pendulum (b) A compound pendulum ?
10. What is the difference between centre of gravity and centre of mass of a body ?
11. What are the reasons for the change of 'g' value at different places on earth ? Where will it share maximum value and where a minimum value ?
12. What is the value of 'g' at the centre of the earth and why is it so ?

Viva - Voce Answers

1. A rigid body making oscillations in a vertical plane, about an axis passing through it and perpendicular to the plane is called a compound pendulum.

   In a simple pendulum we assume that the entire mass is concentrated at the centre of oscillation. In a simple pendulum the oscillations are possible only about one centre of suspension.
2. Very small, less than 5^0.
3. If the moment of inertia of a rigid body about a given axis is I and mass of the body is M, then from $I = Mk^2$ the radius of gyration k is defined by $k = \sqrt{\frac{I}{M}}$. We assume that the entire mass M is situated at a distance 'k' from the axis of rotation.

   k depends upon (a) mass of the body and (b) position of the axis of rotation.
4. There are four such points P, Q, R and S.
5. In the above figure $PR = QS$. If we take a simple pendulum of length $l = PR = QS$, it will have the same time period as the compound pendulum oscillating about any axis passing through P, Q, R and S. This length l is called the length of an equivalent simple pendulum.
6. Distance (d) is always measure from only one end A of the compound pendulum.

   Even when the bas is inverted the distances are measured only from the same end A.
7. About an axis passing through the centre of gravity (C.G.) the time period of compound pendulum will be infinity.
8. About a straight line passing through the point G (where G is centre of mass) at a distance of AG from A and parallel to the T axis.
9. The point from which the pendulum is suspended, or the point through which the axis of rotation passes is called centre of suspension.

   The centre of gravity of the pendulum itself is the centre of oscillation. The distance between these two points is (a) In simple pendulum $l = \frac{gT^2}{4\pi^2}$

   and (b) In compound pendulum is calculated from $T = 2\pi\sqrt{\frac{l + \left(\frac{k^2}{l}\right)}{g}}$
10. Centre of gravity (C.G) is the point through which the resultant of all the gravitational forces of attraction on the body (that is, the weight) acts.

    When a body is acted on by external forces, we can describe the motion - that is its velocity and acceleration - of the body by assuming that the entire mass of the body is concentrated at a point at which the resultant force acts. That point is called the centre Mass (C.M). The translational, rotational and vibrational motions of the body can be conveniently treated with the concept of C.M.

    Either the C.G. or the C.M need not be necessarily situated inside the body.
11. The variation of 'g' on earth is due to the following reasons

(1) The account of mass of the earth attracting the body changes or the distance between the earth and the body changes. (variation of g due to depth and height) (2) The rotation of earth around itself.

The value of 'g' is maximum at the poles and is minimum at the equator.

12. Value of 'g' at the centre of the earth is zero. This is because, there is no mass at the centre.

10. ESTIMATION OF ERROR, GAUSSIAN DISTRIBUTION APPLIED TO SIMPLE PENDULUM

Experiment

Date

Aim :To estimate error and draw the Gaussian distribution curves for the various measurements involved in simple pendulum. Then to calculate the acceleration due to gravity.

Apparatus : A metal bob, weigh less in extensible thread, retort stand, stop clock, cork etc.

Theory : Observations must be taken with utmost care to avoid mistakes and systematic errors like parallax error. Even then some errors will creep into the observations which are to be estimated. Suppose the radius of a simple pendulum bob is measured 10 or 12 times with calipers and the average value of which is found to be 1 cm (say). Let the maximum and minimum values be 0.97 and 1.03 cms. Then the radius of the bob be 1 $\pm$ 0.03 cm. Then the percentage of error in the above observation will be $\frac{0.03}{1} \times 100 = 3\%$.

A physical quantity in general is expressed in the terms of probable error (r). Then a $\pm$ r gives the measure of a quantity a where 'a' is the real or average of the observations 'r' can be calculated from the average error η, by the following formula.

$$r = \pm\ 0.8453\,\eta \qquad \text{.......(1)}$$

The probable error can be calculated from the R.M.S. value of error μ, by the following formula.

$$r = \pm\ 0.6745\,\mu \qquad \text{.......(2)}$$

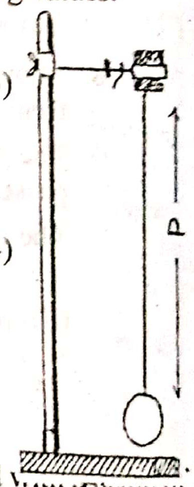
When the number of observations are less η and μ are estimated by the following values.

$$\eta = \frac{\pm\sum|x|}{n(n-1)} \qquad \text{.......(3)}$$

n – number of observations

$$\mu = \pm\sqrt{\frac{\sum x^2}{\sqrt{n(n-1)}}} \qquad \text{.......(4)}$$

μ is also known as standard error or standard deviation.



Procedure : The diameter of the bob of a pendulum is measured by vernier calipers at different positions. A Gaussian distribution curve is plotted. The radius of the bob is estimated. The values are plotted n tables 1and 2.

Simple pendulum is arranged as shown in fig. The length of the thread is measured w... Fig... least count 0.1 cm. The length of the thread is arranged at 40 cm. Then the length of pendulum is given by (40+r), where r is the radius or the bob. Length must be adjusted according to the significant figures. A pin is arranged vertically on the bob and focussed by a telescope. Then the bob is made to vibrate with small amplitude. Time taken for 20 oscillations are to be noted by a step clock of least count 0.2 sec (say).

The counting is started as the pin on the bob crosses the vertical cross wire and is counted as one oscillation when the pin crosses the cross wire in the same direction again and the time taken for 20 such oscillations are measured. Then time period is calculated.

The length of the pendulum is increased in steps of 5 cm. Then time taken for 20 oscillations and time period at each length is calculated. The values are adjusted according to the significant figures. The values or tabulated in the table 3.

l/T^2 value is calculated from the observations.

The maximum error fraction $\left(\frac{\delta L}{l} + \frac{2\delta T}{T}\right)$ = X s estimated.

From each observation maximum error fraction is calculated

$\delta\ (l/T^2) = x\ (l/T^2) = x^l$

Where (l/T^2) is the average value.

The maximum error in that observation = $2x^1$

The observation in which (l/T^2) is greater or less then the average value of (l/T^2) by $2x^1$. Then R.M.S. error is calculated by the formula

$$\mu = \pm\sqrt{\frac{\sum x^2}{\sqrt{n\,(n-1)}}}$$

Then the probable error is estimated by the formula.

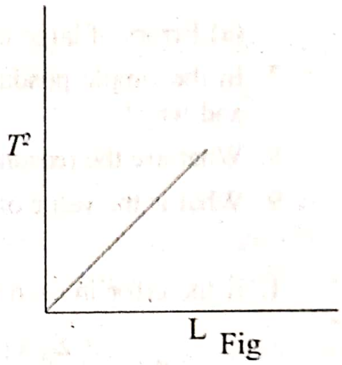
$r = 0.6745\ \mu$

Then the standard value of l/T^2 is given by $(l/T^{2*} \pm r)$

Then the acceleration due to gravity is calculated by the formula

$g = 4\ \pi^2\ (l/T^2 \pm r)$ m/sec²

$l - T^2$ graph is plotted as shown in the figure



Fig

The Gaussian distribution curve is drawn for l/T^2 values.

Result : Acceleration due to gravity g = ... m/sec².

Table – 1

S.No.	M.S.R (A)	Vernier Coincidence (n)	V.S.R. (B) = n x LC	Diameter D = A + nxL.C	radius (r) r = D/2
1					
2					
3					
4					
5					
6					
7					

Average radius r = cm

Table – 2

S.No.	Length of the pendulum (l) cm	Time for 20 oscillations			Time period $T = \frac{t}{20}$ sec	$\frac{l}{T^2}$
		Trial 1	Trial 2	Average (t)		
1						
2						
3						
4						
5						
6						
7						

Note : A table 4 can be prepared similar to table 2 for error in $\left(\frac{l}{T^2}\right)$ and gaussian curve can be drawn.

Viva - Voce

1. What is average error ?
2. What is root mean square error ?
3. What is probable error ?
4. What is Gaussian or Normal distribution ?
5. What is the maximum possible probable error in the simple pendulum formula $g = 4\pi^2\left(\frac{l}{T^2}\right)$
6. When a large number of measurements are taken, which kind of errors will be in large number —

   (a) Errors of large magnitude or (b) Errors of small magnitude.
7. In the simple pendulum experiment, which physical quantity is to be measured more accurately and why?
8. What are the reasons for the change in value of 'g' on earth ?
9. What is the value of 'g' at the centre of the earth ? and why ?

Viva - Voce Answers

1. If the error in each measurement is x, then when are take n different measurements, the average error $\eta = \frac{\pm\Sigma|x|}{\sqrt{n(n-1)}}$
2. When we take n different measurements and the error in each measurement is x, then R.M.S. error $\mu = \frac{\pm\sqrt{\Sigma x^2}}{\sqrt{\sqrt{n(n-1)}}}$.
3. Probable error $r = \pm(0.6745)\ \mu$
4. If dx is the number of errors in the measurements having errors in the range x and $x + dx$ then the graph or distribution drawn between x and N following the equation $dx = Ae^{-h^2x^2}\, dx$ is called Gaussian distribution.
5. $\frac{\Delta l}{l} + 2\left(\frac{\Delta T}{T}\right)$

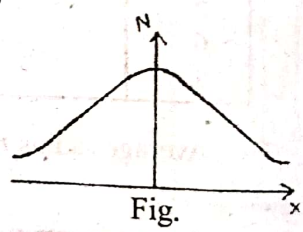


Fig.

6. Small in number.
7. Time should be measured with more accuracy as it occurs in second order T^2 in the formula for $g = 4\pi^2\frac{l}{T^2}$
8. The variation of 'g' on earth is due to the following reasons.

   (i) The amount of mass of the earth attracting the body changes or, the distance between the earth and the box changes. (Variation of g due to depth and height)

   (ii) The rotation of earth around itself.

   The value of 'g' is minimum at the poles and is minimum at the equator.
9. Value of 'g' at the centre of the earth is zero. This is because, there is no mass at the centre.

11. FORCE CONSTANT BY STATIC AND DYNAMIC METHOD

Experiment

Date

Aim : To determine the force constant or spring constant by static and dynamic method using a given spring.

Apparatus : Stretchable spring, weight hanger, clamping stand, weighing balance (or) electronic balance, measuring scale, stoplock, weights.

Formula : (a) Spring constant (or) force constant formula for dynamic method is given by

$$K = \frac{4\pi^2}{(Slop\ of\ graph\ b/n\ M\ and\ T^2)}$$

Where M = mass of spring + mass attached to the spring = $m + m_s$

m_s = Mass of the spring, m is mass attached to the spring

T = Time period of oscillations.

(b) Spring constant (or) force constant formula for static method is given by

$$K = \frac{M\,g}{Slop\ of\ graph\ between\ M\ and\ e}$$

Where M = effective mass of spring = $m + m_s$

e = elongation of the spring

g = Acceleration due to gravity. N/m²

Theory : A spring is an elastic object which stores mechanical energy. This spring has elastic nature which elongates by attaching suitable (or) variable mass to them. The spring constant (or) force constant K of an ideal spring is defined as the force per with length of spring. Force constant (or) spring constant varies from one spring to another. Force constant can be determined both in static (motionless) as well as dynamic (in motion) conditions.

Static method : In this method The spring is fixed at a clamping stand at its one end and mass (or) weight is added on the other end of the spring. Weights are added in equal amounts one by one, the extension is produced in the length of the spring. Entension in the spring is noted using measuring scale. After adding a suitable amounts of weights, the spring will attain a stationary position after some time. At equilibrium there are two equal and opposite force, acting upward and down ward.

i.e At equilibrium upward force = down ward force

fup = k.e

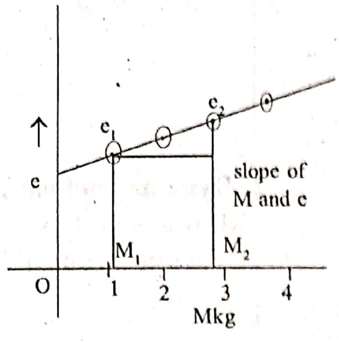
Where e is entension in the length of the spring

fdp = Mg (i.e gravitational pull)

Where M is the effective mass of the spring

$\therefore$ ke = Mg

$$k = \frac{Mg}{e}$$



Draw a graph by taking effective mass of spring (M) on X-axis and elongation 'e' on Y-axis we get a straight line graph as show in fig.

$$\text{Slop of graph} = \frac{e_2 - e_1}{M_2 - M_2}\ N/m$$

Now substitute slop of graph in the k formula we obtain spring constant.

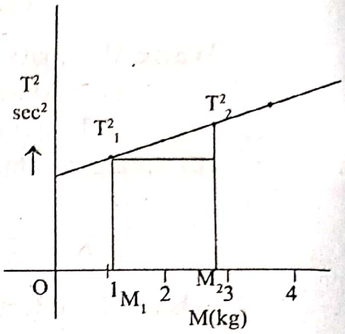
Dynamic method : If the spring is made to oscillate by pulling the weight applied to it down wards, it executes a simple harmonic motion. Basing on this. In this method, the spring is fixed at a clamping stand at its one end and mass (or) weight is added on the other end of the spring oscillates the spring by pulling the weight down ward, then it set up at simple harmonic motion. The time period of the oscillations of spring is

$$T = 2\pi\sqrt{\frac{M}{K}} \qquad T = 2\pi\sqrt{\frac{m+m_s}{K}}$$

$$\text{Now } T^2 = 4\pi^2\frac{(m+m_s)}{K} \quad \text{(or)} \quad k = \frac{4\pi^2}{T^2}(m+m_s)$$

Now using stopclock estimate the time period for 20 oscillation by taking no.of trails. The time period for oscillation noted by changing different weights on the spring.

Now draw a graph b/n M an X-axis and T^2 value on Y-axis weget a straight line



$$\text{Slope of the straight line is} = \frac{T_2^2 - T_1^2}{M_2 - M_1}\ \frac{Sec^2}{kg}$$

From above we calculate the force constant of the spring.

Precautions :

1. Make sure that spring does not have any knickns.
2. note down the correct measurement of extension (e) on the spring.
3. Note down the time period of oscillations without any delay.
4. Make sure that the spring is extensible or not.

Result : Force constant (or) spring constant of the spring by

Static method (K) = ... N/m.

and dynamic method (k) = ... N/m.

Tabular form :

1. Static method :

Mass attached to spring in (grams)	extension of the spring when it is loaded with mass (n)	Length of the spring when nomass is loaded(y)	extension of the spring e = (x–y)

2. Dynamic method :

Mass attached to the spring (grams)	Time for 20 oscillation			Time period T = t/20 (sec)	T^2 (sec²)
	Trail I (sec)	Trail II (sec)	Mean t (sec)		

Viva - Voce

1. Does the force constant of a spring depend upon any other physical quantity, other than the material ? If so, how does it depend ?
2. If we have to consider the mass of the spring ms also then how does the formula for time period

change? Hint : $T = 2\pi\sqrt{\dfrac{m + \left(\dfrac{m_s}{3}\right)}{k}}$

3. Are the motions of spring combinations also simple harmonic or not ?
4. What is meant by elasticity ? What is plasticity ?
5. What is Hooke's law ?
6. What are the reasons for the change in value of 'g' on earth ?
7. What is the value of 'g' at the centre of the earth ? and why ?

Viva - Voce Answers

1. Force constant depends on the length also as $k \alpha \frac{1}{l}$
2. Time period $T = 2\pi\sqrt{\dfrac{m + \left(\dfrac{m_s}{3}\right)}{k}}$ increases
3. The motions of combinations of springs are also simple harmonic.
4. The property of a body by virtue of which its deformation is resisted and the body regains its original size and shape after the deformation force is removed is called elasticity.
5. Within the proportionality limit $\dfrac{stress}{strain}$ = a constant. This is called Hooke's Law. The constant of proportionality is called the coefficient of elasticity.
6. The variation of 'g' on earth is due to the following reasons.

   (i) The amount of mass of the earth attracting the body changes, or the distance between the earth and the body changes. (Variation of g due to depth and height)

   (ii) The rotation of earth around itself. The value of 'g' is minimum at the poles and is minimum at the equator.
7. Value of 'g' at the centre of the earth is zero. This is because, there is no mass at the centre.

12. COUPLED OSCILLATORS

Experiment

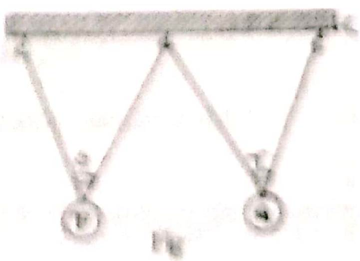
Date

Aim :To determine stiffness of coupling parallel and antiparallel angular frequencies for a coupled oscillator at different coupling constants.

Apparatus : Wooden block with 3 hooks two metallic ball of same mass provide with hooks, stop clock, scale, string etc

Description : One end of the string of about 250 cms length is tied to the hook. 'A' pass the thread through the ring of the sphere or bob of 'P', there it is passed through the hook B, then through the ring of the bob 'Q' and finally it is fixed to through the string. P and Q are adjusted so that they are in the same horizontal level

R and S are two thread knots which fix the position of P and Q. The knot U is below the hook B and it can be moved up and down. The position of U will be fixed unless it is altered



Fig

Procedure : The masses of the bobs are found out by simple balance, ($m_1 = m_2$) The position of the knot U is fixed just below the hook B. The bob 'p' is allowed to oscillate with small amplitude perpendicular to the plane of the strings while Q is at rest. Time taken for twenty oscillations is found out. Then time period T_1 is calculated. Then angular frequency of the bob P, is given by

$$\omega_1 = \frac{2\pi}{T_1}$$

Now keep P at rest and allow the bob 'q' to oscillate in a plane perpendicular to the plane of the strings. Time taken for 20 oscillations and then time period. T_2 of the bob Q is calculated. Then angular frequency of the bob Q is given by

$$\omega_2 = \frac{2\pi}{T_2}$$

It will be found that $\omega_1 = \omega_2$. From the theory the ngular frequency of the parallel oscillations ω_0 will be given by,

$$\omega_0 = \frac{\omega_1 + \omega_2}{2}$$

Bring down the knot 'U' by 4 cm (d) from B. Displace both the bobs through same distance in the same direction normal to the plane containing the strings, then release. The both bobs will oscillate and they will be in the same phase (parallel oscillations). Time taken for 50 oscillations of any bob found out then time perod T_3 is calculated. Then the angular frequency of parallel oscillations $\omega\, n_1$ is given by

$$\omega_{n_1} = \frac{2\pi}{T_3}$$

Displace the bobs P are Q by equal distances parallel to each other in the opposite directions. When they are released they execute antiparallel oscillation in the plane normal to the plane contained by strings in rest position. Time taken for 50 oscillations of any bob is observed and the time period T_4 is calculated. Then angular frequency ωn_2 of antiparallel oscillation is given by

$$\omega_{n_2} = \frac{2\pi}{T_4}$$

Keep one of the bob (say Q) at rest displace P and release both the bobs. Then the amplitude of P gradually decreases, finally comes to rest while the amplitude of Q gradually increases and becomes maximum. Then the amplitude of Q decreases while that of P increases and the process goes on repeating. Determine the time lapsed between the two successive instances of the same bob coming to rest. Let T_5 be the time taken then beat frequency ω is given by

$$\omega = \frac{2\pi}{T_5}$$

Repeat the observations for determining $\omega_{n_1}, \omega_{n_2}$ and ω for d = 8, 12, 16, 20, 24cms. According to theory $\omega = \dfrac{\omega_{n_2} - \omega_{n_2}}{2}$ and verify it.

The coupling constant S at each set is to be calculated by the formula.

$$S = \frac{m}{2}\left(\omega^2_{n_2} - \omega^2_{n_1}\right)$$

All the observations are tabulated in tabular columns.

Observations

d cm	4 cm	8 cm	12 cm	16 cm	20 cm	24 cm
T_3						
ω_{n_1}						
T_4						
ω_{n_2}						
T_5						
ω						
$\frac{\omega_{n_1} - \omega_{n_1}}{2}$						
$S = \frac{m(\omega^2_{n_1} - \omega^2_{n_2})}{2}$						

Result : Stiffness constant at various distance (d) =

Precautions : 1. Amplitude of the oscillations must be small.

2. Time period is observed accurately using telescope.
3. The hooks A, B, C must be arranged symetrically.

Viva - Voce

1. What is the coupled oscillator?
2. What do you understand by normal modes of a coupled oscillator?
3. What do you mean by electronic oscillator?
4. How does the coupled pendulum works?
5. What does coupling means ?

Viva - Voce Answers

1. Coupled oscillators are oscillators connected in such a way that energy can be transofrmed between them.
2. A normal mode of an oscillation of system is in the motion, in which all parts of the system move sinusoidally with in the same frequency and with a fixed phase relation.
3. An elctronic oscillator is an electronic circuit, that produces a periodic oscillating electronic signal, often a sine wave (or) a square wave.
4. The two pendulum that can exchange energy called coupled pendulums. The gravitational force acting on the pendulum creates rotational stiffness that derive each pendulum to return its original position.
5. Coupling is connection between two oscillating systems.

13. VERIFICATION OF LAWS OF VIBRATIONS OF STRETCHED STRING - SONOMETER

Experiment

Date

Aim : To verify the laws of transverse vibrations of stretched strings and b to determine the unknown frequency of a turning fork by sonometer.

Apparatus : Sonometer, turning forks with different frequencies, weight hanger with suitable hanging weights a cork hammer, balance and weight box.

Description : The sonometer consists of a hollow, wooden sounding box and of length more than one meter. On the top of the box and at each near and a wooden prismatic bridge with a metallic edge is fixed. A short peg is fixed at one end at the top of the box to which one end of a string (or wire)

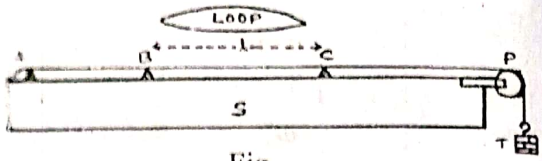


Fig.

is attached while the other end of the string passes over a pulley supporting a weight hanger. By adding a suitable load to the weight hanger, the string can be kept under suitable tension. Between the fixed bride at the ends of the sonometer one or two movable bridges are placed under the string so that the length of the vibrating segment of the string can be adjusted.

Procedure : a) Verification of I law : According to the first law, the frequency of vibration of a stretched string is inversely proportional to its length provided the tension and the mass per unit length remain constant.

Thus if a string of mass per unit length (m) is kept at a constant tension (T), it follows that n $\alpha \frac{1}{l}$

or, nl = constant, where n is the frequency of vibration of the string and l its length.

To verify the I law, the string of the sonometer is kept under a suitable tension. A tuning fork of known frequency is excited by the hammer and its shank is placed on the base of the sounding box. A light paper rider in the form of V shape is placed centrally on the string between the fixed bridge and the movable bridge. The length of the wire is adjusted until the paper rider flutters vigorously and falls down. In this position the frequency of the vibrating segment of the string between the fixed bridge and movable bridge is equal to the frequency of the tuning fork. The length l of the vibrating segment of the string is measured.

Keeping the tension constant and using the same wire, the experiment is repeated with different tuning forks and the results are tabulated as shown in tabular form (1) The first law is verified by showing that $n \times l$ is constant.

b) Verification of II law : According to this law frequency of the vibrating segment of a string is directly proportional to the square root of the tension provided the length of the string and its mass per unit length are constant.

Or, n $\alpha \sqrt{T}$ (when l and m are constant)

Where n is the frequency of the vibrating segment and T is the tension applied.

II law is verified indirectly by keeping 'n' and 'm' constant and showing that $l \; \alpha \sqrt{T}$ or, $\frac{\sqrt{T}}{l}$ is constant.

Thus to verify the II law, a tuning fork is taken and the string is kept under a suitable tension T. The fork is excited and placed on the sounding box. The length l of the vibrating segment of the string between the fixed bridge and the movable bridge is found by the paper rider test method. Keeping the tension constant and using the same string (wire), the experiment is repeated with different tensions. The observation are tabulated as shown in the Table II.

II law is verified by showing that $\frac{\sqrt{T}}{l}$ is constant.

c) Verification of III law : According to this law, the frequency of the vibrating segment of a string is inversely proportional to its mass per unit length provided the length of the string and its tension are kept constant.

Or, n $\alpha \frac{1}{\sqrt{m}}$ (when T and l are constant)

III Law is verified indirectly by keeping 'n' and T constant and showing that

$$l = \alpha \frac{1}{\sqrt{m}}$$

or, $l \times \sqrt{m}$ = constant

To verify the III law, a string (or wire) is kept under a suitable tension. A tuning fork is excited and placed on the sound board. The length of the wire is between the fixed bridge and the movable bridge is found with paper rider test method. A known length of the wire is taken and its mass is found in a bålance from which the mass per unit length of the wire (m) is found.

Using the same tuning fork and the same tension, the experiment is repeated with different materials of wires and the observations are tabulated as shown in table iii.

d) Determination of unknown frequency : The sonometer wire is kept under a suitable tension T. The tuning fork of unknown frequency 'n' is excited and placed on the sounding box. The length of the vibrating segment l of the wire having the same frequency 'n' is found by the paper rider rest method.

Using the same wire, the experiment is repeated with various tension and for each tension, the length 'l' of the wire is found by the paper rider test method.

In each case the tension is found by T = Mg, where M is the load applied at the and of the wire. The observations are tabulated as shown in table (iv) and the average of $\frac{\sqrt{T}}{l}$ is found.

A known length of the wire is taken and its mass is found by the balance from which the mass per unit length of the wire can be determined.

The unknown frequency 'n' of the tuning fork is found by the relation.

$$n = \frac{1}{2\sqrt{m}} \frac{\sqrt{T}}{l}$$

Observatons : Tables i, ii, iii, iv

Precautions : 1. The pulley should be frictionless.

2. The tuning fork should gently be pressed on the sonometer board.

Table (i)

Verification of I law

S.No.	Frequency nHz	Length l			$n \times l$
		I trial	II trial	Mean (l)	= constant

Table (ii)

Verification of II law

S.No.	Tension T	Length of the wire l	$\frac{\sqrt{T}}{l}$ = constant

Table (iii) Verification of III Law

S.No.	Length of wire l cm	m	$l\sqrt{m}$ = constant

Table (iv)

Determination of unknown frequency :

S.No.	Tension T mg	l	$\frac{\sqrt{T}}{l}$

Average $\frac{\sqrt{T}}{l}$

Frequency of the tuning fork n = $\frac{1}{2\sqrt{m}}\left(\frac{\sqrt{T}}{l}\right)$

Result : The laws of transverse vibration are verified

The frequency of the tuning fork = ...Hz.

Viva - Voce

1. To which kind the wave travelling along a stretched string belongs to.?
2. Which of the following properties will be possessed by a wave travelling along a stretched string that is fixed rigidity between two supports ?

   (a) It is a stationary wave, (b) It is a transverse wave, (c) It is polarized, (d) It will have all the above three qualities.

3. In connection with the transverse vibrations of a string, to which number of harmonic the fourth over tone corresponds to ?
4. Om this experiment, several times we use the sentence 'when the string vibrates in one segment'. What is the significance of vibrating in one segment ?

   If it does not vibrate in one segment, what difference will arise ?
5. Of the three different verifications, which one will enable you to measure the velocity (v) of transverse wave along the string ?
6. Of the three different verifications, which one will enable you to measure the frequency (n) of transverse wave along the string ?
7. What is the difference between a progressive wave and a stationary wave ?
8. What is the difference between a longitudinal wave and a transverse wave ?
9. What is natural frequency of a body ? When does resonance occur ?
10. What is the advantage of sound box in the sonometer ?

Viva - Voce Answers

1. Transverse wave.
2. (d) It will have all the above three qualities.
3. Fifth harmonic.
4. When the wire vibrates in one segment only, it will be vibrating with fundamental frequency n, given by $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

   If the wire vibrates in p segments or loops, then it will be the pth harmonic and $n_p = \frac{n}{2l}\sqrt{\frac{T}{m}}$
5. From the verification of the first law, we have $n \times l = a$ constant.

   Now, the velocity $v = 2nl$ can be easily calculated.
6. From the verification of the second law, we have $\frac{\sqrt{T}}{l}$ = a constant. Now the frequency can be calculated from $n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\sqrt{m}}\left(\frac{\sqrt{T}}{l}\right)$
7. A progressive wave always travels in the forward direction only and gets never returned back. A stationary wave will be confined to a limited space.
8. In a longitudinal wave, the particle of the medium will be vibrating in a direction perpendicular to the direction of propagation of the wave.

   In a transverse wave, the particles of the medium will be vibrating in a direction perpendicular to the direction of propagation of the wave.
9. When a body is not subjected to any external force is vibrating on its own accord, these vibrations are called natural vibrations and the frequency of vibration is called the natural frequency.

   When the frequency of the external driving force becomes equal to the natural frequency of the body then resonance occurs.
10. Sound will appear with larger intensity.

14. DETERMINATION OF FREQUENCY OF A BAR – MELDE'S EXPERIMENT

Experiment

Date

Aim : To determine the frequency of an electrically driven tuning fork.

Apparatus : An electrically maintained tuning fork, a light smooth pulley fixed to a stand, a light scale pan, thread, a storage cell, rheostat, plug key and connecting wires.

Description : A fork can be maintained India state of continuous vibration electrically. One terminal of the coil of an electromagnet is connected to the make and break arrangement and the other end of the coil to the cell, rheostat and plug key connected in series. In the normal position when the circuit is closed, the electromagnet attracts the prong of the fork towards it. This breaks electrical circuit and the prong moves back closing the circuit. The electromagnet again attracts the prong towards it. This is repeated again and again and the fork is maintained in a state of continuous vibration.

One end of the thread of length about 3 meters is joined to a screw attached to one prong of the fork and the other end is passed over a smooth pulley and a light pan is fixed at the other end of the thread.

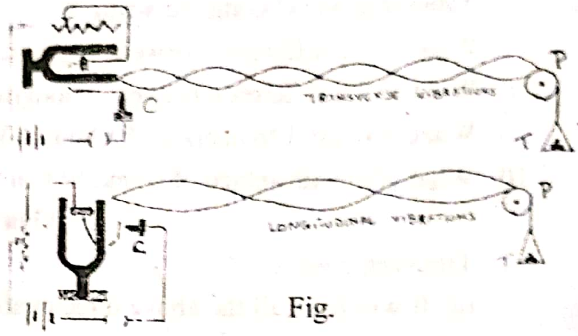


Fig.

When the fork is vibrated electrically, stationary waves of welldefined loops.

Meld's apparatus can be arranged in two modes of vibration a) when the direction of motion of the prong is at right angles to the length of the string, the vibrations of the thread represent the iransverse mode of vibration and b) when the direction of motion of the prong is along the length of the thread represent longitudinal mode of vibration.

Procedure : 1. The apparatus is arranged in transverse mode of vibration of the thread (Fig). A suitable load is placed in the scale pan. The tuning fork is excided electrically. The length of thread is adjusted by moving the pulley untill well defined loops are formed in it. The distance between a definite number of well defined loops (say 3 or 4) is measured with a meter scale from which the average length l of a single loop.is determined.

The total load attached to the thread inclusive of the mass of the pan is noted. If it is Mgm, the tension applied on the string is T = Mg. Where g is acceleration due to gravity.

The mass of the thread (about 5 meters in length) is determined correct to a milligram. The mass per unit length of the string (m) is then determined. The frequency n of the turning fork is founded by the relation.

$$n = \frac{1}{2l} \frac{\sqrt{T}}{m}$$

The experiment is repeated for various lensions and the observations are tabulated in table (i) and n is calculated.

2. The apparatus is arranged in longitudinal mode of vibration of the thread. The experiment is done in similar manner as in 1. The average length l of a loop, the tension T, applied to the thread and the mass per unit length of the thread (m) are found. The frequency of the tuning fork is found by the relation

$$n = \frac{1}{l}\sqrt{\frac{T}{m}}$$

The experiment is repeated with different tensions and the observations are tabulated in Table (ii) and n is calculated.

Precautions : 1. A thin long and inelastic thread should be used.

2. The loops should be well defined and confined to a single plane.

The mean of the two average frequency in the transverse and the longitudinal modes gives the correct frequency of the tuning fork.

Observations : Mass per unit length of the thread (m) = ...gms

Transverse Mode (i)

S.No.	T = Mg	Length of P loops = L	Length of each loop $l = \frac{P}{L}$	$\frac{\sqrt{T}}{l}$

Average $\frac{\sqrt{T}}{l}$ = ...

Then $n = \frac{1}{2\sqrt{m}} \times \frac{\sqrt{T}}{l}$

Longitudinal Mode (ii)

S.No.	T = Mg	Length of P loops = L	Length of each loop $l = \frac{P}{L}$	$\frac{\sqrt{T}}{l}$

Average $\frac{\sqrt{T}}{l}$ =

Then $n = \frac{1}{\sqrt{m}} \times \frac{\sqrt{T}}{l}$

Result : The frequency of the turning fork n =

Viva - Voce

1. In this experiment we use two modes or two different kinds of arrangements. In each mode, what kind of waves (transverse or longitudinal) are formed along the stretched string ?
2. Are the waves formed in this experiment stationary or progressive ?
3. What should be done to change the number of loops formed ?
4. How are the two frequencies (a) The frequency of the tuning fork *(N)* and (b) The frequency of vibration of the stretched thread *(n)* related to each other in

   (i) The transverse mode and

   (ii) The longitudinal mode.

5. How will the $\sqrt{T} - l$ graph look like ?

6. What is the reason for getting $N = \frac{1}{2l}\sqrt{\frac{T}{m}}$ in the transverse mode and $N = \frac{1}{l}\sqrt{\frac{T}{m}}$ in longitudinal mode ? In these two different modes, how will the frequency of vibration of the thread change ?

7. If the number of loops formed in transverse mode is p, under the same tension (T) and for same length (l) of thread, how many loops will be formed in longitudinal mode.

8. What are the different modes used in this experiment ? Explain.

Viva - Voce Answers

1. In both arrangements (modes), the waves along the stretched thread will be transverse waves only.

2. Stationary waves.

3. By changing (1) The tension $T = Mg$ - that is by changing the masses in the pan or (2) By changing the length (l) of the thread between the pully and the prong.

4. In the transverse mode or arrangement only, the frequency of tuning fork (N) will be equal to the frequency of vibration (n) of the thread.

   $N = n$ - in Transverse mode ; $N = 2n$ - in Longitudinal mode

   In both arrangements, the frequency of the tuning fork will not change.

   For the same length of the thread (l) and same tension (T), in the case of the thread.

   (a) In transverse mode $n = N$ of the tuning fork

   (b) In longitudinal mode $n = \frac{N}{2}$

5. In Melde's experiment, the graph between $\sqrt{T}$ and l will be a straight line.

6. (When l & T are kept constant), the frequency of vibration of the thread in transverse mode $= N$ and in longitudinal mode $\frac{N}{2}$ (Where N = frequency of the tuning fork). This is because, in the transverse mode the wire or thread vibrates in accordance with the vibrations of the fork. But in longitudinal mode, while the fork completes one full vibration, the thread will be able to complete only half the vibration.

7. If the number of loops in transverse mode is p, then under same tension (T) and for same length (l) of the thread, the number of loops will be $\frac{p}{2}$ only.

8.(a) Transverse mode - in which the vibrations of the rods or prongs will be perpendicular to the length of the thread.

(b) Longitudinal mode - in which the vibrations of the rods or prongs will be parallel to the length of the thread.

15. DAMPED OSCILLATIONS - TORSIONAL PENDULUM

Experiment

Date

Aim : Study the damped oscillations of a torsional pendulum in the shape of a circular disc in air and water, and determine the logarthmic decrement.

Apparatus : Torsional pendulum in the shape of a cicular disc, lamp and scale arrangement, circular mirror (Concave mirror of focal length 50cm), Stop clock, glass tub with water.

Description : A metal wire of uniform cross section is suspended from a chucknut fixed to L - shaped clamp fixed to a wall. The lower end of the wire is attached to another chucknut fixed to a torsional pendulum consisting of a circular disc. The small circular mirror attached with beeswax to the wire. Lamp and scale arrangement is kept at a distance of 1 met. from

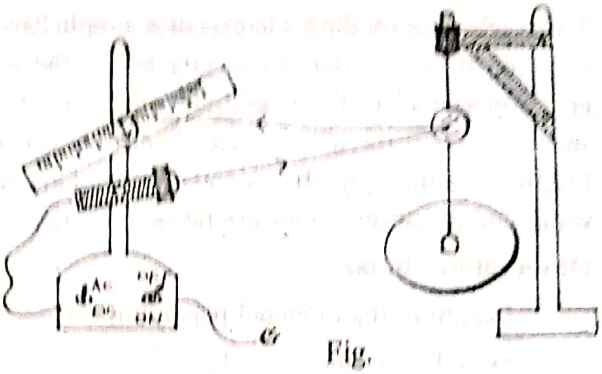


Fig.

the mirror. The illumination of the room is made minimum or semidark. The light from the lamp is focussed on the small circular mirror and a bright circular spot of light is obtained on the transparent scale of lamp and scale arrangement.

Theory : The amplitude of damped vibrations does not remain constant. It decreases exponentially with time. The amplitude of torsional pendulum decreases with time due to viscosity or resistive forces caused by the surrounding air or water or any medium as the energy of vibration is dessipated. If θ_0 is the angular amplitude without damping and θ is the amplitude at time t, the equation. $\theta = \theta_0\, e^{-bt}$

Gives the variation of angular amplitude with time. If r is the resistance per unit velocity and m is the mass of the oscillator $\frac{r}{m} = 2b$ or $b = \frac{r}{2m}$ = coefficient of damping

It T_1 is the period of damped vibration

$$T_1 = \frac{2\pi}{\sqrt{\omega^2 - b^2}}$$

Where ω = angular frequency of the free oscillator $= \frac{2\pi}{T}$

T = period of free oscillation

Let $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ be the successive angular amplitudes at a point on the wire of the torsional pendulum

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \frac{\theta_4}{\theta_5} = d = e^{\lambda}$$

d = decrement, $\lambda = \log_e d$ = logarithmic decrement.

θ_1, θ_2 and θ_2, θ_3 are separated by half period

$$\frac{\theta_1}{\theta_2} = e^{\lambda} = e^{bt/2} \qquad \text{(or)}$$

$$\lambda = \frac{bT}{2}$$

Procedure : A small concave mirror of focal length 50 cm is attached to the wire of torsional pendulum. Lamp and scale arrangement is kept at a distance of 1 met from the mirror. The light from the lamp and scale is focussed on the mirror to get a bright circular spot of light on the plastic transparent scale. The circular disc is rotated about the axis through a small angle and released. Due to the restoring orce due to the elasticity of the wire and moment of interia of the disc, torsional oscillations are produced.

The bright spot on the scale executes simple harmonic motion on the scale as it is formed from the light reflected at the circular mirror attached to the wire of the torstional pendulum. The angular amplitudes are proportional to the amplitudes of the light spot $\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5 on the scale they are noted successively on the left, left etc. The time of twenty oscillations is noted using a stop clock. The wire length is changed by 10 cm and observations are taken. Next the torsional pendulum is suspended in water and the observations are taken as in air.

Observations in air

Length of the torsional pendulum = l = cm

Period of oscillation = T = sec

S.No.	Amplitudes	Ratios of amplitudes	logarthemic decrement $\lambda = \log_e d$
	θ_1	$\frac{\theta_1}{\theta_2} =$	
	θ_2	$\frac{\theta_2}{\theta_3} =$	
	θ_3	$\frac{\theta_3}{\theta_4} =$	
	θ_4	$\frac{\theta_4}{\theta_5} =$	

Average ratio = decrement = d

Logarithmic decrement = $\lambda = \log_e d$

$\lambda = 2.303 \log_{10} d$

Observations in water :

Length of the torsional pendulum = l = cm

Period of oscillation = T = cm

S.No.	Amplitudes	Ratios of amplitudes	logarthemic decrement(λ) $\lambda = \log_e d$
	θ_1	$\frac{\theta_1}{\theta_2} =$	
	θ_2	$\frac{\theta_2}{\theta_3} =$	
	θ_3	$\frac{\theta_3}{\theta_4} =$	
	θ_4	$\frac{\theta_4}{\theta_5} =$	

Average ratio = decrement = d

Logarithmic decrement = $\lambda = \log_e d$

$\lambda = 2.303 \log_{10} d$

Logarithmic decrement can be determined from they observations.

Precautions :

1. The wire should be free from kinks.
2. The disc should execute oscillations in the horizontal plane.
3. The room should be semi dark.

4. The lamp and scale arrangement should not be disturbed after making zero adjustment.
5. The circular mirror attached to the wire should be concave.

Notes : 1. The coefficient of damping b can be found from logarthmic decrement λ usng equation

$$\lambda = \frac{bT}{2}$$

The value of b depends on density and viscosity of medium.

2. The period $T_1 = \frac{2\pi}{\sqrt{\omega^2 - b^2}}$ for damped vibration. Where w is angular frequency of free vibration.

The period $T = \frac{2\pi}{\omega}$ for undamped free vibration. The rigidity modulus of the wire can be determined more accurately correcting for damping of oscillation of torsional pendulum.

3. The amplitude $\theta_0 = \theta_1\left(1 + \frac{\lambda}{2}\right)$ approximately.

Viva - Voce

1. What is damping ? What is the reason for dampng ?
2. In how many different ways the damping can be explained ?
3. In which medium, air or water, the damping is more ?
4. Due to damping, what changes will occur in resonance ?
5. Is damping always disadvantagesous ? or, are there any advantages for which damping can be used ?
6. If damping increases, what will happen to the sharpness of resonance ? (Will it increase or decrease)
7. What are the units for Logarithmic decrement ?
8. Why a lamp and scale arrangement is used in this experiment ? Can you mention any other experiment in which this arrangement is used ?
9. What is the combined arrangement of the disc and the metal wire is called ?
10. In a damped harmonic motion, how does the amplitude change with time ? (Name the functional relation).
11. What is the difference between a simple pendulum and a torsional pendulum ?

Viva - Voce Answers

1. If the amplitude of an oscillator decreases with time and ultimately reduces to zero, the process is called damping. Frictional and viscous forces are responsible for this damping.
2. Damping can be described by three different methods (i) Logarithmic decrement, (ii) Relaxation time and (iii) Quality factor Q.
3. The damping in water is more than in air.
4. As damping increases, sharpness of resonance decreases.
5. Damping is used for advantage in the construction of speedometer, in bringing the oscillation of the pointer of a galvanometer or a ballistic galvanometer to rest immediately.
6. Decreases. The resonance curve gets flattened.
7. This a pure number. $\frac{\theta_1}{\theta_2}$ is a ratio and hence log $\left(\frac{\theta_1}{\theta_2}\right)$ will have no units and dimensions.
8. To measure the angular displacement 'θ' more accurately and conveniently.
   The same arrangement is used in a moving coil galvanometer.
9. Torsional pendulum.
10. As an exponential function.
11. The oscillations of a simple pendulum are ordinary to and fro oscillations ; where as the oscillations of a torsional pendulum are torsional oscillations.

MODEL PAPER – I

B.Sc., First Year, Semester - 1,

PHYSICS

Time : 3 Hrs.] **[Max. Marks : 75**

Section - A (5 x 10 = 50 marks)

Answer ALL questions with internal choice from each unit.

1. Derive the equation of motion of a system of variable mass. (Or)

   Deduce the equations of motion for a rigid rotating body.

2. State and prove Kepler's laws of planetary motion. (Or)

   Define central force and give its characteristics. Explain its conservative nature.

3. State the postulates of the special theory of relativity. Obtain Lorentz transformation equations.

   (Or)

   Describe the Michelson-Morely experiment with necessary theory and derive the expression for fringe shift.

4. Obtain the equation of motion of simple harmonic oscillator. Derive the differential solution of simple oscillator. (Or)

   Obtain the normal mode frequencies of two pendulums connected by a massless spring.

5. What are trasnverse waves ? Derive equation of Motion of a transverse wave in a streteched string.

   (Or)

   Describe any one method for production of ultrasonic waves.

Section - B (5 x 5 = 25 marks)

Answer any FIVE out of the following ten questions.

6. Write a short note on impact parameter.
7. Write a short note on Gyroscope.
8. State Kepler's laws of planetary motion.
9. Explain motion of satellite.
10. Shrot note on Lorentz Contraction.
11. Einstein's Mass-energy relation.
12. Resonance and its examples.
13. What is coupled oscillator give examples ?
14. What are overtones and harmonics ?
15. Give two methods for detection of ultrasonics.

MODEL PAPER – II

B.Sc., First Year, Semester - 1,

PHYSICS

Time : 3 Hrs.] **[Max. Marks : 75**

Section - A (5 x 10 = 50 marks)

Answer ALL questions with internal choice from each unit.

1. What is Ruther Ford scattering ? Obtain an expression for the Ruther Ford scattering cross-section?

(Or)

Derive the Euler's equation of rotational motion for a rigid rotating body.

2. What is central force ? Derive equation of motion of a body under a central force. (Or)

State Kepler's laws of planetary motion. Deduce the third law relating to the time period and semi-major axis.

3. State the postulates of the special theory of relativity. Derive the mass-energy relation. (Or)

Derive Lorentz transformation equations. Explain length contraction in relativity.

4. Discuss the differential equation of a forced damped oscillator and obtain its solution. (Or)

Obtain the normal mode frequencies of N-coupled oscillator.

5. Obtain the modes of vibration of stretched string clamped at both the ends ? (Or)

What are ultrasonic waves ? Explain applications of ultrasonics with necessary theory.

Section - B (5 x 5 = 25 marks)

Answer any FIVE out of the following ten questions.

6. Explain motion of a Rocket.
7. Explain precession of symmetric top.
8. Examples of central forces.
9. Global positioning system.
10. Explain types of frames of references.
11. Shrot note on time dialation.
12. Short note on relaxation time and quality factor.
13. What is difference between harmonics and overtones.
14. Characteristics of simple harmonic oscillator.
15. SONAR.

MODEL PAPER – III

B.Sc., First Year, Semester - 1,

PHYSICS

Time : 3 Hrs.] **[Max. Marks : 75**

Section - A (5 x 10 = 50 marks)

Answer ALL questions with internal choice from each unit.

1. Using system of variale mass, explain the motion of a rocket. Discuss about multistage of rocket.

(Or)

What is rotatioal motion ? Derive the relations of Rotational kinematics.

2. What is central force ? Show that conservative forces as a negative gradient of potential energy.

(Or)

Deduce the Kepler's laws of planetory motions.

3. State the fundamental postulates of special theory of relativity and deduce the lorentz transformation equation. (Or)

Describe the relevant theory and result of Michelson morely experiment. Discuss the negative result of this experiment ?

4. Derive the equation of motion of damped harmonic oscillator and find its solution. (Or)

Give the theory of N-coupled oscillators an extend it to obtain the wave equation.

5. What are transverse waves ? Obtain equation for the velocity of transverse wave in a stretched string.

(Or)

Describe the magnetostriction method of producing ultrasonics.

Section - B (5 x 5 = 25 marks)

Answer any FIVE out of the following ten questions.

6. Short note on scattering cross section.
7. Explain precession of equinoxer.
8. Characteristics of central forces.
9. Expression for variation of mass with velocity.
10. Postulates of special theory of relativity.
11. Explain amplitude resonance.
12. Simple harmonic oscillator examples.
13. Explain melde's strings.
14. Types of crystals used in production of ultrasonics.
15. Applications of ultrasonics.