II BSC FORTH SEMESTER

PAPER-V

PHYSICS STUDY MATERIAL

MODERN PHYSICS

(w.e.f 2020-2021 Batch)

Department of Physics

Sri Y.N.College (A)

Narsapur

This book is intended for the students appearing in Examinations of 2022 May/June

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✦ CONTENTS ✦

1. Model Test Papers 2

QUESTION BANK

UNIT - I

1. Atomic and molecular physics .... 1 - 16

UNIT - II

2. Matter waves & Uncertainty principle .... 17 - 42

UNIT - III

3. Quantum (wave) mechanics .... 43 - 52

UNIT - IV

4. Nuclear physics .... 53 - 73

UNIT - V

5. Nano materials .... 74 - 79
6. Superconductivity .... 80 - 88

12 hours

UNIT - IV

4. Nuclear Physics

Nuclear structure : General properties of Nuclei, Mass defect, Binding energy ; **Nuclear forces :** Characteristics of nuclear forces - Yukawa's meson theory; **Nuclear Models :** Liquid drop model, the Shell model, Magic numbers; **Nuclear Radiation detectors :** G.M. Counter, Cloud chamber, Solid State detector; **Elementary Particles :** Elementary particles and their classification.

7 hours

UNIT - V

5. Nano materials

Nanomaterials - Introduction, Electron confinement, Size effect, Surface to volume ratio, Classification of nano materials - (0D, 1D, 2D); Quantum dots, Nano wires, Fullerene, CNT, Graphene (Mention of structures and properties), Distinct properties of nano materials (Mention - mechanical, optical, electrical and magnetic properties); Mention of applications of nano materials : (Fuel cells, Phosphorus for HD TV, Next Generation Computer chips, elimination of pollutants, sensors).

5 hours

6. Superconductivity

Introduction to Superconductivity, Experimental results - critical temperature, critical magnetic field, Meissner effect, Isotope effect, Type I and Type II superconductors, BCS theory (elementary ideas only), Applications of superconductors.

MODEL TEST PAPER - 1

B.Sc. DEGREE EXAMINATION

SECOND YEAR - SEMESTER- 4

PHYSICS PAPER - 5

MODERN PHYSICS

(For Mathematics Combination)

Time : 3 Hours — Max. Marks : 75

SECTION - A (5 × 10 = 50 Marks)

Answer *ALL* of the following questions.

1. a) Explain Quantum theory of Raman effect.

   Or

   b) Describe spatial quantisation and spinning electron hypothesis.

2. a) State Heisenberg's uncertainty principle and apply in case of i) Position and momentum & ii) Energy and time.

   Or

   b) Explain Heisenberg's uncertainty? Explain gamma ray microscope as a consequence of uncertainty principle.

3. a) Give the physical significance of wave function?

   Or

   b) Describe schrodinger's and Born's interpretation of wave function?

4. a) Mention the basic properties of nucleus with reference to size, charge, mass, nuclear spin, magnetic dipole moment and electric quadrupole moment.

   Or

   b) Explain the principle and working of $G-M$ counter.

5. a) Write about the structure of graphine.

   Or

   b) Give sailent features of BCS theory. Describe briefly the formation of cooper pairs.

SECTION - B (3 × 5 = 15 Marks)

Answer any *THREE* of the following

6. What are the drawbacks (short comings) of Bohr's theory. (or) Explain the limitations of Bohr's theory.
7. Explain wave velocity and group velocity.
8. What are Eigen values and Eigen functions?
9. Write about quadrupole moment of a nucleus.
10. What is Meissner's effect in superconductivity?

SECTION - C (2 × 5 = 10 Marks)

Answer any *TWO* of the following

11. In the normal Zeeman effect, the frequency separation between two consecutive spectral lines $8{\cdot}3 \times 10^8$ Hz. Find the magnetic field.
12. Determine the uncertainty in the velocity of an electron confined to $20Å$ ($h = 6{\cdot}6 \times 10^{-34}$ & $M_e = 9{\cdot}1 \times 10^{-31} kg$).
13. A particle is confined to one dimensional infinite potential well of width $0{\cdot}4 \times 10^{-19}$ m. It is found that when the energy of the particle is 230 ev, its eigen function has 5 antinodes. Find the mass of the particle and show that it can never have energy equal to 1 kev.
14. Calculate the amount of energy released in Joules when 10 micrograms of uranium undergoes fission.
    Avogadro's number = $6{\cdot}023 \times 10^{23}$; Energy released per fission = 200 MeV.
15. Calculate the critical current which can flow through a long thin super conducting wire of diameter 10^{-3} m. Given $H_c = 7{\cdot}9 \times 10^3$ A/m.

ANSWERS

SECTION - A

- **1a.** Refer to L.A.Q. No. 6 in Atomic and Molecular Physics.
- **1b.** Refer to L.A.Q. No. 3 in Atomic and Molecular Physics.
- **2a.** Refer to L.A.Q. No. 5 in Matter Waves and Uncertainty Principles.
- **2b.** Refer to L.A.Q. No. 3 in Matter Waves and Uncertainty Principles.
- **3a.** Refer to S.A.Q. No. 3 in Quantum(wave) Mechanics.
- **3b.** Refer to L.A.Q. No. 1 in Quantum(wave) Mechanics.
- **4a.** Refer to L.A.Q. No. 1 in Nuclear Physics.
- **4b.** Refer to L.A.Q. No. 5 in Nuclear Physics.
- **5a.** Refer to L.A.Q. No. 2 in Nanomaterials.
- **5b.** Refer to L.A.Q. No. 3 in Superconductivity.

SECTION - B

6. Refer to S.A.Q. No. 2 in Atomic and Molecular Physics.
7. Refer to S.A.Q. No. 2 in Mater Waves& Uncertainty priciples
8. Refer to S.A.Q. No. 2 in Quantum(wave) Mechanics.
9. Refer to S.A.Q. No. 4 in Nuclear Physics.
10. Refer to S.A.Q. No. 3 in Superconductivity.

SECTION - C

11. Refer to Problem No. 9 in Atomic and Molecular Physics.
12. Refer to Problem No. 39 in Matterwaves & Uncertainty Principle
13. Refer to Problem No. 4 in Quantum(wave) Mechanics.
14. Refer to Problem No. 13 in Nuclear Physics.
15. Refer to Problem No. 7 in Superconductivity.

MODEL TEST PAPER - 2

B.Sc. DEGREE EXAMINATION

SECOND YEAR - SEMESTER- 4

PHYSICS PAPER - 5

MODERN PHYSICS

(For Mathematics Combination)

Time : 3 Hours Max. Marks : 75

SECTION - A (5 × 10 = 50 Marks)

Answer *ALL* of the following questions.

1. a) Describe stern and Gerlach experiment. What is its importance?

Or

b) Explain Raman effect? Describe the experimental set-up in study of Raman effect in liquids. Write the applications of Raman effect.

2. a) Describe the Davison and Germer experiment on electron diffraction. Discuss the results of the experiment. .

Or

b) Explain Heisenberg's uncertainty principle by illustrating by diffraction through single slit.

3. a) Obtain an expression for the energy of a particle in one dimensional potential well(box) of infinite height.

Or

b) Derive schordinger time independent wave equation and time dependent wave equation.

4. a) What is nuclear binding energy? Draw a binding energy curve. What information do we get from such a curve?

Or

b) Explain the construction and working of wilson cloud chamber.

5. a) What are Nanomaterials. Discuss briefly different types of Nanomaterials.

Or

b) Explain the persistent currents, specific heat, entropy and transition temperatures in connection with super conductors.

SECTION - B (3 × 5 = 15 Marks)

Answer any *THREE* of the following

6. Explain Stokes and anti-stokes lines.
7. Write the properties of matter waves.
8. Give the physical significance of wave function?
9. What are magic numbers? Explain.
10. What are the various applications of Nanotechnology.

SECTION - C (2 × 5 = 10 Marks)

Answer any *TWO* of the following

11. In the normal Zeeman effect, the frequency separation between two consecutive spectral lines is $8{\cdot}3 \times 10^8$ Hz. Find the magnetic field. $\mu_B = 9{\cdot}27 \times 10^{-24}$ J/T.

12. Calculate the wavelength associated with an electron subjected to a potential difference of 1·25 kv.

13. Calculate the energy of an electron moving in one dimension in an infinitely high potential box of widht $2\mathring{A}$ given mass of electron $m = 9{\cdot}1 \times 10^{-31}$ kg and Planck's constant $h = 6{\cdot}62 \times 10^{-34}$ Js

14. What is the mass number of a nucleus whose radius is 3·6 fermi? (given $r_0 = 1{\cdot}2$ fermi)

15. The critical field for niobium is 1×10^4 A/m at 8K and 2×10^5 A/m at OK. Calculate the transition temperature of the element.

ANSWERS

SECTION - A

1a. Refer to L.A.Q. No. 2 in Atomic and Molecular Physics.
1b. Refer to L.A.Q. No. 5 in Atomic and Molecular Physics.
2a. Refer to L.A.Q. No. 2 in Matter Waves and Uncertainty Principles.
2b. Refer to L.A.Q. No. 4 in Matter Waves and Uncertainty Principles.
3a. Refer to L.A.Q. No. 2 in Quantum(wave) Mechanics.
3b. Refer to L.A.Q. No. 1 in Quantum(wave) Mechanics.
4a. Refer to L.A.Q. No. 2 in Nuclear Physics.
4b. Refer to L.A.Q. No. 2 in Nuclear Physics.
5a. Refer to L.A.Q. No. 3 in Nanomaterials.
5b. Refer to L.A.Q. No. 4 in Superconductivity.

SECTION - B

6. Refer to S.A.Q. No. 5 in Atomic and Molecular Physics.
7. Refer to S.A.Q. No. 7 in Mater Waves& Uncertainty priciples
8. Refer to S.A.Q. No. 3 in Quantum(wave) Mechanics.
9. Refer to S.A.Q. No.2 in Nuclear Physics.
10. Refer to S.A.Q. No. 3 in Nanomaterials.

SECTION - C

11. Refer to Problem No. 8 in Atomic and Molecular Physics.
12. Refer to Problem No. 54 in Matterwaves & Uncertainty Principle
13. Refer to Problem No. 2 in Quantum(wave) Mechanics.
14. Refer to Problem No. 22 in Nuclear Physics.
15. Refer to Problem No.3 in Superconductivity.

UNIT- I

1 ATOMIC AND MOLECULAR PHYSICS

LONG ANSWER TYPE QUESTIONS

1. Describe stern and Gerlach experiment. What is its importance?

[ANU J18, M16,15; AdNU O18; AU 18; KU O18, M16,15; RU O17; SKU O17; SVU O17; VSU O18; YVU O18, N17]

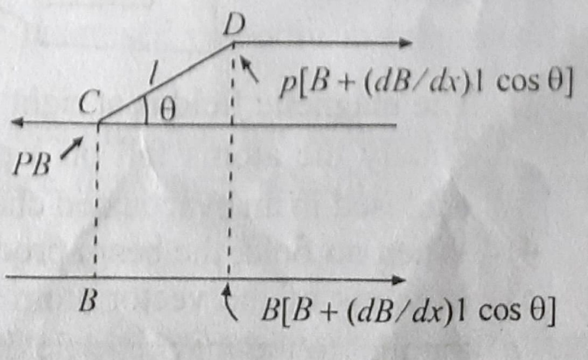
A. **The stern and Gerlach experiment :**

Principle :

1) The experiment is based on the behaviour of a magnetic dipole (atomic magnet) in a non - uniform magnetic field.
2) In a uniform magnetic field (B), the dipole experiences a torque that tends to align the dipole parallel to the field.
3) If the dipole moves in such a field in a direction normal to the field, it will trace a straight line path without any deviation.
4) In an in homogeneous magnetic field, the dipole experiences, in addition, a translatory force.
5) If the atomic magnet flies across such a homogeneous magnetic field normal to the field direction, it will be divided away from its rectlinear path.

Expression :

1) Let the magnetic field vary along the X- direction. So that the field gradient is $\frac{dB}{dx}$ and is positive.
2) CD is the atomic magnet (of polestrength P, length $2l$, dipole moment μ) with its axis inclined at an angle θ to the field direction.
3) If the field strength at the pole C is B, then the field strength at the other pole D will be $[B + \frac{dB}{dx} 2l\cos\theta]$.
4) Hence the forces on the two poles are pB and $p[B + \frac{dB}{dx} 2l\cos\theta]$. $[\because F = mB]$
5) Hence the atomic magnet experiences not only a torque ($\tau = p \cdot 2lB = \mu B$) but also a translatory force.



$$F_x = \frac{dB}{dx} p2l\cos\theta$$

$$\therefore F_x = \frac{dB}{dx} \mu \cos\theta \rightarrow (1)$$

6) Let V = velocity of the atomic magnet of mass m as it enters the field, L = length of the path of the atom in the field and t = the time of travel of the atom through the field $= \frac{L}{V}$

7) The accelaraton given to the atom along the field direction, by the transalatory force is $a_x = \frac{F_x}{m} \rightarrow (2)$

8) The displacement of the atom along the field direction, on emerging out of the field $= D = \frac{1}{2}\left[\frac{F_x}{m}\right]t^2 = \frac{1}{2}\left[\frac{F_x}{m}\right]\frac{L^2}{V^2}$.

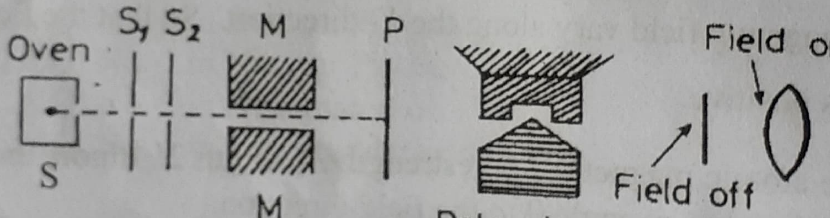
$$= \frac{1}{2}\frac{dB}{dx}\frac{\mu\cos\theta}{m}\frac{L^2}{V^2} \rightarrow (3)$$

9) If μ is resolved component of the magnetic moment in the field direction, $\mu_l = \mu\cos\theta$.

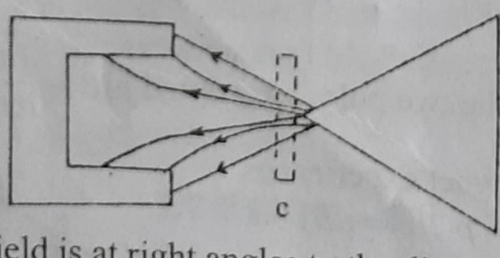
$$\therefore\ D = \frac{1}{2}\frac{dB}{dx}\frac{\mu_l}{m}\frac{L^2}{V^2} \rightarrow (4)$$

Experimental Arrangement :

1) Silver is boiled in an electric oven S. Atoms of silver stream out from an opening in the oven. By the use of slits S_1 and S_2, a sharp linear beam of atoms is obtained.
2) These atoms then pass through a very homogeneous magnetic field between the shaped poles of a magnet MM.



a b d



c

3) The magnetic field is at right angles to the direction of movement of the atoms. Finally the atoms fall on a photographic plate P. The whole arrangement is enclosed in an evaccuated chamber.
4) When no field, the beam produces a narrow continuous line on the plate.
5) In terms of the vector atom model, those atoms, with electron spins directed parallel to the magnetic field, will experience a force in one direction, whereas those with oppositely directed spins will experience a force in the opposite direction.

6) According to this, the beam of atoms should split into two beams in its passage through the inhomogeneous magnetic field. This splitting of the beam into two parts of approximately equal intensity was actually observed in these experiments.

7) On applying the inhomogeneous magnetic field, it was found the stream of silver atoms splits into two separate lines. Knowing $\frac{dB}{dx}$, L, V and D, μ_l was calculated.

Importance : This experiment gave a very direct confirmation of the essential features of vector atom model i.e., Spatial quantisation, spinning electron and quantised atomic magnetic moment.

2. Describe spatial quantisation and spinning electron hypothesis.

[AU 18, 17; KU J15]

A. The vector atom model is an extension of the Rutherfold - Bohr - Sommerfeld atom model on new lines. It explains the draw backs of previous models and also to cover new fields of experimental observation. The two essential features of vector atom model are 1) Spatial quantisation and 2) Spinning electron.

1. Spatial quantisation :

1) Bohr - Sommerfeld's orbits are all quantised as regards their magnitude (i.e., their size and shape).
2) The angular momentum of the electron in stationary orbits is quantised. But quantum theory demands more than this.
3) Now, the quantisation of orientation of orbits in space is introduced. This makes the orbits vector quantities. A preferred direction is required for quantisation of direction.
4) The external magnetic induction field direction is taken as preferred direction.
5) The projection of the orbital angular momentum of electron in the field direction must be quantised.
6) The stern - Gerlach experiment is proof of space quantisation.

2. Spinning electron hypothesis :

1) The spin motion was given to electron by uhlenbeck and Gouds midt, to account for the structure of spectral line and to explain anomalous Zeeman effect.
2) The electron spins about its own axis. Further it has orbital motion.
3) According to quantum theory, the spin of the electron also should be quantised. Now the electron has extra spin angular momentum and spin magnetic moment. The orbital and spin motions are quantised not only in magnitude but also in direction.
4) A new quantum number known as spin quantum number is introduced. The two motions are quantised vectors. Hence the model is called vector Atom model.

3. Explain about the vector atom model.

[ANU J18, O17, J16; AdNU O17; AU 17; BRAU O18; KU O17; RU O18; SKU O18; VSU O18, S17]

A. **Quantum numbers associated with vector atom model :** The quantum numbers associated with each electron in a atom are given below.

1) **Principle (or) total quantum number (n) :**It belongs to principle orbit of an electron. The total quantum number n can have only non zero positive integer values i.e., $n = 1, 2, 3, 4 \ldots \infty$. The energy levels corresponding to $n = 1, 2, 3, 4, \ldots$ are denoted by $K, L, M, N \ldots$ respectively. It indicates the size of the orbit and indicates energy levels.

2) **Orbital quantum number (l) :** It defines the shape of the orbit occupied by the electron. Hence it is called orbital quantum number. l can have any integral values from 0 to $(n-1)$ i.e., $l = 0, 1, 2, 3, \ldots (n-1)$. The particular l value defines the subshell. The subshells with $l = 0, 1, 2, 3, \ldots$ are called $S, P, d, f, \ldots$ subshells respectively. The number of electrons in a sub shell is equal to $2(2l + 1)$.

3) **Spin quantum number (s) :** This quantum number s concerns spins of an electron about its own axis. This spin produces a spin magnetic moment which can be either parallel or antiparallel to the surrounding magnetic field. Electron has two spin values. If the electron spin in clockwise $s = + 1/2$ and the electron spin in anticlockwise $s = - 1/2$.

4) **Total angular quantum number (j) :** The resultant angular momentum of the electron due to both orbital and spin motions is called total angular quantum number. It's numerical value is vector sum of l and s.
$j = l + s$, when l, s are parallel and $j = l - s$, when l, s are antiparallel.

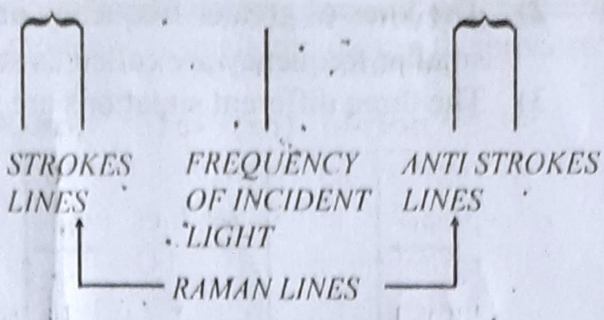
5) **Orbital magnetic quantum number (m_l) :** To explain the effect of magnetic field on orbital motion, a quantum number m_l known as orbital quantum number is introduced. i.e., $m_l = l \cos\theta$. But $\cos\theta$ lies from + 1 to − 1. Hence permitted values of m_l are from $+l$ to $-l$. The number of possible m_l values are $(2l + 1)$ including zero.

6) **Magnetic spin quantum number (m_s) :** It is defined as the component of s in the field direction. i.e., $m_s = s\cos\theta$. The possible values of m_s are from $+s$ to $-s$ excluding zero. The number of possible values of m_s are $(2s + 1)$.

7) **Total magnetic quantum number (m_j) :** It is defined as the component of j on the field direction i.e., $m_j = j\cos\theta$
The possible values of m_j are $+j$ to $-j$ excluding zero. The number of possible values of m_j are $(2j + 1)$ excluding zero.

4. What is Raman effect? Give the experimental arrangement for the Raman effect. Obtain the expression for Raman shift. (or) Describe the Raman effect experiment. Write the applications of Raman effect. **[ANU O18,J18,J16,15,12; AdNU N17; AU 18,17; BRAU O18, O17; KU O18, O17,15; RU O18, O17; SKU O18, O17; SVU O17; VSU S17; YVU N17]**

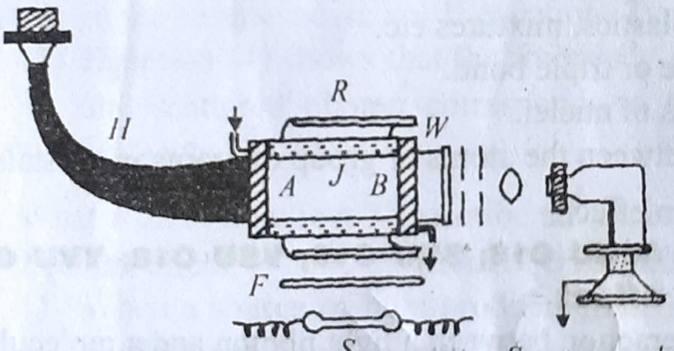
A. 1) When monochromatic beam of incident light is passed through a gas, liquid or transparent solid body, the light is scattered. In 1928, Sir C.V Raman observed that the spectrum of scattered light, (in addition to the frequency of incident light) consists of frequencies greater and smaller that of the incident beam frequency. This is known as Raman effect.

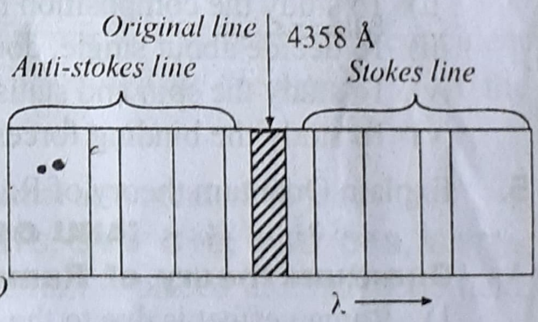


2) The spectrum of the scattered light is called Raman spectrum. The new lines are called Raman lines.
3) The lines of smaller frequency are called as stokes lines while the lines of greater frequency are called as anti stokes lines. The lines are shown in fig.
4) The displacement of the lines, are independent of the frequency of incident light but are functions of the scattering substance. So Raman displacements are characteristic of scattering substance.

Experimental arrangement for Raman spectroscopy :

1) Scientist Wood's apparatus, is used for studying Raman effect in liquids.
2) It consists of a glass tube *AB* of length 10 to 15cm and 1 to 2cm in diameter, containing the pure experimental liquid free from dust and air bubbles.
3) The tube is closed at one end by an optically plane glass plate *W* and at the other end it is drawn into a horn (*H*) and blackened on the outside.
4) Light from a mercury arc *S* is passed through a filter *F* which allows only monochromatic radiation of $\lambda = 4358$ Å to pass through it.
5) The Raman tube is surrounded by a water-jacket (*J*) through which water is circulated to prevent over heating of the liquid.
6) A semi cylindrical aluminium reflector *R* is used to increase the intensity of illumination.



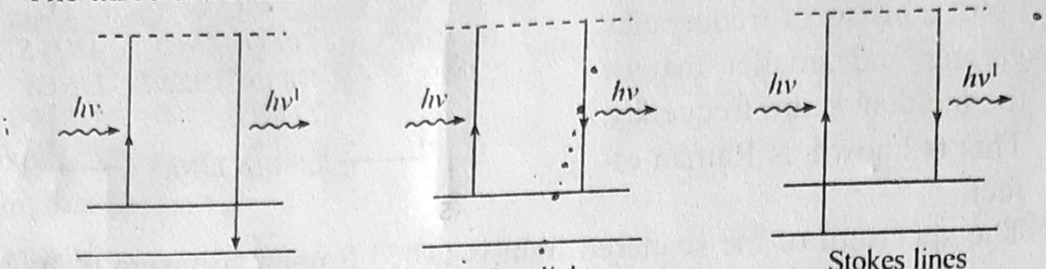


7) The scattered light coming out of *W* is converging on the slit of a spectrograph
8) The spectrograph must have a large resolving power. A short focus camera is used to photograph the spectrum.
9) On developing the photographic plate, it exhibits a number of stoke's lines, a few anti-stokes lines and a strong unmodified line.

Expression of Raman shift :

1) Raman observed that spectrum of scaltered light consists of frequencies greater and smaller than of the incident beam frequency.

2) The lines of greater frequency are called as anti-stokes lines while the lines of smallar frequency are called as stokes-lines.
3) The three different situations are shown below.



Anti - stokes lines $\lambda^{1} < h\nu$ | Ray, light $\lambda^{1} = \lambda$ | Stokes lines $\lambda^{1} > \lambda$

4) The difference of frequency between actual frequency of incident light and frequency of either stokes or anti stokes lines, called Raman shift.
5) The wave number displacement in Raman effect is given by $\Delta\nu = \nu_0 - \nu_s$ for stokes lines and $\Delta\nu - \nu_{a\cdot s} - \nu_0$ for anti stokes lines.
6) But $\nu_0 = \frac{1}{\lambda_0}$, $\nu_s = \frac{1}{\lambda_s}$ and $\nu_{a\cdot s} = \frac{1}{\lambda_{a\cdot s}} \Rightarrow \Delta\nu = \frac{1}{\lambda_0} - \frac{1}{\lambda_s} \rightarrow (1)$

and $\Delta\nu = \frac{1}{\lambda_{a\cdot s}} - \frac{1}{\lambda_0} \rightarrow (2)$
7) Equations (1) and (2) are expressions of Raman shift.

Characteristics of Raman lines :

1) The stokes lines are always more intense than antistokes lines.
2) The Raman lines are symmetrically displaced about the parent line.
3) The frequency difference between the modified and parent line represents the frequency of the corresponding of infrared absorption line.

Applications of Raman effect :

i) To study the molecular structure of crystals and compounds.
ii) To study the composition in plastics, mixtures etc.
iii) To decide about single, double or triple bond.
iv) To study the spin and statistics of nuclei.
v) To study the binding forces between the atoms or group of atoms in crystals.

5. Explain Quantum theory of Raman effect.

[ANU 017; AdNU 018; SVU 018; VSU 018; YVU 018]

A. **Quantum theory of Raman effect :**

1) Raman effect is due to the interaction between a light photon and a molecule of the scatterer.
2) Suppose a photon of frequency ν_1 is incident on a molecule of mass 'm' and there is a collision between the two.
3) Let V_1 and V_2 be velocities E_1 and E_2 be intrinsic energies of molecule before and after collision.
4) Let ν_2 be the frequency of the scattered photon.
5) Applying the principle of conservation of energy,

$$E_2 + \frac{1}{2}mV_2^2 + h\nu_2 = E_1 + \frac{1}{2}mV_1^2 + h\nu_1 \rightarrow (1)$$

6) We may assume that the K.E. of the molecule is unaltered during the process.

Hence, $E_2 + h\nu_2 + E_1 + h\nu_1 \Rightarrow \nu_2 - \nu_1 = \dfrac{E_1 - E_2}{h}$

$\therefore\ \nu_2 = \nu_1 + \dfrac{E_1 - E_2}{h} \rightarrow (2)$

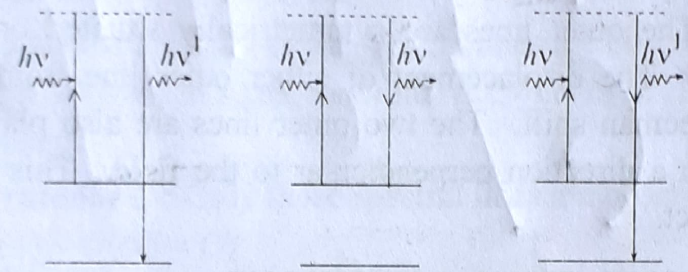
7) Three cases may arise.

i) If $E_1 = E_2$, then, $\nu_1 = \nu_2$. This represents the unmodified line.

ii) If $E_2 > E_1$, then, $\nu_2 < \nu_1$. This represents the stokes line. It means that the molecule has absorbed some energy from the incident photon. Consequently the scaottered photon has lower energy or longer wave length.

iii) If $E_2 < E_1$, then, $\nu_2 > \nu_1$. This represents the antistokes line. It means that the molecule was previously in the excited state and it handed over some of its intrinsic energy to the incident photon. The scattered photon thus the greater energy or shorter wave length.

8) The three different situations are shown in figures.



Anti-stokes lines $\lambda^1 < h\nu$ | Ray leigh lines $\lambda^1 = \lambda$ | stokes lines $\lambda^1 > \lambda$

9) Since the molecules possess quantised energy levels,
We can write $E_1 - E_2 = nh\nu_c \rightarrow (3)$

Where $n = 1, 2, 3 \ldots$ etc., and $\nu_c =$ the characteristic frequency of the molecule.

10) In the simplest case $n = 1$, equation (2) reduces to $\nu_2 = \nu_1 \pm \nu_c \rightarrow (4)$

11) Equation (4) shows that the frequency difference $\nu_1 - \nu_2$ between the incident and scattered photon corresponds to the characteristic frequency ν_c of the molecule.

6. What is Zeeman effect ? Describe the experimental arrangement to study the Zeeman effect. **[BRAU O18; KU M16; SVU O18; VSU O18, S17]**

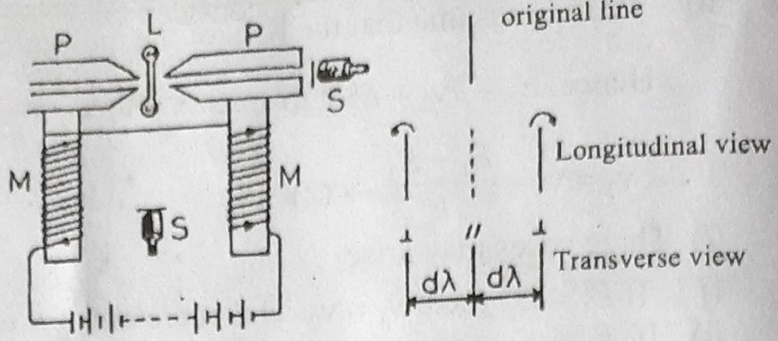
A. 1) When a source of light producing line spectrum is placed in a magnetic field, the spectral lines are split up into components. This effect is called zeeman effect.

2) When the splitting occurs into two or three components, it is called normal zeeman effect.

3) The splitting of a spectral line into more than three components in ordinary weak magnetic fields is called anomalous zeeman effect.

Experimental arrangement :

1) MM is an electromagnet capable of producing a very strong magnetic field.
2) It consists of pole pieces PP have longitudinal holes drilled through them.
3) L is a source of light, emitting line spectrum is placed between P and P.



4) The spectral lines are observed with high resolution spectrograph.
5) In the absence of magnetic field, the position of spectral line is noted.
6) In the presence of magnetic field, the spectral line is split into two components on eitherside of original line, in the direction of field. The original line is not present. Both the lines are circularly polarised in opposite directions. This is called normal longitudinal zeeman effect.
7) The spectral line is then viewed transversely, the single spectral line split up into three components when the field is applied. The central line has same wavelength as the original line and is plane polarised with vibrations parallel to the field. The outer lines are symmetrically situated on either side of the central line. The displacement of either outer line from the central line is known as zeeman shift. The two outer lines are also plane polarised having vibrations in a direction perpendicular to the field. This is called transverse zeeman effect.

SHORT ANSWER TYPE QUESTIONS

1. Explain the spectral terms.

A. In describing the electron configuration, small letters are used to represent the values of l,

l :	0	1	2	3	4	5
notion :	s	p	d	f	g	h

Thus, if an electron is in a shell for $l = 0$, it is called s – electron, for $l = 1$, p – electron, and so on.

To represent the configuration, following two points are considered.

i) The value of total quantum number n is written as a prefix to the letter representing its l values.

ii) The number of electrons with the same n and p values is written at the upper right hand side of letter representing their l value.

The electron configuration of sodium and chlorine are expressed as,

sodium (11 electrons) $1s^2, 2s^2, 2p^6, 3s^1$

chlorine (17 electrons) $1s^2, 2s^2, 2p^6, 3s^2, 3d^5$

The total orbital momentum of an atom L, are expressed by capital letters as shown below

L :	0	1	2	3	4	5
notation :	S	P	D	F	G	H

To express the spectral terms, we consider the following two points.

i) The value of total angular momentum of the atom (J) is written as the subscript at the lower right of the letter representing the particular L value of the atomic state.

ii) The number of possible values of J for given values of L is written as a subscript at the upper left hand of the letter representing the L values.

For $L = 1$, and $S = \frac{1}{2}$, the skectral terms are $^2P_{1/2}$ and $^2P_{3/2}$

Hence, $L = 1$ so the state is P. For $S = 1/2$, the multiplicity is $(2S + 1) = 2$, Now $J = L \pm S = 1 \pm \frac{1}{2} = \frac{1}{2}$ and $\frac{3}{2}$.

2. Write about selection rules.

A. The spectral lines are governed under certain principles known as selection rules. There are three selection rules as described below.

a) The selection rule for L : Only those lines are observed for which the value of L changes by $\pm$ 1 (i.e., $\Delta L = \pm 1$)

b) The selection rule for J : Only those spectral lines are observed when the transitions takes place between states for which $\Delta J = \pm 1$ or 0. The transition $O \rightarrow O$ is not allowed.

c) The selection rule for S : Only those spectral lines are observed for which the value of S changes by $O(\Delta S = O)$

d) Selection rules for magnetic quantum numbers : In the presence of magnetic field, the orbital magnetic quantum number m_l and spin magnetic quantum number m_s play an important part in the transition. Their selection ruels are $\Delta m_l = 0$ or ± 1 and $\Delta m_s = 0$.

In consequence, $\Delta m_J = 0 \pm 1$.

3. Write about intensity rules.

A. The intensity rules are

i) The intensity of transition is strong for which L and J change in the same sense.

ii) The intensity of the transition is weak for which L and J change in opposite sense.

iii) The intensity of a line is strong when the transition is in the decreasing sense. i.e., from $L \rightarrow (L - 1)$.

iv) The intensity of a line is weak when the transition is in the increasing sense i.e. from $L \rightarrow (L + 1)$

v) The oppositely directed transition does not occur.

The above rules can be summarized as

$\Delta L = -1$ $\Delta J = -1$ most intense line

$\Delta L = -1$ $\Delta J = 0$ less intense line

$\Delta L = +1$ $\Delta J = +1$ weaker

$\Delta L = +1$ $\Delta J = 0$ weaker

$\Delta L = -1 \quad \Delta J = +1$ no transition.

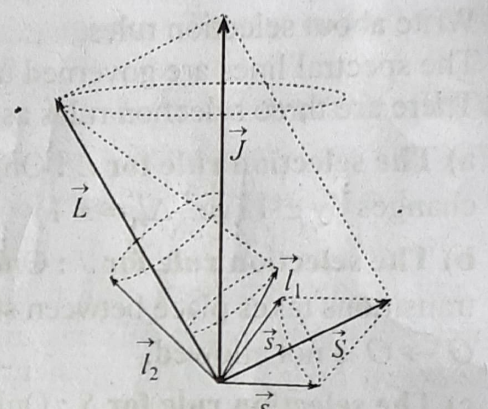
$\Delta L = +1 \quad \Delta J = -1$ no transition.

4. Explain $L-S$ and J-J Coupling. **[ANU O18, O17, M15, ANU Revised MP; AdNU O18; AU 17; BRAU O17; KU O18, O17 J16, 13; RU O17; SKU O18]**

A. **Coupling schemes :**

- The orbital and spin angular momenta of electrons in an atom can be added together in 2 ways. They are (1) $L-S$ coupling or the Russel-Sanunders coupling and (2) the J-J coupling.

1) ***L* – *S* coupling :**



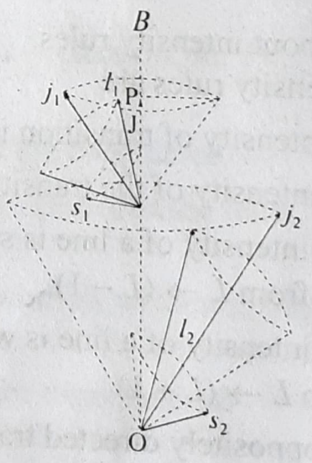
i) The coupling which occurs most frequently is called $L-S$ coupling.

ii) In this type, all the orbital angular momentum vectors of the various electrons combine to form a resultant 'L' and independently, all their spin angular momentum vectors combine to form a resultant 'S'.

iii) These resultants L and S then combine to form the total angular momentum J of the atom.

iv) Symbolically, we can represent the coupling as $L = (l_1 + l_2 + l_3 + \ldots)$; $S = (S_1 + S_2 + S_3 + \ldots)$ and $J = L + S$. L is always an integer including zero. When $L > S, J$ can we have $(2S+1)$ values and when $L < S, J$ can have $(2L+1)$ values.

2) **The *j* – *j* coupling :**



i) This method is employed when interaction between the spin and orbital vectors of each electron is stronger than the interaction between either the spin vectors or the orbital vectors of the different electrons.

ii) The orbital and spin angular momenta of each electron in the atom are added to obtain the resultant angular momentum of the electron.

iii. Thus $j_1 = (l_1 + s_1), j_2 = (l_2 + l_2)$, $j_3 = (l_3 + s_3) \ldots$ and $J = j_1 + j_2 + j_3 = \Sigma j$.

5. What is Raman effect ? Explain. Write its applications.

[ANU O17; AU 18; BRAU O18; RU O18; SKU O17, YVU O18]

A. When monochromatic beam of incident light is passed through a gas, liquid or transparent solid body the light is scattered. In 1928, Sir C.V Raman observed that the spectrum of scattered light, (in addition to the frequency of incident light) consists of frequencies greater and smaller that of the incident beam frequency. This is known as Raman effect.

Applications of Raman effect :

i) To study the molecular structure of crystals and compounds .
ii) To study the composition in plastics, mixtures etc.
iii) To decide about single, double or triple bond.
iv) To study the spin and statistics of nuclei.
v) To study the binding forces between the atoms or group of atoms in crystals.

6. Explain Zeman effect and Stark effect. **[ANU O18; ADNU N17; BRAU O/N17; [RU O17; SKU N18, N/D17]**

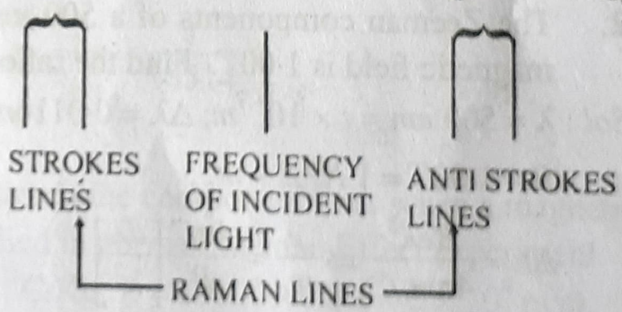
A. **Zeeman effect :**

1) When a source of radiation, giveing line spectrum is placed in a magnetic field, the spectral lines are split up into a number of component lines, symmetrically distributed about the original line. Doublets, triplets and even more complex systems are observed. This is known as Zeeman effect.
2) If the magnetic field is very strong each spectral line is split up into two components in the longitudinal view and in three components in transverse view. This is known as Normal Zeeman effect.
3) When the magnetic field is comparitively weak, each line splits into more than three components. This is known as Anomalous Zeeman effect.

Stark effect : The splitting of spectral lines due to the action of external electric field is called the stark effect.

7. Explain stokes and anti-stokes lines. **[ANU M15]**

A. 1) When monochromatic beam of incident light is passed through a gas, liquid or transparent solid body, the light is scattered. In 1928, Sir C.V Raman observed that the spectrum of scattered light, (in addition to the frequency of incident light) consists of frequencies greater and smaller that of the incident beam frequency.. This is known as Raman effect. The spectrum of the scattered light is called Raman spectrum. The new lines are called Raman lines.

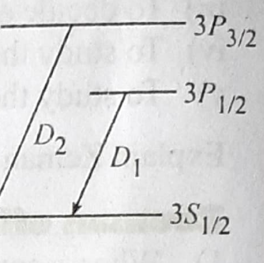


2) The lines of smaller frequency are called as stokes lines while the lines of greater frequency are called as anti stokes lines.
3) The lines are shown in fig.
4) The displacement of the lines, are independent of the frequency of incident light but are functions of the scattering substance. So Raman displacements are characteristic of scattering substance.

8. Write about the five structure of sodium D-lines.

A. The yellow lines of sodium are split due to spin-orbit coupling of L and S.

The transition which gives rise to the doublet is from the $3P$ to the $3S$ level, levels which would be the same in the hydrogen atom. The fact is that the $3S$ (orbital quantum number = 0) is lower than the $3P(l = 1)$. The $3S$ electron penetrates the $1S$ shell more and is less effectively shielded than $3P$ electron, so the $3S$ level is lower. The fact that there is a doublet shows the smaller dependence of the atomic energy levels on the total angular momentum. The $3P$ level is split into states with total angular momentum $j = \frac{3}{2}$ and $j = \frac{1}{2}$ by the magnetic energy of the electron spin in the presence of the internal magnetic field caused by the orbital motion. This effect is called spin-orbit effect.



Hence we get D_1, D_2 lines due to the transitions shown in the fig.

PROBLEMS

1. An element is placed in a magnetic field of flux density 0·3 weber/m^2. Calculate the Zeeman shift of a spectral line of wavelength 4500Å. **[ANU M15]**

Sol : $B = 0{\cdot}3\text{Weber/m}^2$, $\lambda = 4500\text{Å} = 4{\cdot}5 \times 10^{-7}$ m,

$e = 1{\cdot}6 \times 10^{-19}\, C$

$m_0 = 9{\cdot}1 \times 10^{-31} Kg\,;\; C = 3 \times 10^8$ m/s

$$\Delta\lambda = \frac{eB\lambda^2}{4\pi m_0 C} = \frac{1{\cdot}6 \times 10^{-19} \times (0{\cdot}3) \times (4{\cdot}5 \times 10^{-7})^2}{4 \times 3{\cdot}14 \times (9{\cdot}1 \times 10^{-31}) \times (3 \times 10^8)} = 0{\cdot}02838\text{Å}.$$

2. The Zeeman components of a 500 nm spectral line are 0·0116 nm apart when the magnetic field is 1·00T. Find the ratio e/m_0 for the electron.

Sol : $\lambda = 500\, nm = t \times 10^{-7} m$; $\Delta\lambda = 0{\cdot}0116 nm = 0{\cdot}000116 \times 10^{-7} m$,

$B = 1{\cdot}00T = 1\, N/A - m.$

$$\Delta\lambda = \frac{Be\lambda^2}{4\pi m_0 C} \Rightarrow \frac{e}{m_0} = \frac{4\pi C}{B}\left[\frac{d\lambda}{\lambda^2}\right]$$

$$\frac{e}{m_0} = \frac{4\times(3{\cdot}14)\times(3\times10^8)}{1}\left[\frac{0{\cdot}000116\times10^{-7}}{(5\times10^{-7})^2}\right] \therefore \frac{e}{m_0} = 1{\cdot}75\times10^{11}\; C/Kg.$$

3. Calculate the frequency of vibration of CO molecule and spacing between its vibrational energy levels, given that the force constant K of the bond in CO is 187 Nm^{-1} and reduced mass of the CO molecule μ is $1{\cdot}14 \times 10^{-26}$ Kg. **[ANU J15]**

Sol : The frequency of vibration is given by

$$\nu_v = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{1}{2\pi} = \frac{187}{\sqrt{\left[\frac{187}{1{\cdot}14\times10^{-26}}\right]}} = 2{\cdot}04\times10^{13}\;\text{Hz}.$$

The separation ΔE between vibrational energy levels in CO is given by

$$\Delta E = E_{v+1} - E_v = h\nu_v = (6{\cdot}63 \times 10^{-34}) \times (2{\cdot}04 \times 10^{13})\ \mathrm{J} = 8{\cdot}44 \times 10^{-2}\ \mathrm{eV}.$$

4. A sample was excited by the 4358 $\mathring{A}$ line of mercury. A Raman line was observed at 4447 A°. Calculate the Raman shift in cm^{-1}.

[ANU O17; KU O18, O17, M15; SVU O17; VSU O18]

Sol : Given $\lambda_o = 4358\ \mathring{A} = 4358 \times 10^{-8}$ cm.

$\lambda_s = 4447\ \mathring{A} = 4447 \times 10^{-8}$ cm.

Raman shift, $\Delta\lambda = \dfrac{1}{\lambda_0} - \dfrac{1}{\lambda_s}$

$$= \frac{1}{4358 \times 10^{-8}} - \frac{1}{4447 \times 10^{-8}} = \frac{10^8}{4358} - \frac{10^8}{4447} = 22946 - 22487$$

$\therefore\ \Delta\lambda = 459\ cm^{-1}$.

5. The exciting line in an experiment is 5460 $\mathring{A}$ and the stokes line is at 5520 $\mathring{A}$. Find the wavelength of antistokes lines.

[ANU J16, M15; KU J15]

Sol : The wave number displacement in Raman effect is given by $\Delta\nu = \nu_0 - \nu_s$ for anti stokes lines. $= \nu_{a \cdot s} - \nu_0$

$$\nu_0 = \frac{1}{\lambda_0} = \frac{1}{5460 \times 10^{-8}\ cm} = 18315 cm^{-1}$$

$$\nu_s = \frac{1}{\lambda_s} = \frac{1}{5220 \times 10^{-8} cm} = 18116\ cm^{-1}$$

$\therefore\ \Delta\nu = \nu_0 - \nu_s = 18315 - 18116 = 199\ cm^{-1}$

For anti- stokes lines, $\nu_{a \cdot s} = \Delta\nu + \nu_0 = 199 + 18315 = 18514\ cm^{-1}$

$\therefore$ Wave length of antistokes line is $\lambda_{a \cdot s} = \dfrac{1}{\nu_{a \cdot s}} = \dfrac{1}{18514}$

$= 5 \cdot 401 \times 10^{-5}\ cm = 5401\ \mathring{A}$.

6. Calculate the wavelength separation between the component lines when a magnetic field of flux density 5 weber / m^2 is applied in normal Zeeman Effect experiment. (Specific charge of electron = $1{\cdot}76 \times 10^{11}$C/Kg, $\lambda = 6000^{\circ}A$ and $c = 3 \times 10^8$ m/s).

[KU J16]

Sol : Given $\dfrac{e}{m_o} = 1{\cdot}76 \times 10^{11}$ C/Kg

$\lambda = 6000\mathring{A} = 6 \times 10^{-7}$m

$C = 3 \times 10^8$m/s ; $B = 5$ weber / m^2

$$\Delta\lambda = \frac{eB\lambda^2}{4\pi m_o C} = \frac{1{\cdot}76 \times 10^{11} \times 5 \times (6 \times 10^{-7})^2}{4 \times 3{\cdot}14 \times 3 \times 10^8}$$

$\therefore\ \Delta\lambda = 8{\cdot}4 \times 10^{-11}$m $= 0{\cdot}84\mathring{A}$.

7. Calculate the wavelength separation between the unmodified line of wavelength $\lambda = 6000\mathring{A}$ and the modified line when a magnetic field of flux density 1 weber/m^2 is applied in normal Zeeman Effect. $e = 1{\cdot}6 \times 10^{-19}\ c$, $m_0 = 9{\cdot}1 \times 10^{-31}$Kg and $c = 3 \times 10^8$m/s. **[KU M16]**

Sol: Given $\lambda = 6000\mathring{A} = 6 \times 10^{-7}$m

$B = 1\ wb/m2\ ;\ e = 1{\cdot}6 \times 10^{-19C}$

$m_0 = 9{\cdot}1 \times 10^{-31}$Kg; $C = 3 \times 10^8$m/S

$$\Delta\lambda = \frac{eB\lambda^2}{4\pi m_0 C} = \frac{1{\cdot}6 \times 10^{-19} \times 1\ (6 \times 10^{-7})^2}{4 \times 3{\cdot}14 \times 9{\cdot}1 \times 10^{-31} \times 3 \times 10^8}$$

$$\therefore\ \Delta\lambda = 0{\cdot}1679 \times 10^{-10}\text{m} = 0.168\mathring{A}$$

8. In the normal Zeeman effect, the frequency separation between two consecutive spectral lines is 8.3×10^8 Hz. Find the magnetic field. $\mu_B = 9.27 \times 10^{-24}\ J/T$.

Sol : Given $\Delta\nu = 8.3 \times 10^8\ Hz$; $\mu_B = 9.27 \times 10^{-24}\ J/T$ **[ANU J15]**

$h = 6.63 \times 10^{-34}\ JS\ ;\ B = ?$

Formula, $\Delta\nu = \dfrac{Be}{4\pi m} = \dfrac{\mu_B B}{h}\quad [\because\ \mu_B = \dfrac{eh}{4\pi m}]$

$$B = \frac{h\Delta\nu}{\mu_B} = \frac{6.63 \times 10^{-34} \times 8.3 \times 10^8}{9.27 \times 10^{-24}} \quad \therefore\ B = \frac{6.63 \times 8.3}{9.27} \times 10^{-2} = 5.936 \times 10^{-2}\ T$$

9. In the normal Zeeman effect, the frequency separation between two consecutive spectral lines is $8.3 \times 10^8\ Hz$. Find the magnetic field.

Sol : Given $\Delta\nu = 8{\cdot}3 \times 10^8\ Hz$

$e = 1{\cdot}6 \times 10^{-19}C$

$m_0 = 9{\cdot}1 \times 10^{-31}\ Kg\ ;\ B = ?$

Formula, $d\nu = \dfrac{Be}{4\pi m_0}$

$$B = \frac{4\pi m_0 d\nu}{e} = \frac{4 \times 3{\cdot}14 \times 9{\cdot}1 \times 10^{-31} \times 8{\cdot}3 \times 10^8}{1{\cdot}6 \times 10^{-19}}$$

$$= \frac{4 \times 3{\cdot}14 \times 9{\cdot}1 \times 8{\cdot}3 \times 10^{-4}}{1{\cdot}6} \quad \therefore\ B = 592{\cdot}9 \times 10^{-4}T.$$

10. Find the possible spectral terms for $3p'$ electron in L-S coupling scheme.

Sol : $3p^1$ electron has $L = 1,\ S = \dfrac{1}{2}$

$J = L \pm S.$

$$\therefore\ J = 1 \pm \frac{1}{2} = \frac{3}{2} \text{ or } \frac{1}{2}$$

Spectral terms are $2_{P_{3/2}}$ and $2_{P_{1/2}}$

11. A material was excited by a radiation having a wavelength of 4358 $\mathring{A}$. A Raman line (stokes line) was observed at 4400 $\mathring{A}$. Calculate the Raman shift.

[ANU July 18]

Sol : $\lambda_0 = 4358\,\mathring{A} = 4358 \times 10^{-8}$ cm ; $\lambda_S = 4400\,\mathring{A} = 4400 \times 10^{-8}$ cm

Raman shift $\Delta\lambda = \frac{1}{\lambda_0} - \frac{1}{\lambda_S} = \frac{1}{4358 \times 10^{-8}} - \frac{1}{4400 \times 10^{-8}}$

$\Delta\lambda = 22,946 - 22,723$; $\therefore$ $\Delta\lambda = 223\ Cm^{-1}$.

12. If a magnetic field of $0{\cdot}027\ T$ is applied find separation of spectral lines ($e = 1{\cdot}6 \times 10^{-19}\ C,\ M_e = 9{\cdot}1 \times 10^{-31}\ Kg$).

[ANU July 18]

Sol : $\Delta\lambda = \frac{Be\lambda^2}{4\pi M_e C}$

Here $B = 0{\cdot}027\ T$; $e = 1{\cdot}6 \times 10^{-19} C$; $M_e = 9{\cdot}1 \times 10^{-31} Kg$; $C = 3 \times 10^8\ ms^{-1}$

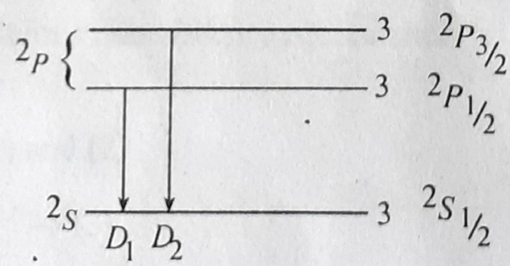
$$\Delta\lambda = \frac{(0{\cdot}027)\,(1{\cdot}6 \times 10^{-19})\,(\lambda^2)}{(4 \times 3{\cdot}14) \times (9{\cdot}1 \times 10^{-31})\,(3 \times 10^8)} = \frac{0{\cdot}0432}{342{\cdot}888} \times 10^4 \lambda^2$$

$$\therefore\ \Delta\lambda = \frac{432}{342{\cdot}888}\lambda^2 = 1{\cdot}25988\ \lambda^2.$$

13. Calculate the spectral lines for $2S$, $2P$ electron using $L - S$ coupling. **[SVU 18]**

Sol :

Level	l	S	Multiplicity $(2S+1)$	$J = l \pm \frac{1}{2}$	Full Notation
S	0	$\frac{1}{2}$	2	$\frac{1}{2}$	${}^2S_{\frac{1}{2}}$
P	1	$\frac{1}{2}$	2	$\frac{3}{2}, \frac{1}{2}$	${}^2P_{\frac{3}{2}}$, ${}^2P_{\frac{1}{2}}$



$\therefore$ Spectral lines for $2S$, $2P$ electron using $L - S$ couplingare D_1 and D_2.

14. Calculate the change in angular frequency of an electron due to magnetic field 1 weber / m^2.

Sol : $\Delta v = \frac{Be}{4\pi m_o}$

Here $B = 1\ Wb/m^2$; $e = 1{\cdot}6 \times 10^{-19} C$; $m_o = 9{\cdot}1 \times 10^{-31}\ Kg$

$$\Delta\nu = \frac{(1)(1{\cdot}6 \times 10^{-19})}{(4 \times 3{\cdot}14)(9{\cdot}1 \times 10^{-31})} = \frac{1{\cdot}6 \times 10^{12}}{12{\cdot}56 \times 9{\cdot}1}$$

$$\therefore \Delta\nu = 1{\cdot}3998 \times 10^{10}\ Hz.$$

15. A sample was excited by 4360 Å line of mercury a Raman line was observed at 4450 Å. Calculate the Raman shift in cm^{-1}.

Sol : Raman shift $\Delta\lambda = \frac{1}{\lambda_o} - \frac{1}{\lambda_s}$

Here $\lambda_o = 4360\ \text{Å} = 4360 \times 10^{-8}\ cm$; $\lambda_S = 4450\ \text{Å} = 4450 \times 10^{-8}\ cm$

$$\Rightarrow \Delta\lambda = \frac{1}{4360 \times 10^{-8}} - \frac{1}{4450 \times 10^{-8}} = \left(\frac{4450 - 4360}{4360 \times 4450}\right) 10^{8}\ cm^{-1}$$

$$= \frac{90}{4360 \times 4450} \times 10^{8}\ cm^{-1} \Rightarrow \Delta\lambda = \frac{90{,}0000}{4360 \times 4450} \times 10^{4}\ cm^{-1} = 0{\cdot}0463869 \times 10^{4}\ cm$$

$$\therefore \Delta\lambda = 463{\cdot}869\ cm^{-1}.$$

16. Calculate the specific charge of electron. Bohr magnetron of electron $\mu_B = 9{\cdot}274 \times 10^{-24}\ Amp - m^2$. **[ANU 018]**

Sol : $\mu_B = \frac{eh}{4\pi m} \Rightarrow \frac{e}{m} = \frac{4\pi\mu_B}{h}$

Here $\mu_B = 9{\cdot}274 \times 10^{-24}\ Amp - m^2$; $h = 6{\cdot}63 \times 10^{-34}$

$$\Rightarrow \frac{e}{m} = \frac{4 \times 3{\cdot}14 \times 9{\cdot}274 \times 10^{-24}}{6{\cdot}63 \times 10^{-34}} = 17{\cdot}5 \times 10^{10}$$

$$\therefore \frac{e}{m} = 1{\cdot}75 \times 10^{11}\ c/kg.$$

UNIT- II

2 MATTER WAVES & UNCERTAINTY PRINCIPLE

1. What are matterwaves ? Explain the de-Broglie concept of matterwaves. Obtain an expression for the wavelength of matterwaves. Give the properties of matter waves. **[ANU O18, J16; AdNU N17; AU 18, 17; BRAU O18, O17; RU O18, O17; SKU O18. VSU O18, S17]**

A. **Matter waves :** The waves associated with the material particles are called matter waves or de-Broglie waves.

de-Broglie concept of matterwaves :

1) According to de-Broglie a moving particle has wave properties associated with it.
2) A material particle of mass m moving with a velocity V behaves like a wave of wavelength λ.

   de-Broglie wavelength $(\lambda) = \dfrac{h}{mV} = \dfrac{h}{p}$

   Here $P = mV$ is the momentum of the particle and h is the Planck's constant. The corresponding wavelength is called the de-Broglie wavelength.

Expression for the wavelength of matterwaves :

1) According to Planck's quantum theory

   $E = h\nu = \dfrac{hc}{\lambda} \rightarrow (1)$

   Where c is the velocity of light in vaccum and λ is the wavelength.
2) According to Einstein's mass-energy equivalence

   $E = mc^2 \rightarrow (2)$
3) From equations (1) and (2)

   $mc^2 = \dfrac{hc}{\lambda}\ ;\ \lambda = \dfrac{h}{mc} \rightarrow (3)$

   where $mc = p$ (Momentum associated with photon)
4) Consider a material particle of mass m moving with a velocity V, then the wavelength associated with this particle can be expressed as

   $\lambda = \dfrac{h}{P} = \dfrac{h}{mV} \rightarrow (4)\ \ (\because\ P = mV)$
5) If E is the kinetic energy of the material particle, then

   $E = \dfrac{1}{2}mV^2 = \dfrac{P^2}{2m}$

$\Rightarrow P = \sqrt{2mE}$ $\therefore$ de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mE}} \rightarrow (5)$

6) When a charged particle carrying a charge q is accelerated by a potential difference V, then its K.E is given by $E = qV$

7) Hence, de-Broglie wavelength $(\lambda) = \dfrac{h}{\sqrt{2mqV}}$

Properties :

1) Lighter is the particle, smaller is the mass (m) and larger is the wavelength of the matter wave

$$\lambda = \frac{h}{mV} = \frac{h}{p}$$

2) Smaller is the velocity (V) of the particles, larger is the wavelength.
3) Matter waves are exhibited by any particle that is in motion.
4) Matter waves are associated with charge less particles also like neutron.
5) The velocity of matter waves is not a constant.
6) The velocity of matter waves can be greater than the velocity of light.
7) Matter waves are not produced by any distrubance created in any material medium.

2. Describe the Davisson and Germer experiment and its result.

[ANU J18, O17, J15, M16; AdNU O18 N17; AU. 17; BRAU O18, O17; KU O18, M16, J15; RU O18, O17; SKU O18, O17; SVU O18, O17; VSU S17; YVU O18, N17]

Or

What are matter waves ? How they are experimentally verified ? Explain.

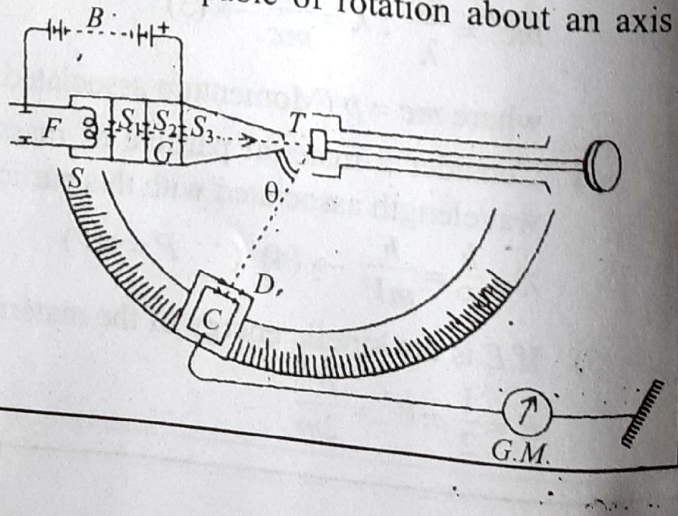
A. **Matter Waves :**

The waves associated with material particles are called matter waves.

Description : As shown in figure the arrangement consists of the following.

1) An electron gun consists of a tungsten filament F. This filament is electrically heated to dull red to emit electrons by thermionic emission. These electrons are accelerated through the potential difference generated by the battery B. The electron beam is collimated into a narrow fine beam by means of suitable slits S_1, S_2, S_3.

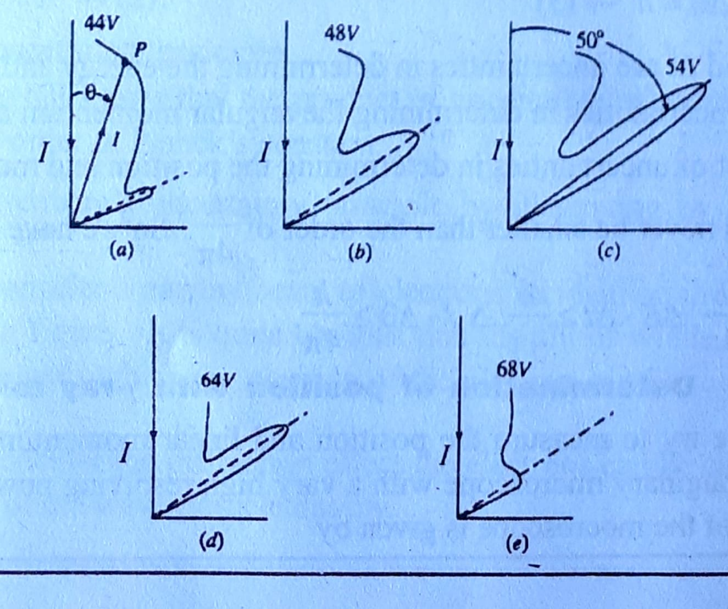
2) The outcoming electron beam is directed to fall on the large single crystal of nickel. This crystal is the target T and is capable of rotation about an axis perpendicular to the plane of the page. The electron beam scattered in different directions. The angular distribution of scattered electrons and their respective intensities are measured with the help of a Faraday cylinder c, a circular scale ss and a galvanometer G.M.



3) Faraday cylinder C called the collector, acts as an electron detector and is connected to a sensitive galvanometer. The Faraday cylinder consists of two cylinders D and C insulated from each other and provided with co-axial slits. A retarding potential is maintained between the outer and inner cylinder only fast moving electrons coming from electron gun G and scattered by T can enter into the inner cylinder. Any secondary electrons produced in collisions with the target atoms are reflected away by the Faraday cylinder. Thus, only those electrons having nearly the same velocity as that coming out from the electron gun and incident on the target may enter into the collector c. These electrons are detected by the sensitive galvanometer. The galvanometer deflection is directly proportional to the intensity of electron current.
4) The Faraday cylinder can be rotated along a graduated circular scale ss, so that it can receive the scattered electrons at all angles between 20° and 90°. Thus the angular distribution of scattered electrons can be determined.
5) The accelerating potential V provided by the battery can be changed from 30 V to 600 V. The retarding potential will be $\frac{9}{10}^{th}$ of the accelerating potential each time.
6) The whole apparatus was completely enclosed and highly evacuated.

Experimental procedure :

1) Let us consider the normal incidence of the electron on the target crystal surface. The surface of the target (nickel) acts as a diffraction grating to the incident electron beam. For each azimuth of the crystal, a beam of low voltage electrons is made to fall normally on the crystal surface. The collector is moved to various positions along the circular scale ss. At each position, the deflection in the galvanometer is noted.
2) This deflection gives a measure of the intensity of the diffracted beam of electrons. The angle between the incident beam and the beam entering the collector is known as co-lattitude angle. It is also a angle of diffraction. The intensity of scattered electron beam was measured at different angles. The experiment is repeated for several accelerating voltages V.



3) It is observed that a bump begins to appear in the curve for 44 volts electrons. According to de-broglie, the wave length associated with moving electron is given by

$$\lambda = \frac{12 \cdot 26}{\sqrt{V}} \mathring{A}$$

4) The bumb becomes most prominent at $V = 54$ volt.

$$\therefore \lambda = \frac{12 \cdot 26}{\sqrt{54}} = 1 \cdot 67\mathring{A}$$

5) From $X-$ ray analysis, A nickel crystal acts as a plane diffraction grating ($d = 0 \cdot 91\mathring{A}$). According to the diffracted electron beam at ($\theta = 50^{o}$).

$$\theta^{1} = \frac{180^{o} - 50^{o}}{2} = 65^{o}$$

6) Using Bragg's equation (taking $n = 1$)

$$\lambda = 2d \sin \theta^{1} = 2\,(0 \cdot 91\mathring{A}) \sin 65^{o}$$
$$\lambda = 1 \cdot 65\mathring{A}$$

Result : This is in good agreement with de - broglie wave length, hence confirms the de-broglie concept of matter waves.

3. State Heisenberg's uncertainity? Explain gamma ray microscope as a consequence of uncertainty principle.

Or

State and prove Heisenberg uncertainty principle with example.

[ANU J18, O17; SKU O17; VSU O18, S17]

A. **Heisenberg uncertainty principle :**

1) "It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy".

2) Taking Δx as the error in determining its position and ΔP the error in determining its momentum at the same instant, these quantities are related as follows.

$$\Delta x \cdot \Delta P \approx h \rightarrow (1)$$

Similarly $\Delta E \cdot \Delta t \approx h \rightarrow (2)$

$$\Delta J \cdot \Delta \theta \approx h \rightarrow (3)$$

where ΔE and Δt are uncertainties in determining the energy and time. While ΔJ and $\Delta\theta$ are uncertainties in determining the angular momentum and angle.

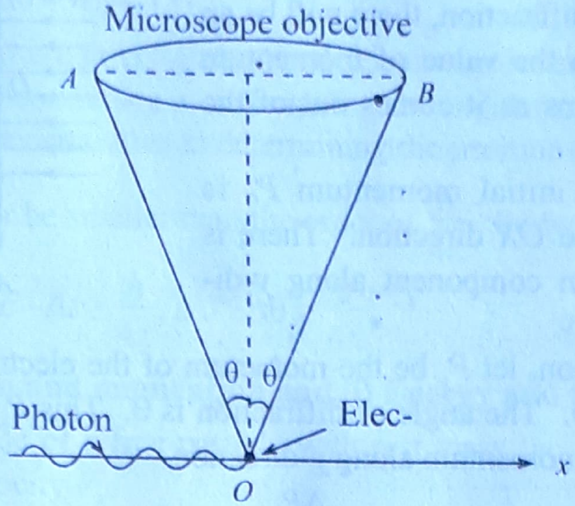
3) The product of uncertainties in determining the position and momentum of the particle can never be smaller than the order of $\frac{h}{4\pi}$. So we have

$$\Delta P \cdot \Delta x \geq \frac{h}{4\pi}, \Delta E \cdot \Delta t \geq \frac{h}{4\pi}, \Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}.$$

Illustration : Determination of position with γ-ray microscope :

1) Suppose we try to measure the position and linear momentum of an electron using an imaginary microscope with a vary high resolving power. The resolving power of the mocroscope is given by

$$\Delta x = \frac{\lambda}{2\sin\theta} \rightarrow (1)$$



where Δx is the minimum distance between two points in the field of view θ is the semivertical angle of cone of radiation entering the microscope.

2) If position of electron changed by Δx, the microscope will not be able to detect it. Thus Δx is the uncertainty in the measurement of electron position.

3) The incident photon has momentum $\frac{h}{\lambda}$. It collides with electron. The scattered photon may enter the objective lens anywhere between OA and OB.

4) The component of this momentum along OA is $\frac{-h}{\lambda}\sin\theta$ and that along OB is $\frac{h}{\lambda}\sin\theta$.

5) Hence the uncertainty in the momentum measurment in the x-direction is

$$\Delta P_x = \frac{h}{\lambda}\sin\theta - \left(\frac{-h}{\lambda}\sin\theta\right) = \frac{2h}{\lambda}\sin\theta$$

$$\therefore\ \Delta x \cdot \Delta P_x = \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta \approx h$$

$$\Delta x \cdot \Delta P_x \approx h \rightarrow (2)$$

This is uncertainty principle.

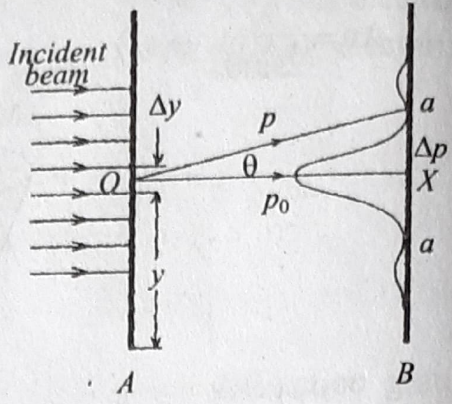
6) Equation (2) shows that the product of uncertainty in position and momentum is of the order of Planck's constant.

4. Explain Heisenberg's uncertainty principle by illustrating by diffraction through single slit.

A. 1) Let us consider a narrow beam of electrons incident on the slit of width Δy as shown in figure. It is quite obvious that the slit of width $d = \Delta y$ gives us the accuracy with which the position of the electron can be measured in the y-direction.

2) Let us consider the diffraction effect the electrons can be considered as waves because of the wave-particle duality.

3) The diffraction pattern will be as shown in the figure. As there is a spread of the wave due to diffraction, there will be an uncertainty in the value of momentum of the electrons as it comes out of the slit.



4) The original initial momentum P_0 is only along the OX direction. There is no momentum component along y-direction initially.

5) After diffraction, let P_y be the mometum of the electron as it reaches the first minimum at a. The angle of diffraction is θ. This P_y itself will be the uncertainty. ΔP_y is momentum along y-direction.

For small θ values, $\tan\theta \approx \theta \approx \dfrac{\Delta P_y}{P_0} \rightarrow (1)$

6) From the diffraction formula

$n\lambda = d\sin\theta \rightarrow (2)$

For first order diffractions $n = 1$ and $d = \Delta y$ we have

$\sin\theta = \dfrac{\lambda}{\Delta y}$ and for small θ.

$\sin\theta \approx \theta \approx \dfrac{\lambda}{\Delta y} \rightarrow (3)$

7) From equations (1) and (3), we have $\dfrac{\Delta P_y}{P_0} \approx \dfrac{\lambda}{\Delta y}$

$\Delta P_y \cdot \Delta y \approx \lambda P_0 \rightarrow (4)$

8) From de-Broglies hypothesis we have $\lambda = \dfrac{h}{P_0}$

$\Delta P_y \cdot \Delta y \approx \left(\dfrac{h}{P_0}\right) P_0$

$\Delta P_y \, \Delta y \approx h \rightarrow (5)$

The product of the uncertainty in mmentum and uncertainty in position is of the order of Planck's constant. This is nothing but the Heisenberg's uncertainty principle.

5. State Heisenberg's uncertainty principle and apply in case of
i) Position and momentum & ii) Energy and time.

[ANU 018; AdNU 18; AU 18; BRAU 18; KU 18; SKU 18]

A. **Heisenberg's uncertainty principle :**

1) "It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy".

2) Taking Δx as the error in determining its position and ΔP the error in determining its momentum at the same instant, these quantities are related as follows.

$$\Delta x \cdot \Delta P \approx h \rightarrow (1)$$

3) Similarly $\Delta E \cdot \Delta t \approx h \rightarrow (2)$

$$\Delta J \cdot \Delta\theta \approx h \rightarrow (3)$$

where ΔE and Δt are uncertainties in determining the energy and time. While ΔJ and $\Delta\theta$ are uncertainties in determining the angular momentum and angle.

4) The product of uncertainties in determining the position and momentum of the particle can never be smaller than the order of $\frac{h}{4\pi}$. So we have

$$\Delta P \cdot \Delta x \geq \frac{h}{4\pi}, \Delta E \cdot \Delta t \geq \frac{h}{4\pi}, \Delta J \cdot \Delta\theta \geq \frac{h}{4\pi}.$$

i) In case of Position and momentum and ii) Energy and time :

5) Consider the case of a free particle with rest mass 'm_o' moving along X – direction with velocity V_x.

The kinetic energy is given by $E = \frac{1}{2} m_o V_x^2 = \frac{P_x^2}{2m_o} \rightarrow (1)$

6) If ΔP_x and ΔE be the uncertainties in momentum and energy respectively, then differentiating equation (1),

We have $\Delta E = \frac{1}{2m_o}(2P_x \Delta P_x) \Rightarrow P_x \Delta P_x = m_o \Delta E$

$$\therefore \Delta P_x = \frac{m_o}{P_x}\Delta E = \frac{1}{V_x}\Delta E \rightarrow (2) \quad [\because P_x = m_o V_x]$$

7) Let the uncertainty in the time intervel for measurement at point x is Δt, then uncertainty Δx in position is $\Delta x = V_x \Delta t \rightarrow (3)$

8) From equations (2) and (3), $\Delta x \, \Delta P_x = \Delta t \, \Delta E \rightarrow (4)$

We know that $\Delta x \, \Delta P_x \geq \frac{h}{4\pi}$

$$\therefore \Delta t \, \Delta E \geq \frac{h}{4\pi} \rightarrow (5)$$

This is the Time-energy uncertainty principle.

6. Estimate the wavelength of a matter wave and discuss the complementary principle of Bohr. **[ANU O18; SVU 18]**

A. **Complementary principle of Bohr :** "Any new theory in physics must reduce to well-established corresponding classical theory when the new theory is applied to the special situation in which the less general theory is known to be valid".

Explanation :

1) According to classical theory, the frequency of the spectral line is the same as the orbital frequency of the electron $\left(\nu = \frac{\omega}{2\pi}\right)$.

2) But in Bohr's theory, the frequency of the spectral line is determined by the difference in energy between two orbital states : $\nu = (E_i - E_f)/h$.

3) Let us consider an atom of effectively infinite mass.

Then $E_n = \frac{-me^4}{8\varepsilon_o n^2 h^2} \rightarrow (1)$

4) If n is sufficiently great, the energy change ΔE corresponding to a change of n by Δn is obtained by differentiating equation (1)

$$\Delta E_n = \frac{me^4}{4\varepsilon_o^2 h^2 n^3} \Delta n \rightarrow (2)$$

$$\therefore \nu = \frac{\Delta E_n}{h} = \frac{me^4}{4\varepsilon_o^2 h^3 n^3} \Delta n \rightarrow (3)$$

5) According to Bohr's first postulate, $\frac{nh}{2\pi} = mr^2\omega$

or $nh = 2\pi mr^2\omega$

$$\therefore \nu = \frac{me^4}{4\varepsilon_o^2 (2\pi mr^2\omega)^3} \Delta n = \frac{me^4}{32\pi^3 \varepsilon_o^2 m^3 r^6 \omega^3} \Delta n \rightarrow (4)$$

6) Now for the equilibrium in the orbit, we have,

$$mr\omega^2 = \frac{1}{4\pi\varepsilon_o} \frac{e^2}{r} \text{ or } \frac{1}{r^3} = \frac{4\pi\varepsilon_o m\omega^2}{e^2}$$

$$\therefore \nu = \frac{me^4}{32\pi^3 \varepsilon_o^2 m^3 \omega^3} \left[\frac{4\pi\varepsilon_o m\omega^2}{e^2}\right]^2 \Delta n \text{ or } \nu = \frac{\omega}{2\pi} \Delta n$$

Now, If $\Delta n = 1$, $\nu = \frac{\omega}{2\pi}$

7) Thus the frequency given by the quantum theory for two very large quantum numbers and separated by units becomes identical with the orbital frequency and hence with the classical frequency.

8) Bohr's correspondence principle explained the polarised spectral lines and velocity, where as Bohr's general principle has not explained.

SHORT ANSWER TYPE QUESTIONS

1. Show that the phase velocity of de-Broglie waves is greater than the velocity of light.

A. 1) De-Broglie wavelength $(\lambda) = \frac{h}{mV} \rightarrow (1)$

where m is the mass of the particle and V be the velocity of the particle.

2) This wave will have its frequency ν and velocity ω that are related to its wavelength by

$\omega = \nu\lambda \rightarrow (2)$

3) From quantum theory, the energy of the particle is

$E = h\nu \rightarrow (3)$

and from relativity $E = mc^2 \rightarrow (4)$

4) From equations (3) and (4) $h\nu = mc^2$

$\nu = \frac{mc^2}{h} \rightarrow (5)$

5) From equations (5) and equation (2), we get $\omega = \left(\frac{mc^2}{h}\right)\lambda \rightarrow (6)$

6) and from (6) and (1) $\omega = \left(\frac{mc^2}{h}\right)\frac{h}{mV} = \frac{c^2}{V}$; $\omega = \frac{c^2}{V} \rightarrow (7)$

7) Now according to relativity, the particle velocity V should always be less than the velocity of light c. That is the velocity of de-Broglie wave ω is always greater than the velocity of light c.

2. Explain wave velocity and group velocity **[ANU J18, J15; KU J16, M15]**

A. **Wave velocity (V_p) or Phase velocity :**

The wave velocity is the velocity with which the planes of constant phase advance through the medium.

Phase velovity $(V_p) = \frac{\omega}{k}$

Phase velocity of de-Broglie waves :

1) A particle of mass m moving with a velocity V is associated with it whose wavelength is given by

$$\lambda = \frac{h}{mV} \rightarrow (1)$$

2) Let E be the total energy of particle. Then according to quantum expression

$$E = h\nu$$

$$\nu = \frac{E}{h}$$

3) $\therefore$ Angular frequency $(\omega) = 2\pi\nu = \frac{2\pi E}{h}$

$$\omega = \frac{2\pi mc^2}{h} \quad (\because E = mc^2)$$

4) Now de-Broglie wave velocity V_p is given by

$$V_p = \frac{\omega}{k} = \left(\frac{2\pi mc^2}{h}\right) \times \frac{h}{2\pi(mV)} ; V_p = \frac{c^2}{V}$$

where K is propagation constant.

$$K = \frac{2\pi}{\lambda}$$

Group velocity (V_g) :

The velocity with which the group of waves travels is called group velocity.

$$V_g = \frac{d\omega}{dk}$$

Group velocity of de-Broglie waves :

1) A particle moving with a velocity V is supposed to consist of a group of waves, according to de-Broglie hypothesis.

Group velocity $(V_g) = \frac{d\omega}{dk}$

2) The angular frequency and wave number of the de-Broglie waves associated with a parcticle of rest mass m_0 moving with the velocity V is given by

$$\omega = 2\pi\nu = \frac{2\pi mc^2}{h} = \frac{2\pi m_0 c^2}{h\sqrt{1-\frac{V^2}{c^2}}} \rightarrow (1)$$

$$\text{and } k = \frac{2\pi}{\lambda} = \frac{2\pi mV}{h} = \frac{2\pi m_0 V}{h\sqrt{1-\frac{V^2}{c^2}}} \rightarrow (2)$$

3) Differentiating equations (1) and (2) we have

$$\frac{d\omega}{d\nu} = \frac{2\pi m_0 V}{h\left[1-\frac{V^2}{c^2}\right]^{\frac{3}{2}}} \rightarrow (3) \text{ and } \frac{dV}{d\nu} = \frac{2\pi m_0}{h\left[1-\frac{V^2}{c^2}\right]^{\frac{3}{2}}} \rightarrow (4)$$

4) Group velocity $(V_g) = \frac{d\omega}{dk} = \frac{d\omega}{d\nu} \times \frac{d\nu}{dk} = V$

Hence the de-Broglie wave group associated with a moving particle travels with same velocity as the particle.

3. Derive the Relation between phase velocity and group velocity.

A. 1) Wave velocity $(V_p) = \frac{\omega}{k} \rightarrow (1)$

2) Group velocity $= \frac{d\omega}{dk} \rightarrow (2)$

3) The wave number $(k) = \frac{2\pi}{\lambda}$

$$\therefore \frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2} \rightarrow (3)$$

Also $\omega = 2\pi\nu = \frac{2\pi V_p}{\lambda}$; $\frac{d\omega}{d\lambda} = 2\pi\left[\frac{-V_p}{\lambda^2} + \frac{1}{\lambda}\cdot\frac{dV_p}{d\lambda}\right]$

$$\frac{d\omega}{d\lambda} = \frac{-2\pi}{\lambda^2}\left[V_p - \lambda\frac{dV_p}{d\lambda}\right] \rightarrow (4)$$

4) Dividing equation (4) by equation (3), we get

$$\frac{d\omega}{d\lambda}\cdot\frac{d\lambda}{dk} = \frac{\frac{-2\pi}{\lambda^2}\left[V_p - \lambda\frac{dV_p}{d\lambda}\right]}{\frac{-2\pi}{\lambda^2}} \; ; \; \frac{d\omega}{dk} = V_p - \lambda\frac{dV_p}{d\lambda} \; ; \; V_g = V_p - \lambda\frac{dV_p}{d\lambda} \rightarrow (5)$$

5) Equation (5) represents a relationship between group velocity V_g and phase velocity V_p.

4. State and explain Heisenberg's uncertainty principle. **[ANU O18; AdNU N17; AU 18, 17; KU J15, M15; SVU O18; YVU O18]**

A. **Heisenberg uncertainty principle :**

1) "It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy".
2) Taking Δx as the error in determining its position and ΔP the error in determining its momentum at the same instant, these quantities are related as follows.

$$\Delta x \cdot \Delta P \approx h \rightarrow (1)$$

3) Similarly $\Delta E \cdot \Delta t \approx h \rightarrow (2)$

$$\Delta J \cdot \Delta\theta \approx h \rightarrow (3)$$

where ΔE and Δt are uncertainties in determining the energy and time. While ΔJ and $\Delta\theta$ are uncertainties in determining the angular momentum and angle.

4) The product of uncertainties in determining the position and momentum of the particle can never be smaller than the order of $\frac{h}{4\pi}$. So we have

$$\Delta P \cdot \Delta x \geq \frac{h}{4\pi}, \Delta E \cdot \Delta t \geq \frac{h}{4\pi}, \Delta J \cdot \Delta\theta \geq \frac{h}{4\pi}.$$

5. Explain the consequencies of uncertainty principle. (or) Explain complementarity principle of bohr. **[ANU O17; SKU O17; SVU O18]**

A. 1) The uncertainty relation in a way explains the possibility of wave-particle duality present in nature.
2) If we treat the entity completely as a particle there will be no question of considering its wave properties and vice versa.
3) A single experiment can never probe simultaneously into the two different natures.
4) If we develop a very careful experiment to reveal out its wave character, an electron for example will hide its particle character which becomes fuzzy.
5) On the other hand if we develop a very careful experiment to reveal out its particle nature, the same electron will now hide its wave character will become fuzzy.
6) "The two views are however complementary to eachother and not contradictory. Both are necessary to have a comprehensive understanding of the system". This is called Bohr's principle of complementary.

6. Discuss importance of uncertainty principle. Write its applications.

A. **Importance :**

1) This theory also applicable big bodiės like sun and moon other than small particles like electrons, protons. In the case of big bodies comparing experimental errors, uncertainty values are very small. Thus uncertainty principle does not effected.
2) Uncertainty principle explains particle & wave nature of matter, radiation in new concepts.

Applications :

1) It is used to explain Non-existance of electrons and existance of protons and neutrons in nucleus.
2) It is used to binding energy of an electron in atoms.
3) It is used determine the radius of the Bohr's first orbit.
4) It is used to frequency of radiation of light from an excited atom.

7. Write properties of matter waves.

A. **Properties :**

1) Lighter is the particle, smaller is the mass (m) and larger is the wavelength of the matter wave $\lambda = \frac{h}{mV} = \frac{h}{p}$.
2) Smaller is the velocity (V) of the particles, larger is the wavelength.
3) Matter waves are exhibited by any particle that is in motion.
4) Matter waves are associated with charge less particles also like neutron.
5) The velocity of matter waves is not a constant.
6) The velocity of matter waves can be greater than the velocity of light.
7) Matter waves are not produced by any distrubance created in any material medium.

PROBLEMS

1. Calculate the de-Broglie wavelength associated with a proton moving with a velocity equal to 1/20 th of the velocity of light.

Sol : Velocity of proton $(v) = \frac{1}{20} \times$ Velocity of light

$$= \frac{1}{20} \times 3 \times 10^8 = 1{\cdot}5 \times 10^7 \text{ m/s}$$

Mass of the proton $= 1{\cdot}67 \times 10^{-27}$ Kg

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}62 \times 10^{-34}}{1{\cdot}67 \times 10^{-27} \times 1{\cdot}5 \times 10^7} = 2{\cdot}64 \times 10^{-14} \text{ m.}$$

2. Calculate the energy of the proton in terms of electron volt whose de-Broglie wavelength is 1Å. (Proton mass $= 1{\cdot}66 \times 10^{-27}$ Kg. **[SKU 018]** Planck's constant $(h) = 6{\cdot}6 \times 10^{-34}$ J – sec)

Sol : Given that $\lambda = 1\text{Å} = 1 \times 10^{-10}$ m

$h = 6{\cdot}6 \times 10^{-34}$ J – sec, $m = 1{\cdot}66 \times 10^{-27}$ Kg

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{2m\lambda^2}$$

$$\therefore\ E = \frac{(6{\cdot}6 \times 10^{-34})^2}{2 \times 1{\cdot}66 \times 10^{-27} \times (10^{-10})^2} = 13{\cdot}61 \times 10^{-21} \text{ Joules}$$

$$E = \frac{13{\cdot}61 \times 10^{-21}}{1{\cdot}6 \times 10^{-19}} = 8{\cdot}51 \times 10^{-2} \text{ eV.}$$

3. What would be the wavelength of quantum of radiant energy emitted, if an electron transmitted into radiation and converted into one quantum ?

Sol : According to Planck, the energy E associated with one quantum $E = h\nu$

$$E = mc^2$$

$$\therefore\ h\nu = mc^2 \text{ or } \frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}\ ;\ \lambda = 0.0244\times10^{-10}\ \text{m} = 0.0242\ \text{Å}.$$

4. Compute the de-Broglie wavelength of 10^{11} kev neutron. Mass of neutron may be, taken as 1.675×10^{-27} Kg.

Sol : Given that K.E of neutron $= 10^{11}$ kev$= 10^{14}$ ev$= 10^{14}\times1.6\times10^{-19}\ \text{J} = 1.6\times10^{-5}\ \text{J}$

$$\text{Now } \frac{1}{2}mv^2 = 1.6\times10^{-5}\ \text{J}$$

$$v = \left(\frac{2\times1.6\times10^{-5}}{1.675\times10^{-27}}\right)^{\frac{1}{2}}\ \text{m/s}$$

$$\text{Again } \lambda = \frac{h}{mv} = \frac{6.625\times10^{-34}}{1.675\times10^{-27}}\times\left(\frac{1.675\times10^{-27}}{2\times1.6\times10^{-5}}\right)^{\frac{1}{2}}$$

$$\lambda = 2.86\times10^{-18}\ \text{m}.$$

5. Compute the de-Broglie wavelength of a proton whose K.E is equal to the rest mass energy of the electron. (Rest mass energy of electron = 0.512 MeV)

Sol : Rest mass energy of electron = 0.512 MeV

$$\text{Energy of proton } (E) = \frac{1}{2}mv^2 = 0.512\ \text{MeV}$$

$$E = 0.512\times10^{6}\times1.6\times10^{-19} = 8.193\times10^{-14}\ \text{J}$$

$$m = 1.673\times10^{-27}\ \text{Kg}$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times8.193\times10^{-14}}{1.673\times10^{-27}}} = 9.897\times10^{6}\ \text{m/s}$$

$$\text{Now } \lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(1.673\times10^{-27})(9.897\times10^{6})} = 0.0004004\times10^{-10}\ \text{m}$$

$$\lambda = 0.0004004\ \text{Å}.$$

6. Calculate the wavelength of an electron subjected to a potential difference of 1000 kv. **[VSU 018]**

Sol : $V = 1000$ V

$$\lambda = \frac{12.26}{\sqrt{V}}\ \text{Å} = \frac{12.26}{\sqrt{1000}} = \frac{12.26}{10\sqrt{10}} = \frac{12.26}{31.62} = 0.3877\ \text{Å}.$$

7. What voltage must be applied to an electron microscope to produce electrons of wavelength 0·5Å ?

Sol : $\lambda = 0{\cdot}5\ \text{Å} = 0{\cdot}5 \times 10^{-10}$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{h^2}{2m\lambda^2} \quad (\because E = eV)$$

$$eV = \frac{h^2}{2m\lambda^2}$$

$$V = \frac{h^2}{2m\lambda^2 e} = \frac{(6{\cdot}62 \times 10^{-34})^2}{2 \times 9 \times 10^{-31} \times (0{\cdot}5 \times 10^{-10})^2 \times 1{\cdot}6 \times 10^{-19}}$$

$V = 608{\cdot}6$ volts.

8. An electron initially at rest is accelerated through a potential difference of 54V. Calculate the velocity of the electron and de-Broglie wavelength. **[KU 018]**

Sol : $\lambda = \frac{12{\cdot}26}{\sqrt{V}} = \frac{12{\cdot}26}{\sqrt{54}} = 1{\cdot}666\ \text{Å}$

De-Broglie wavelength $\lambda = \frac{h}{mv}$

$$v = \frac{h}{m\lambda} = \frac{6{\cdot}62 \times 10^{-34}}{9{\cdot}1 \times 10^{-31} \times 1{\cdot}666 \times 10^{-10}}$$

$v = 4{\cdot}371 \times 10^{6}$ m/s.

9. What voltage must be applied to an electron microscope to produce electrons of wavelength 0·40 Å ?

Sol : $\lambda = 0{\cdot}40\ \text{Å} = 0{\cdot}4 \times 10^{-10}$ m ; $h = 6{\cdot}6 \times 10^{-34}$

$e = 1{\cdot}6 \times 10^{-19}$, $m = 9{\cdot}1 \times 10^{-31}$ kg

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}} \quad (\because E = eV)$$

$$V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{(6{\cdot}6 \times 10^{-34})^2}{2 \times 9{\cdot}1 \times 10^{-31} \times 1{\cdot}6 \times 10^{-19} \times (0{\cdot}4 \times 10^{-10})^2} \ ; \ V = 935 \text{ volts.}$$

10. With what velocity must an electron travel its momentum is equal to that of a photon with a wavelength of $\lambda = 5200$ Å ?

Sol : $\lambda = 5200\ \text{Å} = 5200 \times 10^{-10}$ m

Momentum of electron $(P) = mv \rightarrow (1)$

Momentum of photon $\frac{hv}{c} = \frac{h}{\lambda} \rightarrow (2)$

$$mv = \frac{h}{\lambda};\ v = \frac{h}{m\lambda}$$

$$v = \frac{6{\cdot}6 \times 10^{-34}}{(9{\cdot}1 \times 10^{-31}) \times 5200 \times 10^{-10}} = 1395 \text{ m/s.}$$

11. Find the wavelength associated with 1 gram of mass having a velocity 2000 m/sec ($h = 6{\cdot}62 \times 10^{-34}$ $J{\cdot}s$)

Sol : $m = 1$ gm $= 10^{-3}$ Kg

$v = 2000$ m/s $h = 6{\cdot}62 \times 10^{-34}$ J − s

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}62 \times 10^{-34}}{10^{-3} \times 2000} = 3{\cdot}31 \times 10^{-34} \text{ m}$$

$\lambda = 3{\cdot}31 \times 10^{-24}$ Å ($\because$ 1Å $= 10^{-10}$ m).

12. Find the energy of the neutron in ev whose de-Broglie wavelength is 1Å.

[SKU N17]

Sol : $\lambda = 1\text{Å} = 10^{-10}$ m, $h = 6{\cdot}6 \times 10^{-34}$ $J{\cdot}s$, $m = 1{\cdot}674 \times 10^{-27}$ Kg

$$\lambda = \frac{h}{mv} \text{ and } E = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2E}{m}}$$

$$mv = m \times \sqrt{\frac{2E}{m}} = \sqrt{2Em}$$

$$\lambda = \frac{h}{\sqrt{2Em}} \Rightarrow E = \frac{h^2}{2m\lambda^2}$$

$$E = \frac{(6{\cdot}6 \times 10^{-34})^2}{2 \times 1{\cdot}674 \times 10^{-27} \times (10^{-10})^2} = 13{\cdot}01 \times 10^{-21} \text{ Joules}$$

$$E = \frac{13{\cdot}01 \times 10^{-21}}{1{\cdot}6 \times 10^{-19}} = 8{\cdot}131 \times 10^{-2} \text{ ev.}$$

13. Calculate the wavelength associated with an electron having K.E equal to 1·512 MeV. Rest mass energy = 0·512 MeV.

Sol : K.E of the electron = 1·512 MeV

Rest mass energy = 0·512 MeV

$$mc^2 = m_0c^2 + \text{K.E} = 0{\cdot}512 + 1{\cdot}512 = 2{\cdot}024 \text{ MeV}$$

$$mc^2 = 2{\cdot}024 \times 10^6 \times 1{\cdot}6 \times 10^{-19} \text{ Joule}$$

$$m = \frac{2{\cdot}024 \times 10^6 \times 1{\cdot}6 \times 10^{-19}}{(3 \times 10^8)^2} = 3{\cdot}598 \times 10^{-30} \text{ Kg}$$

We know that $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{m} = \frac{9{\cdot}1 \times 10^{-31}}{3{\cdot}598 \times 10^{-30}} \quad (\because m_0 = 9{\cdot}1 \times 10^{-31} \text{ Kg})$$

$$\sqrt{1-\frac{v^2}{c^2}} = 0{\cdot}2528$$

$$1-\frac{v^2}{c^2} = (0{\cdot}2528)^2 = 0{\cdot}06395$$

$$\frac{v^2}{c^2} = 1 - 0{\cdot}06395 = 0{\cdot}93605$$

$$v^2 = 0{\cdot}93605 \times (3 \times 10^8)^2 \; ; \; v = 2{\cdot}902 \times 10^8 \text{ m/s}$$

$$\text{Now } \lambda = \frac{h}{mv} = \frac{6{\cdot}63 \times 10^{-34}}{3{\cdot}598 \times 10^{-30} \times 2{\cdot}902 \times 10^8} = 0{\cdot}006348 \times 10^{-10} \text{ m}$$

$$\lambda = 0{\cdot}006348 \text{ Å}.$$

14. An electron falls from rest through a potential difference of 100 V. What is the de-Broglie wavelength ? (Planck's constant $= 6{\cdot}6 \times 10^{-34}$ Js, mass of electron $= 9{\cdot}1 \times 10^{-31}$ Kg, charge of the electron $= 1{\cdot}6 \times 10^{-19}$c.

Sol : Given $V = 100$ volts

$$h = 6{\cdot}6 \times 10^{-34} \text{ Js}$$

$$m = 9{\cdot}1 \times 10^{-31} \text{ Kg}, \; e = 1{\cdot}6 \times 10^{-19} \text{ c}$$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6{\cdot}6 \times 10^{-34}}{\sqrt{2 \times 9{\cdot}1 \times 10^{-31} \times 1{\cdot}6 \times 10^{-19} \times 100}} = 122 \times 10^{-10} \; m$$

$$\lambda = 1{\cdot}22 \text{ Å}.$$

15. What is the de-Broglie wavelength of neutron of energy 28·8 ev ? (Mass of the neutron $= 1{\cdot}678 \times 10^{-27}$ Kg)

Sol : $m = 1{\cdot}678 \times 10^{-27}$ Kg

$$E = 28{\cdot}8 \text{ ev} = 28{\cdot}8 \times 1{\cdot}6 \times 10^{-19} \text{ J}$$

$$h = 6{\cdot}6 \times 10^{-34} \text{ Js}$$

$$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda = \frac{6{\cdot}6 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}678 \times 10^{-27} \times 28{\cdot}8 \times 1{\cdot}6 \times 10^{-19}}}$$

$$\lambda = \frac{6{\cdot}6 \times 10^{-11}}{\sqrt{3{\cdot}2 \times 1{\cdot}678 \times 28{\cdot}8}} = \frac{6{\cdot}6 \times 10^{-11}}{\sqrt{154{\cdot}64}} \Rightarrow \lambda = \frac{6{\cdot}6 \times 10^{-11}}{12{\cdot}435} = 0{\cdot}53 \times 10^{-11} \text{ m.}$$

16. Electrons are accelerated through 344 volts and are reflected from a crystal. The first reflection maximum occurs when glancing angle is 60°. Determine the spacing of the crystal.

Sol : $\lambda = \frac{h}{mV} = \frac{h}{\sqrt{2meV}}$

From Bragg's law

$$2d\sin\theta = n\lambda \;;\; 2d\sin 60^\circ = \lambda\frac{h}{\sqrt{2meV}}$$

$$d = \frac{1}{\sqrt{3}} \times \frac{6{\cdot}62\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times1{\cdot}6\times10^{-19}\times344}} \;;\; d = 0{\cdot}5\times10^{-10}\text{m} = 0{\cdot}5\text{ Å}$$

17. What is the de-Broglie wave length of an electron which has been accelerate from rest through a P.D of 120 V ?

Sol : $V = 120$ V ; $\lambda = \frac{12{\cdot}25}{\sqrt{V}}$ Å $\Rightarrow \lambda = \frac{12{\cdot}25}{\sqrt{120}} = \frac{12{\cdot}25}{10{\cdot}95} = 1{\cdot}118$ Å.

18. What voltage must be applied to an electron microscope to produce electrons of wavelength 0·80 Å ?

Sol : $\lambda = 0{\cdot}8\text{ Å} = 0{\cdot}8\times10^{-10}$ m ;

$h = 6{\cdot}6\times10^{-34}$ Js, $e = 1{\cdot}6\times10^{-19}$ C, $m \doteq 9{\cdot}1\times10^{-31}$ Kg

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$V = \frac{h^2}{2me\lambda^2} = \frac{(6{\cdot}6\times10^{-34})^2}{2\times9{\cdot}1\times10^{-31}\times1{\cdot}6\times10^{-19}\times(0{\cdot}8\times10^{-10})^2} \;;\; V = 480 \text{ volts.}$$

19. What is the de Broglie wavelength of an electron accelerated through 30,000 V?

Sol : Given that $V = 30{,}000$ V

$$\lambda = \frac{12{\cdot}25}{\sqrt{V}}\mathring{A} = \frac{12{\cdot}35}{\sqrt{30{,}000}} \;;\; \lambda = \frac{12{\cdot}35}{173{\cdot}2} = 0{\cdot}071\text{Å}.$$

20. Calculate the wavelength associated with electrons whose speed is 0 · 01 the speed of light ($h = 6\cdot62\times10^{-34}$ Joule sec, $m = 9{\cdot}11\times10^{-31}Kg$)

Sol : Velocity of electron $= 0\cdot01 \times$ velocity of light $= \frac{1}{100}\times3\times10^8 = 3\times10^6$ m/s

$m = 9\cdot11\times10^{-31}$ Kg, $h = 6\cdot62\times10^{-34}$ $J{\cdot}s$

$$\lambda = \frac{h}{mv} = \frac{6\cdot62\times10^{-34}}{9\cdot11\times10^{-31}\times3\times10^6} = \frac{6\cdot62}{27\cdot33}\times10^{-9}$$

$\lambda = 0\cdot2422\times10^{-9}$ m.

21. Find the de-broglie wave length of an electron which is accelerated through a potential difference of 1000 volts. (Mass of electron $= 9{\cdot}11\times10^{-31}$ Kg, $e = 1{\cdot}632\times10^{-19}$ c) **[KU 017]**

Sol : $\lambda = \frac{h}{m_0 v}$ but $v = \sqrt{\frac{2eV}{m_0}}$

$$\lambda = \frac{h}{\sqrt{2eVm_0}} = \frac{6{\cdot}62\times10^{-34}}{\sqrt{2\times1{\cdot}632\times10^{-19}\times1000\times9{\cdot}11\times10^{-31}}}$$

$$\lambda = \frac{0{\cdot}1225}{\sqrt{10}}\times10^{-10}\,m = \frac{0{\cdot}1225}{3{\cdot}1623}\times10^{-10}\,m \;;\; \lambda = 0{\cdot}0395\times10^{-10}\,m = 0{\cdot}0395\,\mathring{A}.$$

22. Compare the uncertainties in the velocities of an electron and a proton confined to 1 mm box. **[KU 017]**

Sol : Let $(\Delta V)_e$ and $(\Delta V)_p$ be the uncertainties in the velocity of electron and proton.

$$(\Delta V)_e = \frac{\Delta P}{m_e}\ ; (\Delta V)_p = \frac{\Delta P}{m_p}$$

$$\frac{(\Delta V)_e}{(\Delta V)_p} = \frac{m_p}{m_e} = \frac{1{\cdot}67 \times 10^{-27}\ \text{Kg}}{9{\cdot}1 \times 10^{-31}\ \text{Kg}} = 1835.$$

23. The speed of an electron is 300 m/s and it is measured with an accuracy of 0·005%. Calculate the certainty with which we can locate the position of the electron.

Sol : Given $V = 300$ m/s, $m = 9{\cdot}1 \times 10^{-31}$ Kg, $h = 6{\cdot}62 \times 10^{-34}$ Js

Momentum of the electron $= mv = 9{\cdot}1 \times 10^{-31} \times 300$

$$\Delta P_x = \left(\frac{0{\cdot}005}{100}\right) mv = 5 \times 10^{-5} \times 9{\cdot}1 \times 10^{-31} \times 300$$

$$\therefore\ \Delta x = \frac{h}{\Delta P_x} = \frac{6{\cdot}62 \times 10^{-34}}{5 \times 10^{-5} \times 9{\cdot}1 \times 10^{-31} \times 300}$$

$\Delta x = 0{\cdot}04849\ m.$

24. A microscope is used to locate an electron in an atom to within a distance of 0·2 Å. Calculate the uncertainty in the momentum of the electron located in this way ? And also calculate the uncertainty in the velocity.

Sol : $h = 6{\cdot}62 \times 10^{-34}$ J sec

$\Delta x = 0{\cdot}2\ \text{Å} = 0{\cdot}2 \times 10^{-10}$ m

$$\Delta P = \frac{h}{\Delta x} = \frac{6{\cdot}62 \times 10^{-34}}{0{\cdot}2 \times 10^{-10}}$$

$\Delta P = 3{\cdot}31 \times 10^{-23}$ Kg – m/s

$$\Delta V = \frac{\Delta P}{m_0} = \frac{3{\cdot}31 \times 10^{-23}}{9{\cdot}1 \times 10^{-31}}$$

$\Delta V = 3{\cdot}64 \times 10^{7}$ m/s.

25. How does the concept of Bohr's orbit violate the uncertainty relation ? Explain.

Sol : $\Delta E \cdot \Delta t = \hbar$

According to Bohr's theory an electron revolves in a quantised orbit and hence possesses well defined energy, with O uncertainty i.e., $\Delta E = O$

$$\therefore\ \Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{O} = \propto$$

This means that all energy states of atom must have infinite life time, but according to experimental observations, the excited states of an atom has a life time of 10^{-8} sec. Thus the concept of Bohr's orbit is in violation of uncertainty principle.

26. An electron has a speed of $3{\cdot}5 \times 10^7$ cm/sec accurated to $0{\cdot}0098\%$ with what fundamental accuracy can we locate the position of electron.
(Mass of electron $= 9{\cdot}11 \times 10^{-31}$ Kg)

Sol : $V = 3{\cdot}5 \times 10^7$ m/s, $m = 9{\cdot}11 \times 10^{31}$ Kg, $\Delta P = 0{\cdot}0098\%$

$$mv = 9{\cdot}1 \times 10^{-31} \times 3{\cdot}5 \times 10^5 = 3{\cdot}185 \times 10^{-25} \text{ Kg m/s}$$

$$\Delta P = \left(\frac{0{\cdot}0098}{100}\right) mv = 98 \times 10^{-6} \times 3{\cdot}185 \times 10^{-25}$$

$$\Delta P\, \Delta x = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi\Delta P} = \frac{6{\cdot}63 \times 10^{-34}}{2 \times 3{\cdot}14 \times 98 \times 10^{-6} \times 3{\cdot}185 \times 10^{-25}}$$

$$\Delta x = 3{\cdot}382 \times 10^{-6} \text{ m}$$

27. Show that if a component of angular momentum of the electron in a hydrogen atom is known to be $2h$ within 5% error. Its angular orbital position in the plane perpendicular to that component cannot be specified at all.

Sol : Let ΔJ be the uncertainty in the angular momentum and $\Delta\theta$ be the uncertainty in the angle

$$\Delta J\, \Delta\theta = h$$

$$\text{Given } \Delta J = \frac{5}{100} \times 2h = \frac{h}{10}$$

$$\Delta\theta = \frac{h}{\Delta J} = \frac{h}{h/10} = 10 \text{ rad} > 2\pi \text{ rad}$$

This is not possible.

28. If the uncertainty in the location of a particle is equal to its de-Broglie wavelength. Show that the uncertainty in its velocity is equal to its velocity.

Sol : $\Delta P \cdot \Delta x = h$; $\Delta(mv)\Delta x = h$

$$\Delta V \cdot \Delta x = \frac{h}{m}$$

From data $\Delta x = \lambda$

$$\Delta V = \frac{h}{m\lambda} = \frac{h}{m\left(\frac{h}{mv}\right)} \qquad \left(\because \lambda = \frac{h}{mv}\right)$$

$\Delta V = V$ i.e., the velocity of the particle.

29. Assume that an electron is inside a nucleus of radius 10^{-15} m. Using uncertainty principle, estimate the K.E of the electron in ev.

Sol : The maximum uncertainty in the position $= 2 \times 10^{-15}$ m
(or) $\Delta x = 2 \times 10^{-15}$ m

$$\Delta P = \frac{h}{2\pi\Delta x} \Rightarrow \Delta P = \frac{6{\cdot}63 \times 10^{-34}}{2 \times 3{\cdot}14 \times 2 \times 10^{-15}} = 5{\cdot}276 \times 10^{-20} \text{ Kg m/s.}$$

Now the momentum itself must be at least comparable in magnitude. Hence

$P \approx 5{\cdot}276 \times 10^{-20}$ Kg m/s

$$K = \frac{P^2}{2m_0} = \frac{(5{\cdot}276 \times 10^{-20})^2}{2 \times 9{\cdot}1 \times 10^{-31}} = 1{\cdot}530 \times 10^{-9} \text{ J}$$

$$K = \frac{1{\cdot}530 \times 10^{-9}}{1{\cdot}6 \times 10^{-19}} \text{ eV}$$

$K = 9563 \times 10^6$ eV ; $K = 9563$ MeV.

30. Calculate the smallest possible uncertainty in the position of an electron moving with velocity 3×10^7 m/s.

Sol : $(\Delta x)_{Min} (\Delta P)_{Max} = \dfrac{h}{2\pi}$

$$P = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$(\Delta x)_{Min} \times \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \approx \frac{h}{2\pi}$$

$$(\Delta x)_{Min} = \frac{h\sqrt{1 - \frac{v^2}{c^2}}}{2\pi\, m_0 v} = \frac{6{\cdot}62 \times 10^{-34} \sqrt{1 - \frac{(3 \times 10^7)^2}{(3 \times 10^8)^2}}}{2 \times 3{\cdot}14 \times 9 \times 10^{-31} \times 3 \times 10^7}$$

$(\Delta x)_{Min} = 3{\cdot}8 \times 10^{-12} m = 0{\cdot}038$ Å.

31. An electron has a speed of 3×10^2 m/s accurate to $0{\cdot}01\%$ with what fundamental accuracy can be locate the position of the electron ? **[KU M15]**

Sol : $v = 3 \times 10^2$ m/s

$P = mv = 9{\cdot}1 \times 10^{-31} \times 3 \times 10^2 = 27{\cdot}3 \times 10^{-29}$ Kg m/s

$$\Delta P = \frac{27{\cdot}3 \times 10^{-29} \times 0{\cdot}01}{100} = 27{\cdot}3 \times 10^{-33} \text{ Kg m/s}$$

$$\Delta x = \frac{h}{\Delta P} = \frac{6{\cdot}6 \times 10^{-34}}{27{\cdot}3 \times 10^{-33}} = 2{\cdot}4 \times 10^{-2} \text{ m.}$$

32. If the uncertainty in position of an electron is 4×10^{-10} m. Calculate the uncertainty in its momentum $h = 6{\cdot}624 \times 10^{-34}$ Joule sec. **[SVU O18]**

Sol : $h = 6{\cdot}624 \times 10^{-34}$ J – s

$\Delta x = 4 \times 10^{-10}$ m

$\Delta x\, \Delta P_x = h$

$$\Delta P_x = \frac{h}{\Delta x} = \frac{6{\cdot}624 \times 10^{-34}}{4 \times 10^{-10}} = 1{\cdot}656 \times 10^{-24} \text{ Kg/sec.}$$

33. An electron is confined to a box of length 10^{-9} m. Calculate the minimum uncertainty in its velocity. (Mass of the electron = $9{\cdot}1 \times 10^{-31}$ Kg, $h = 6{\cdot}6 \times 10^{-34}$ Js)

Sol : $(\Delta x)_{Max} = 10^{-9}$ m, $h = 6{\cdot}6 \times 10^{-34}$ Js

$$(\Delta P_x)_{Min} = \frac{h}{(\Delta x)_{Max}} = \frac{6{\cdot}6 \times 10^{-34}}{10^{-9}} = 6{\cdot}6 \times 10^{-25} \text{ Kg m/s}$$

But $(\Delta P_x)_{Min} = m\,(\Delta V_x)_{Min}$

$$(\Delta V_x)_{Min} = \frac{(\Delta P_x)_{Min}}{m} = \frac{6{\cdot}6 \times 10^{-25}}{9 \times 10^{-31}} \text{ ; } (\Delta V_x)_{Min} = 7{\cdot}3 \times 10^{5} \text{ m/s.}$$

34. Find the uncertainty in the momentum of a particle when position is determined with in 0·01 cm.

Sol : $\Delta x = 0{\cdot}01$ cm $= 0{\cdot}01 \times 10^{-2}$ m $= 10^{-4}$ m

$h = 6{\cdot}6 \times 10^{-34}$ Js

$\Delta x \cdot \Delta P_x = h$

$$\Delta P_x = \frac{h}{\Delta x} = \frac{6{\cdot}6 \times 10^{-34}}{10^{-4}}$$

$\Delta P_x = 6{\cdot}6 \times 10^{-30}$ Kg m/sec.

35. An electron has a speed of 4×10^{5} m/s within the accuracy of 0·01%. Calculate the uncertainity in the position of the electron.

Sol : $V = 4 \times 10^{5}$ m/s, $m = 9{\cdot}11 \times 10^{-31}$ Kg

$P = mV = 9{\cdot}11 \times 10^{-31} \times 4 \times 10^{5} = 36{\cdot}44 \times 10^{-26}$ Kg m/s

$$\Delta P = \left(\frac{0{\cdot}01}{100}\right) mV = 10^{-4} \times 36{\cdot}44 \times 10^{-26}$$

$$= 36{\cdot}44 \times 10^{-30} \text{ Kg m/s}$$

$\Delta x \cdot \Delta P = h$

$$\Delta x = \frac{h}{\Delta P} = \frac{6{\cdot}6 \times 10^{-34}}{36{\cdot}44 \times 10^{-30}}$$

$\Delta x = 0{\cdot}18 \times 10^{-4}$ m.

36. The speed of an electron is 10^{5} m/s and it is measured with an accuracy of 0·005%. Calculate the certainty with which we can locate the position of the electron.

Sol : $V = 10^{5}$ m/s, $m = 9{\cdot}1 \times 10^{-31}$ Kg, $h = 6{\cdot}62 \times 10^{-34}$ J-S

$$\Delta P_x = \left(\frac{0{\cdot}005}{100}\right) \text{mV} = \frac{0{\cdot}005}{100} \times 9{\cdot}1 \times 10^{-31} \times 10^{5}$$

$\Delta P_x \cdot \Delta x = h$

$$\Delta x = \frac{h}{\Delta P_x} = \frac{6{\cdot}62 \times 10^{-34}}{0{\cdot}005 \times 9{\cdot}1 \times 10^{-28}}$$

$\Delta x = \frac{6.62}{0.0455} \times 10^{-6}$; $\Delta x = 145.49 \times 10^{-6}$ m.

37. An electron has a speed of 600 $m\,s^{-1}$ with an accuracy of 0.005%. Calculate the cetainity with which we can locate the position of the electron. Given that $h = 6 \cdot 6 \times 10^{-34}$ *Js*, $m = 9 \cdot 1 \times 10^{-31}$ Kg. **[ANU J15]**

Sol : Given that $v = 600$ m/s, $m = 9 \cdot 1 \times 10^{-31}$ Kg

$h = 6 \cdot 6 \times 10^{-34}$ J-S.

Momentum of the electron $= m\,v = 9 \cdot 1 \times 10^{-31} \times 600$

$$\Delta P_x = \left(\frac{0.0005}{100}\right) m\,v = 5 \times 10^{-5} \times 9 \cdot 1 \times 10^{-31} \times 600$$

$$\therefore \ \Delta x = \frac{h}{\Delta Px} = \frac{6 \cdot 6 \times 10^{-34}}{5 \times 10^{-5} \times 9 \cdot 1 \times 10^{-31} \times 600} = 0 \cdot 02254 \text{ m.}$$

38. If the uncertainity in the momentum of an electron is 2×10^{-32} Kg – m/sec, find the uncertainity in its position.

Sol : Given that $\Delta P_x = 2 \times 10^{-32}$ Kg - m/sec

$\Delta x \cdot \Delta p_x \approx h$

$$\Delta x = \frac{h}{\Delta p_x} \approx \frac{6.62 \times 10^{-34}}{2 \times 10^{-32}}$$

$\Delta x = 3.31 \times 10^{-2}$ m ; $\Delta x = 3.31$ cm.

39. Determine the uncertainty in the velocity of an electron and proton confined to 20Å ($h = 6.62 \times 10^{-34}$ & $M_e = 9.1 \times 10^{-31} Kg$ $m_P = 1.67 \times 10^{-27}$ *Kg*). **[ANU O18, J18, J16]**

Sol : Given $\Delta X = 20 Å = 20 \times 10^{-10}$m

$h = 6.62 \times 10^{-34}$ Js ; $M_e = 9.1 \times 10^{-31}$ Kg

$$\Delta P = \frac{h}{\Delta X}$$

$$\text{i) } V_e = \frac{h}{M_e \Delta X} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 20 \times 10^{-10}}$$

$$V_e = \frac{6.6}{9.1 \times 2} \times 10^6 = 0.3626 \times 10^6 \ \therefore \ V_e = 3.626 \times 10^5 \text{ m/s}$$

$$\text{ii) } V_P = \frac{h}{m_P \Delta X} = \frac{6.62 \times 10^{-34}}{(1.67 \times 10^{-27}) \times (20 \times 10^{-10})} = 1.982 \times 10^2 \ m/s.$$

40. Calculate the de Broglie wavelength of α- particle accelerated through a potential difference of 6 KV. Given that $h = 6.62 \times 10^{-34}$ *Js*, $m_p = 1.67 \times 10^{-27}$ Kg. **[ANU M16]**

Sol : Given $V = 6KV = 6 \times 10^3$ *Volt*

$h = 6.62 \times 10^{-34}$Js; $m_p = 1.67 \times 10^{-27}$Kg

$m_\alpha = 4 \times m_p = 4 \times 1.67 \times 10^{-27}$

$$\lambda = \frac{h}{\sqrt{2m_\alpha eV}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times (4 \times 1.6 \times 10^{-27}) \times (1.6 \times 10^{-19}) \times 6 \times 10^3}}$$

$$\therefore \lambda = \frac{6.62 \times 10^{-34} \times 10^{23}}{\sqrt{8 \times 1.67 \times 1.6 \times 6 \times 10^3}} = \frac{6.62 \times 10^{-11}}{\sqrt{128.256}} = 0.01848 \times 10^{-11} = 0{\cdot}0018\ \mathring{A}.$$

41. Find the de-Broglie wavelength of a neutron of energy 12.8 MeV. Given mass of neutron = $1{\cdot}675 \times 10^{-27}$Kg and $h = 6{\cdot}625 \times 10^{-34}$J-sec. **[KU M16]**

Sol : Given $E = 12.8\text{MeV} = 12{\cdot}8 \times 10^{-13}$J ;

$m = 1{\cdot}675 \times 10^{-27}$Kg

$h = 6{\cdot}625 \times 10^{-34}$J-s

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}675 \times 10^{-27} \times 12{\cdot}8 \times 10^{-13}}}$$

$$\therefore\ \lambda = \frac{6.625 \times 10^{-14}}{\sqrt{42{\cdot}88}} = \frac{6{\cdot}625 \times 10^{-14}}{6{\cdot}548}\ \therefore\ \lambda = 0{\cdot}2693 \times 10^{-14}\text{m}.$$

42. Calculated de-broglie wavelength associated with a human body having mass 60 Kg moving with a velocity 1m/sec.

Sol : Given $m = 60$ Kg

$V = 1m/S$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6{\cdot}62 \times 10^{-34}}{60 \times 1}$$

$\lambda = 0{\cdot}1103 \times 10^{-34}$ m.

43. The uncertainty in position of an electron is 2×10^{10}m. Then find the uncertainty in its momentum ($h = 6{\cdot}63 \times 10^{-34}\ JS$). **[AdNU N17]**

Sol : Given that $\Delta x = 2 \times 10^{10} m$

$h = 6{\cdot}63 \times 10^{-34}\ Js$

$$\Delta x{\cdot}\Delta P = h \Rightarrow \Delta P = \frac{h}{\Delta x} = \frac{6{\cdot}63 \times 10^{-34}}{2 \times 10^{10}}\ ;\ \Delta P = 3{\cdot}315 \times 10^{-44}\ Kg\ m/s.$$

44. Find the de-Broglie wavelength of electrons, when they are accelerated through a potential difference of 150V. **[VSU S17]**

Sol : $V = 150$ V

$$\lambda = \frac{12{\cdot}25}{\sqrt{V}} \Rightarrow \lambda = \frac{12{\cdot}25}{\sqrt{150}} = \frac{12.25}{12.24} = 1{\cdot}0008\ \mathring{A}.$$

45. Calculate the energy of electron having de Broglie wavelength of $4^o A$. Given that the mass of electron is $9{\cdot}1 \times 10^{-31}$ Kg. **[KU M13]**

Sol : $\lambda = 4^o A = 4 \times 10^{-10}$ m, $m = 9{\cdot}1 \times 10^{-31}$ Kg.

$$E = \frac{h^2}{2m\lambda^2} = \frac{(6{\cdot}62 \times 10^{-34})^2}{2 \times 9{\cdot}1 \times 10^{-31} \times (4 \times 10^{-10})^2}$$

$$E = \frac{(6{\cdot}62)^2}{18{\cdot}2 \times 16} \times 10^{-17} = 0{\cdot}1505 \times 10^{-17} \text{ J.}$$

46. If the uncertainty in the position of a proton is 5×10^{-14} m, find the uncertainty in its momentum.

Sol : $\Delta x = 5 \times 10^{-14}$ m, $h = 6{\cdot}6 \times 10^{-34}$

$\Delta p{\cdot}\Delta x \approx h$

$$\Delta p = \frac{h}{\Delta x} = \frac{6{\cdot}6 \times 10^{-34}}{5 \times 10^{-14}} = 1{\cdot}32 \times 10^{-24} \text{ Kg m/s.}$$

47. If the uncertainty in position of electron is 0.1Å calculate the uncertainty in its momentum ($h = 6{\cdot}624 \times 10^{-34}\ J - S$).

Sol : Given that $\Delta x = 0{\cdot}1\ \mathring{A} = 0{\cdot}1 \times 10^{-10} = 10^{-11}$ m

$h = 6{\cdot}624 \times 10^{-34}\ Js$

$\Delta P{\cdot}\Delta x = h$

$$\Delta P = \frac{h}{\Delta x} = \frac{6{\cdot}624 \times 10^{-34}}{10^{-11}}$$

$\Delta P = 6{\cdot}624 \times 10^{-23}$ Kg m/s.

48. Calculate the deBroglie wavelength associated with a proton moving with a velocity of 2200 ms^{-1}.

Sol : Velocity (v) = 2200 ms^{-1}

$h = 6{\cdot}625 \times 10^{-34}\ Js$; $m = 1{\cdot}67 \times 10^{-27}$ Kg

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}625 \times 10^{-34}}{1{\cdot}67 \times 10^{-27} \times 2200}$$

$$\lambda = \frac{6{\cdot}625}{1{\cdot}67 \times 22} \times 10^{-9} = 0{\cdot}180 \times 10^{-9}\ m.$$

49. If the uncertainty in position of an electron is 4×10^{-10} m and uncertainty in its momentum is $1{\cdot}65 \times 10^{-24}$ Kg m/s, Calculate Plancks constant.

Sol : Given that $\Delta x = 4 \times 10^{-10}$ m

$\Delta P = 1{\cdot}65 \times 10^{-24}$ Kg m/s

$\Delta x{\cdot}\Delta P = h$

$h = 4 \times 10^{-10} \times 1{\cdot}65 \times 10^{-24}$

$h = 6{\cdot}624 \times 10^{-34}\ Js.$

50. If the uncertainty in the momentum of an electron is $1{\cdot}65 \times 10^{-24}$ Kg m/s. Calculate the uncertainty in its position.

Sol : Given that $\Delta P = 1{\cdot}65 \times 10^{-24}$ Kg m/s

$$\Delta P \cdot \Delta x \approx h$$

$$\Delta x = \frac{h}{\Delta P} = \frac{6 \cdot 62 \times 10^{-34}}{1 \cdot 65 \times 10^{-24}} = 4 \cdot 012 \times 10^{-10} \text{ m}.$$

51. Calculate energy of electron of wavelength $0 \cdot 3\ \AA$ in eV. **[ANU O/N 17]**
(Given mass of electron $9 \cdot 11 \times 10^{-31}\ Kg$, $h = 6 \cdot 63 \times 10^{-34}\ J.S$)

Sol : Given $\lambda = 0 \cdot 03\ \AA = 0 \cdot 03 \times 10^{-10}\ m = 3 \times 10^{-12} m$

$m = 9 \cdot 11 \times 10^{-31}\ Kg$; $h = 6 \cdot 63 \times 10^{-34}\ J.S$

$$E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6 \cdot 63 \times 10^{-34})^2}{2 \times 9 \cdot 11 \times 10^{-31} \times (3)^2 \times 10^{-24}} = \frac{(6 \cdot 63)^2 \times 10^{-68}}{2 \times 9 \cdot 11 \times 9 \times 10^{-55}} J = \frac{(6 \cdot 63)^2 \times 10^{-13}}{2 \times 9 \cdot 11 \times 9} J$$

$$E = \frac{(6 \cdot 63)^2}{2 \times 9 \cdot 11 \times 9} \times 10^{-13} \times \frac{1\ eV}{1 \cdot 6 \times 10^{-19}}$$

$$= 0 \cdot 1675 \times 10^6\ eV.$$

52. The uncertainty in position of an electron is 2×10^{-10}m. Then find the uncertainty in its momentum ($h = 6 \cdot 63 \times 10^{-34}\ JS$). **[YVU N17]**

Sol : Given that $\Delta x = 2 \times 10^{-10} m$

$h = 6 \cdot 63 \times 10^{-34}\ Js$

$$\Delta x \cdot \Delta P = h \Rightarrow \Delta P = \frac{h}{\Delta x} = \frac{6 \cdot 63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$\Delta P = 3 \cdot 315 \times 10^{-24}\ Kg\ m/s.$$

53. If neutron is travelling with a De-Broglie wavelength of $1\AA$ then find its momentum. **[AdNU 18]**

Sol : Given that $\lambda = 1\AA = 10^{-10}\ m$

$h = 6 \cdot 63 \times 10^{-34}$ J-S

$P = ?$

$$P = \frac{h}{\lambda} = \frac{6 \cdot 63 \times 10^{-34}}{10^{-10}} = 6 \cdot 63 \times 10^{-24} \text{ kg-m/s}.$$

54. Calculate the wave length associated with electron subjected to a potential difference of $1 \cdot 25$ kv. **[VSU O18]**

Sol : $V = 1 \cdot 25$ kv $= 1 \cdot 25 \times 10^3$ v

$$\lambda = \frac{12 \cdot 26}{\sqrt{V}} \text{Å} = \frac{12 \cdot 26}{\sqrt{1.25 \times 10^3}} = \frac{12 \cdot 26}{\sqrt{1250}} = \frac{12 \cdot 26}{35 \cdot 36} = 0 \cdot 3467 \text{ Å}$$

55. What is the energy of Gamma ray proton of wavelength 1Å? $h = 6 \cdot 63 \times 10^{-34}$.

Sol : Given $\lambda_P = 1\text{Å} = 10^{-10}$ m

$h = 6 \cdot 63 \times 10^{-34}$; $c = 3 \times 10^8$ m/s

$$E = \frac{hc}{\lambda} = \frac{(6{\cdot}63 \times 10^{-34})(3 \times 10^{8})}{10^{-10}}$$

$$= 6{\cdot}63 \times 3 \times 10^{-16} J = 6{\cdot}63 \times 3 \times 10^{-16} \times \frac{1\, ev}{1{\cdot}6 \times 10^{-19}} = 12{\cdot}43 \times 10^{3}\, eV$$

$\therefore\ E = 12{\cdot}43\, KeV.$

56. What is the ratio of kinetic energies of electron and proton to have equal wave-length of 10Å? **[ANU 018]**

Sol : Given $\lambda_P = \lambda_e = 10$Å

$$m_e = 9{\cdot}1 \times 10^{-31}\, kg$$

$$m_p = 1{\cdot}67 \times 10^{-27}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow E = \frac{h^2}{2m\lambda^2} \Rightarrow E \propto \frac{1}{m} \quad [\therefore\ h, \lambda \text{ are constant}]$$

$$\frac{E_e}{E_p} = \frac{m_p}{m_e} = \frac{1{\cdot}67 \times 10^{-27}}{9{\cdot}1 \times 10^{-31}} = 1835{\cdot}16.$$

UNIT- III

3 QUANTUM(WAVE) MECHANICS

LONG ANSWER TYPE QUESTIONS

1. What is Wave function. Derive schrodinger time independent wave equation and time dependent wave equation. **[ANU O18,O17; AdNU O18, N17; AU O18,17; BRAU O18, O17; KU O18, O17, M16, 15, J16, 15; RU O18, O17; SKU O18; SVU O18, O17; VSU O18, O17; YVU O18, O17]**

A. **Wave Function :** A function describing the probability of finding the particles within a matter wave is called Wave function. It is denoted by ψ . It is written in the form $\psi = \psi_0 e^{-i\omega t}$.

Schrodinger's time independent equation :

1) Let us consider a system of stationary waves associated with a material particle of mass m. Let (x, y, z) be the co-ordinates of the particle. The differential equation of a wave motion is given by

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) = v^2 \nabla^2 \psi \rightarrow (1)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ [$\because \nabla^2$ being the Laplacian operator] and v is the wave velocity.

2) Solution of equation (1) is given by

$\psi = \psi_0 \sin \omega t = \psi_0 \sin 2\pi \nu t \rightarrow (2)$

where ν = frequency of the stationary wave associated with the particle.

3) Differentiating equation (2), twice we get

$$\frac{\partial \psi}{\partial t} = \psi_0 (2\pi \nu) \cos 2\pi \nu t \text{ and } \frac{\partial^2 \psi}{\partial t^2} = -\psi_0 (2\pi \nu)^2 \sin 2\pi \nu t$$

$$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2 \nu^2 \psi = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right) \psi \rightarrow (3) \quad \left(\because \nu = \frac{v}{\lambda}\right)$$

4) Substituting equation (3) in equation (1)

$$v^2 \nabla^2 \psi = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right) \psi \text{ or } \nabla^2 \psi + \left(\frac{4\pi^2}{\lambda^2}\right) \psi = 0 \rightarrow (4)$$

5) According to de-broglie hypothesis, the wavelength associated with the material particle is given by

$$\lambda = \frac{h}{mv}$$

$$\therefore \nabla^2 \psi + \left(\frac{4\pi^2}{h^2} m^2 v^2\right) \psi = 0 \rightarrow (5)$$

6) If E and V be the total energy and potential energy of the particle, then its kinetic energy $\frac{1}{2}mv^2$ is given by $\frac{1}{2}mv^2 = E - V$

$m^2v^2 = 2m(E - V) \rightarrow (6)$

7) From equations (5) and (6)

$$\nabla^2\psi + \left[\frac{4\pi^2}{h^2} \times 2m(E - V)\right]\psi = 0$$

$$\nabla^2\psi + \left[\frac{8\pi^2 m}{h^2}(E - V)\right]\psi = 0 \rightarrow (7)$$

Equation (7) is known as schrodinger time independent wave equation.

8) Substituting $\hbar = \frac{h}{2\pi}$ in equation (7), the schrodinger wave equation can be written as

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \rightarrow (8)$$

Equation (8) is the schrodingers time independent equation.

9) For a free particle $V = 0$, hence the schrodinger wave equation for a free particle can be express as

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \rightarrow (9)$$

10) Equation (8) can be written as

$$\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + (E - V)\psi = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi - V\psi = -E\psi$$

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi \rightarrow (10)$$

or $H\psi = E\psi \rightarrow (11)$

where $H = \frac{-\hbar^2}{2m}\nabla^2 + V$ is the Hamitonian and E is the total energy of the system.

Schrodinger's time dependent equation :

1) The wave function ψ including time (t) can be written as

$$\psi = \psi_0(x, y, z)\, e^{-i\omega t} = \psi_0\, e^{-i\omega t} \rightarrow (12)$$

2) Differentiating w.r.to time, we get

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)\, e^{-i\omega t} = (-i\omega)\,\psi_0\, e^{-i\omega t} \rightarrow (13)$$

But $E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega \rightarrow (14)$

Hence $\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar}\psi = \frac{E}{i\hbar}\psi \quad \left(\because -i = \frac{-i \times i}{i} = \frac{1}{i}\right)$

$E\psi = i\hbar\frac{\partial\psi}{\partial t} \rightarrow (15) \qquad (\because \sqrt{-1} = i)$

3) Substituting equation (15) in the schrodinger's time independent equation

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

We have $\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$

$$\nabla^2\psi = \frac{-2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right]$$

$$\frac{-\hbar^2}{2m}\Delta^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \rightarrow (16)$$

This is known as schrodinger's time dependent equation.

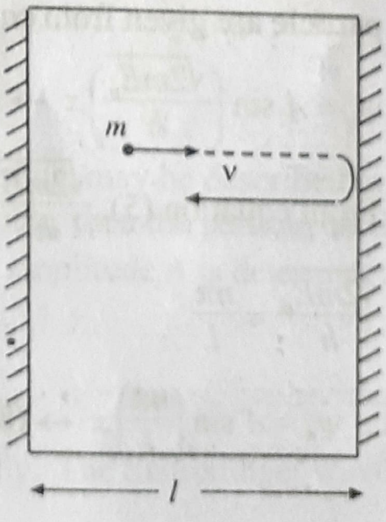
4) Equation (16) can be written as $\hat{H}\psi = \hat{E}\psi \rightarrow (17)$

where $\hat{H} = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$ = Hamiltonian operator

and $\hat{E} = i\hbar\frac{\partial}{\partial t}$ = Energy operator.

2. Obtain an expression for the energy of a particle in one dimensional potential box (or) well of infinite height. **[ANU O18, J18, O17; AdNU O18, N17; AU 18, 17; BRAU O17; KU O18, O17, J16, M15; RU O18, O17; SKU O18; VSU O18, S17; YVU O18]**

A. 1) Let us consider a particle of mass m in a one dimentional box extending from $x = 0$ to $x = l$, with a width l and having infinetely hard walls at $x = 0$ and $x = l$ as shown in figure.



2) The total energy E of the particle remains constant because, it will not loose energy in collision. In side the box we can take the potential energy $V = 0$ Schrodinger's equation

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

reduces in one dimension with $V = 0$

$$\therefore \frac{\partial^2\psi}{\partial x^2} + \left(\frac{2mE}{\hbar^2}\right)\psi = 0 \rightarrow (1)$$

3) The solution of this equation is

$$\psi = A\sin\left(\frac{\sqrt{2mE}}{\hbar}\right)x + B\cos\left(\frac{\sqrt{2mE}}{\hbar}\right)x \rightarrow (2)$$

where A and B are constants.

4) As the walls of the box are of infinite height at $x = 0$ and $x = l$ and as the particle can not have infinite energy, it can not move out of the walls.

That is the wave function should also be zero in this region.

$\psi = 0$ for $x = l$ and $x = 0 \rightarrow (3)$

Because of this boundary condition that $\psi = 0$ at $x = 0$ we have $B = 0$ in equation (2)

$$\therefore\ \psi = A\sin\left(\frac{\sqrt{2mE}}{\hbar}\right)x \rightarrow (4)$$

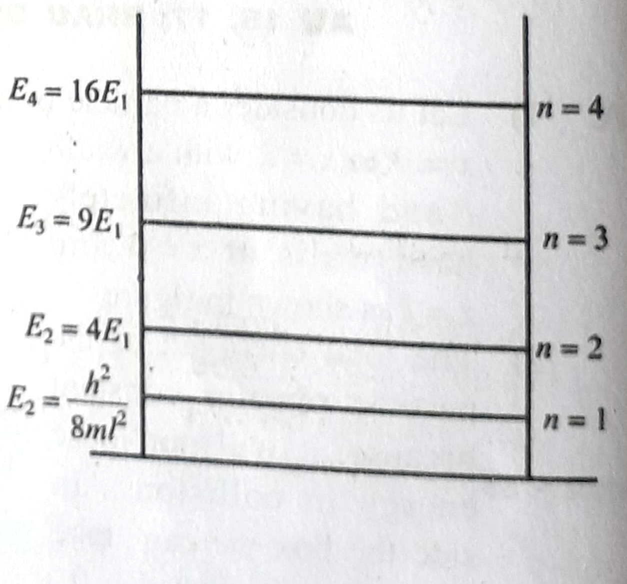
5) Now $\psi = 0$ at $x = l$, we have from equation (4)

$$\left(\frac{\sqrt{2mE}}{\hbar}\right)l = n\pi \rightarrow (5) \text{ where } n = 1, 2, 3 \ldots.$$

6) The energy $E_n = \dfrac{n^2\pi^2\hbar^2}{2ml^2} = \dfrac{n^2\pi^2h^2}{8\pi^2 ml^2} \qquad \left(\because \hbar = \dfrac{h}{2\pi}\right)$

$$E_n = \frac{n^2h^2}{8ml^2} \rightarrow (6) \quad (n = 1, 2, 3 \ldots.)$$

These are the eigen values and correspond to the energies of the system. The energy is quantised in units of $\dfrac{h^2}{8ml^2}$.



7) Now, the wave functions of the particle are given from equation (4)

$$\psi_n = A\sin\left(\frac{\sqrt{2mE_n}}{\hbar}\right)x \rightarrow (7)$$

8) From equation (5), $\dfrac{\sqrt{2mE}}{\hbar}l = n\pi$

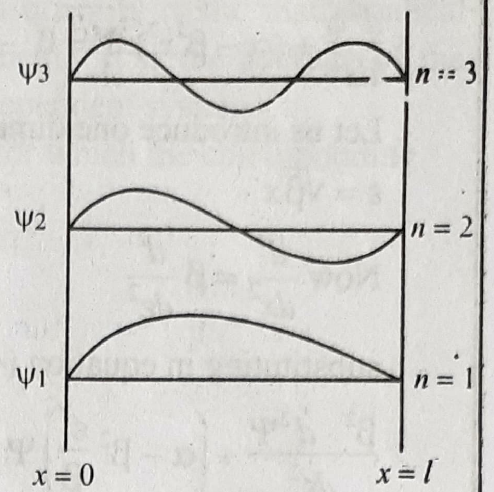
$$\frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{l}$$

$$\therefore\ \psi_n = A\sin\left(\frac{n\pi}{l}\right)x \rightarrow (8)$$

9) The normalisation condition is that $\int_{-\infty}^{\infty} |\psi_n|^2\,dx = 1$, but the particle can exist between $x = 0$ and $x = l$.

10) Hence $\displaystyle\int_{x=0}^{x=l} |\psi_n|^2\,dx = 1 \Rightarrow \int_{x=0}^{x=l}\left[A^2\sin^2\left(\frac{n\pi}{l}\right)x\right]dx = 1$

$$A^2 \int_{x=0}^{x=l} \left[\sin^2\left(\frac{n\pi}{l}\right)x\right]dx = 1 \Rightarrow A^2\left[\frac{l}{2}\right] = 1$$



$$A = \sqrt{\frac{2}{l}} \rightarrow (9)$$

$\therefore$ The wave function for a particle in a box are

$$\psi_n = \sqrt{\frac{2}{l}}\,\sin\left(\frac{n\pi}{l}\right)x$$

where $n = 1, 2, 3 \ldots$

3. Write the application of Schrodinger wave equation to one dimensional harmonic oscillator and derive equation for the energy of the oscillator.

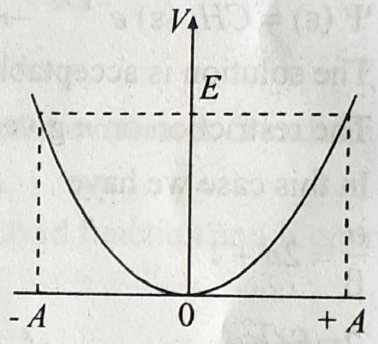
A. Let us consider a linear harmonic oscillator.

For simple harmonic motion, $f(x) = -kx$

Where K is a constant known as force per unit displacement.

$$\text{Again, } f(x) = m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0 \rightarrow (1)$$



This is classical equation of simple harmonic motion. The potential energy function $v(x)$ is given by

$$f(x) = -Kx = -\frac{dV(x)}{dx}$$

$$dV(x) = Kx\,dx$$

On integrating we get $V(x) = \frac{1}{2}Kx^2 \rightarrow (2)$

The plot of V against x is a parabola. Now the particle may be described in parabolic potential wall. If E be the total energy of the particle, then the particle oscillates back and forth between $x = -A$ and $x = +A$. The amplitude A is determined by energy E, such that

$$E = \frac{1}{2}KA^2$$

Let us consider the problem quantum mechanically. The Schrodinger wave equation for harmonic oscillator is given by

$$\frac{d^2\phi}{dx^2} + \frac{2m}{\hbar}\left(E - \frac{1}{2}Kx^2\right)\Psi = 0 \rightarrow (3)$$

Let us put, $\frac{2mE}{\hbar^2} = \alpha$, and $\sqrt{\frac{mK}{\hbar^2}} = \beta$

substituting these values in equation (3), we get

$$\frac{d^2\phi}{dx^2} + (\alpha - \beta^2 x^2)\,\Psi = 0 \rightarrow (4)$$

Let us introduce one dimensional independent variable and such that

$\varepsilon = \sqrt{\beta}x$

Now $\frac{d^2}{dx^2} = \beta \frac{d^2}{d\varepsilon^2}$

substituting in equation (4), we get

$$\frac{\beta^2 \cdot d^2\Psi}{d\varepsilon^2} + \left(\alpha - \beta^2 \frac{\varepsilon^2}{\beta}\right)\Psi = 0$$

$$\frac{d^2\Psi}{d\varepsilon^2} + \left(\frac{\alpha}{\beta} - \varepsilon^2\right)\Psi = 0 \rightarrow (5)$$

The solution of equation (5) can be expressed in terms of Hermite polynomial $H_n\,(\varepsilon)$

The general solution eqn. (5) is given by

$$\Psi\,(\varepsilon) = CH_n\,(\varepsilon)\,e^{-\varepsilon^2/2} \rightarrow (6)$$

The solution is acceptable only for $n = 0, 1, 2 \ldots.$

The restriction on n gives a corresponding restriction on E.

In this case we have

$$\frac{\alpha}{\beta} = 2n + 1$$

$$\frac{2mE/\hbar^2}{\sqrt{mK/\hbar^2}} = 2n + 1 = 2\left(n + \frac{1}{2}\right)$$

$$E = \left(n + \frac{1}{2}\right)\hbar\frac{\sqrt{K}}{m}$$

But $\frac{1}{2\pi}\sqrt{\frac{K}{m}} = w$, where w is the angular frequency of the oscillator.

$$\therefore\; E = \left(n + \frac{1}{2}\right)hW = \left(n + \frac{1}{2}\right)\hbar\nu \rightarrow (7) \quad (\because W = 2\pi\nu)$$

$$E = \left(n + \frac{1}{2}\right)\hbar\nu, \text{ where } n = 0, 1, 2, \ldots$$

Eqn. (7) shows the wave mechanical oscillator can take only certain discrete energie separated by intervals $h\nu$. Where h is planks constant and ν is the frequency of the oscillator.

SHORT ANSWER TYPE QUESTIONS

1. What are the basic posulates of quantum mechanics ? Explain.

[ANU O18, J18; AU 17; BRAU O18; KU O18; SKU O18; SVU O18; VSU O18, S17; YVU O18, N17]

A. 1) A wave function ψ will be associated with a physical system and any state of the system is fully described by this wave function as far as possible.

2) Every dynamical variable is associate with the corresponding operator.
3) The expected or average value of a dynamical quantity is the mathematical expectation for the result of a single measurement. It is the average of the result of a large number of measurements on independent systems.
The expectation value of dynamical variable G for which the corresponding operator G is given by

$$< G > = \int_{-\infty}^{\infty} \psi^* \, G \, \psi \, dV$$

For example for energy $< E > = \int_{-\infty}^{\infty} \psi^* \left(i\hbar \frac{\partial}{\partial t} \right) \psi \, dV$

$$< E > = i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial t} \, dV$$

2. What are Eigen values and Eigen functions ?

[ANU O18, J18, O17; KU O17; RU O18, O17; SKU O18]

A. 1) There is a class of function ψ which when operated by an operator $\hat{O}$ are merely multiplied by some constant say λ.
i.e., $\hat{O}\,\psi(x) = \lambda\,\psi(x)$
Then we say that the number λ is an eigen value of the operator $\hat{O}$ and the operand $\psi(x)$ is an eigen function of $\hat{O}$.
2) For example, since the operand sin $4x$ is well behaved function and if operated by an operator $\left(\frac{-d^2}{dx^2}\right)$ gives the following result.

$$\frac{-d^2}{dx^2} \sin 4x = 16 \sin 4x$$

We say then the number 16 is an eigen value of the operator $\frac{-d^2}{dx^2}$ and the operand sin $4x$ is an eigen function of the operator $\frac{-d^2}{dx^2}$.

In operator form schroedinger wave equation can be written as $H\psi = E\psi$
where $H = \frac{-\hbar^2}{2m}\Delta^2 + V$ and $E = i\hbar \frac{\partial}{\partial t}$.

3. Give the physical significance of wave function ?

[AdNU O18, N17; AU 18, 17; KU J16, M15; SKU O18; SVU O18, O17; YVU O18, N17]

A. The probability that a particle will be found at a given place in space at a given instant of time is represented by $\psi(x, y, z, t)$. It is called wave function.
1) The wave function ψ has no meaning. But $\psi^* \psi = |\psi|^2$ represents the proability density of the particle associated with the wave

$$\iiint |\psi|^2 \, dx \, dy \, dz = 1$$

2) ψ satisfies the above requirement is said to be normalised wave function.

3) It must be finite every where.
4) It must be single valued.
5) It must be continuous.

PROBLEMS

1. An electron is bound by a potential which closely approches an infinite square well of width 10×10^{-10} m, calculate the lowest three permissible quantum energies possessed by electron. **[M16]**

Sol : $l = 10 \times 10^{-10}$, $h = 6{\cdot}62 \times 10^{-34}$ Js, $m = 9{\cdot}1 \times 10^{-31}$ Kg

In the case of a potential well

$$E_n = \frac{n^2h^2}{8ml^2} \text{ where } n = 1, 2, 3 \ldots.$$

$$E_1 = \frac{h^2}{8ml^2} = \frac{(6{\cdot}62 \times 10^{-34})^2}{8 \times 9{\cdot}1 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$E_1 = 0{\cdot}6 \times 10^{-19} \text{ Joules} = \frac{0{\cdot}6 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 0{\cdot}375 \text{ ev}$$

$$\therefore\ E_2 = n^2E_1 = 4 \times 0{\cdot}375 = 1{\cdot}500 \text{ ev} \qquad (\because\ n = 2)$$

$$E_3 = n^3E_1 = 27 \times 0{\cdot}375 = 10{\cdot}125 \text{ ev } (\because\ n = 3)$$

2. Calculate the energy of an electron moving in one dimension in an infinitely high potential box of width 2Å given mass of electron $m = 9{\cdot}1 \times 10^{-31}$ Kg and Planck's constant $h = 6{\cdot}62 \times 10^{-34}$ Js **[VSU O18]**

Sol : $l = 2\text{Å} = 2 \times 10^{-10}\text{m}$; $h = 6{\cdot}62 \times 10^{-34}$ Js ; $m = 9{\cdot}1 \times 10^{-31}$ Kg

$$E_n = \frac{n^2h^2}{8ml^2} \text{ where } n = 1, 2, 3 \ldots.$$

The least energy of the particle may be obtained by substituting $n = 1$

$$\therefore\ E = \frac{h^2}{8ml^2} = \frac{(6{\cdot}62 \times 10^{-34})^2}{8 \times 9{\cdot}1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$E = 15{\cdot}04 \times 10^{-19} \text{ J}$$

$$E = \frac{15{\cdot}04 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 9{\cdot}4 \text{ ev}$$

3. Find the least energy of an electron moving in one dimension in an infinitely high potential box of width 1Å, given mass of the electron $9{\cdot}11 \times 10^{-31}$ Kg and $h = 6{\cdot}63 \times 10^{-34}$ Js **[KU O18, J15, M15]**

Sol : $l = 1\text{Å} = 10^{-10}$ m, $m = 9{\cdot}11 \times 10^{-31}$ Kg, $h = 6{\cdot}63 \times 10^{-34}$ Js

Energy of the electron in one dimensional potential box of width l is given by

$$E_n = \frac{n^2h^2}{8ml^2} \text{ where } n = 1, 2, 3 \ldots.$$

Least energy can be obtained by substituting $n = 1$

$$E_1 = \frac{h^2}{8ml^2} = \frac{(6{\cdot}63 \times 10^{-34})^2}{8 \times 9{\cdot}11 \times 10^{-31} \times (10^{-10})^2} = 60{\cdot}3 \times 10^{-19}\ J$$

$$E_1 = \frac{60{\cdot}3 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 37{\cdot}68 \text{ ev.}$$

4. A particle is confined to one dimensional infinite potential well of width $0{\cdot}4 \times 10^{-19}$ m. It is found that when the energy of the particle is 230 ev, its eigen function has 5 antinodes. Find the mass of the particle and show that it can never have energy equal to 1 kev.

Sol : According to data the particle is in quantum state with $n = 5$. E_1 and E_5 be the energies in first and fifth states respectively. Then

$$E_5 = 5^2 E_1 = 230 \text{ ev} = 230 \times 1{\cdot}6 \times 10^{-19}\ J;\ E_1 = \frac{230 \times 1{\cdot}6 \times 10^{-19}}{25} = 14{\cdot}7 \times 10^{-19}\ J$$

We know that $E_1 = \dfrac{h^2}{8ma^2}$

$$m = \frac{h^2}{8E_1 a^2} = \frac{(6{\cdot}62 \times 10^{-34})^2}{8 \times 14{\cdot}7 \times 10^{-19} \times (0{\cdot}4 \times 10^{-19})^2} = 2{\cdot}3 \times 10^{-11}\ Kg$$

If $E_n = 1\ kev = 10^3$ ev

$$n^2 = \frac{E_n}{E_1} = \frac{10^3 \times 1{\cdot}6 \times 10^{-19}}{14{\cdot}7 \times 10^{-19}} = 108{\cdot}7 \qquad \therefore\ n = 10{\cdot}4$$

$\therefore$ $n = 10{\cdot}4$ is not an integer, hence $E_n = 1$ kev is not a permitted value of energy for the particle.

5. A particle is moving in one dimensional potential box (of infinite height) of width 25Å. Calculate the probability of finding the particle with in an interval of 5Å at the centre of the box when it is in its state of least energy.

Sol : The probability of a particle at centre of a box, $P = \dfrac{2}{l}\Delta x$

Here $l = 25°A = 25 \times 10^{-10}$ m and $\Delta x = 5°A = 5 \times 10^{-10}$ m

$$P = \frac{2 \times 5 \times 10^{-10}}{25 \times 10^{-10}} = 0{\cdot}4.$$

6. For a wave function ψ show that $[y,z] = 0$. **[ANU J16]**

Sol : The particle moves inside a box along X- direction of infinite square well.

The box is supposed to have walls of infinite height at $x = 0$ and $x = L$.

The normalised wave function of the particle in x box of infinite square well is,

$$\psi_v = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

The particle can not exist outside the box and so its wave function ψ is o for $x \leq 0$ and $x \geq L$. Hence ψ is for $y = 0$ and $z = 0$.

$\therefore$ For a wave function ψ, we have $[Y, Z] = 0$.

7. Find the least energy of an electron moving in one dimension in an infinitely high potential box of width $2{\cdot}5\ \AA$. **[ANU O/N 17]**

A. Given $l = 2{\cdot}5\ \AA = 10^{-10}\ m$

$m = 9{\cdot}11 \times 10^{-31}\ Kg\ ;\ h = 6{\cdot}63 \times 10^{-34}\ Js$

Energy of the electron in one dimensional potential box of width l is given by

$$E_n = \frac{n^2 h^2}{8ml^2} \text{ where } n = 1, 2, 3 \ldots.$$

Least energy can be obtained by substituting $n = 1$,

$$E_1 = \frac{h^2}{8ml^2} = \frac{(6{\cdot}63 \times 10^{-34})^2}{8 \times 9{\cdot}11 \times 10^{-31} \times (2{\cdot}5 \times 10^{-10})^2} = 9{\cdot}650 \times 10^{-19}\ J$$

$$\therefore\ E_1 = \frac{9{\cdot}650 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 6{\cdot}03125\ eV.$$

8. A particle is moving in a one dimensional infinite high box of width $15\ \AA$. Calculate the probability of finding the particle within an interval of $1\ \AA$ at the state of least energy. **[VSU 17]**

Sol : Probability of the particle $P = \frac{2}{l}\Delta x$

Here $l = 15\ \AA\ ;\ \Delta x = 1\ \AA$

$$\therefore\ P = \frac{2 \times 1}{15} = 0{\cdot}133.$$

9. A particle is moving in one dimensional potential box (of infinite height) of width 'l' $\AA$. Calculate the probability of finding the particle of $\Delta x\ \AA$ at the centre of the box when it is in state of least energy.

Sol : We know that wave function of a particle enclosed with in an infinite potential box is given by

$$\psi_n(x) = \sqrt{\frac{2}{l}}\ \sin\frac{n\pi x}{l}$$

when the particle is in the state of least energy $n = 1$. Hence in this case

$$\psi_1(x) = \sqrt{\frac{2}{l}}\ \sin\frac{\pi x}{l}$$

At the centre of the box $x = \frac{l}{2}$. The probability of finding the particle in the unit interval at the centre of box is given by

$$|\psi_1(x)|^2 = \left[\sqrt{\frac{2}{l}}\ \sin\frac{\pi\left(\frac{l}{2}\right)}{l}\right]^2 = \frac{2}{l}\sin^2\frac{\pi}{2} = \frac{2}{l}$$

The probability P in the interval Δx is given by $P = |\psi(x)|^2\ \Delta x = \frac{2}{l}\Delta x$.

UNIT - IV

4 NUCLEAR PHYSICS

LONG ANSWER TYPE QUESTIONS

1. Mention the basic properties of nucleus with reference to size, charge, mass, nuclear spin, magnetic dipole moment and electric quadrupole moment.

[ANU J18, O17; AU 17; KU O17; RU O17; SKU O18; SVU O18; VSU O18]

A. The basic properties of nucleus of an atom are given below.

i) Nucleus size : The size of the nucleus is estimated by the study of proton and neutron scattering experiments. The mean radius of the nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10}m. The nuclear radius is given by $R = R_0 A^{1/3}$ where $R_0 = 1{\cdot}3 \times 10^{-15}$ m and A is mass number.

ii) Nuclear charge : The charge of the nucleus is due to the protons contained in it. Each proton has a positive charge of $1{\cdot}6 \times 10^{-19} c$. The nuclear charge is Ze where Z is the atomic number of the nucleus.

iii) Nuclear mass : The mass of the nucleus is sum of the masses of protons and neutrons. It is expressed in atomic mass units (a.m.u). 1a.m.u = $1{\cdot}66 \times 10^{-27}$ kg. It is given by $Zm_p + Nm_n$ where m_p, m_n are mass of proton and neutron. Z is atomic number and N is number of neutrons.

iv) Nuclear density (d) : The ratio of total mass of the nucleus to volume of the nucleus is called nuclear density.

i.e., $d = \frac{M}{V}$. But mass of the nucleus $M = mA$ and $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi A R_o^{\frac{1}{3}}$

Where m = mass of nucleon, A is Atomic number, R is radius of nucleus.

So, $d = \frac{3m}{4\pi R_0^{\frac{1}{3}}}$.

v) Nuclear Spin : Both the proton and neutron, like the electron, have an intrinsic spin. The spin angular momentum is calculated by $L_s = \sqrt{l\,(l+1)}\,\frac{h}{2\pi}$.

In addition to the spin angular momentum, the protons and neutrons in the nucleus have an orbital angular momentum. The resultant angular momentum of the nucleus is obtained by adding the spin and orbital angular momenta of all the nucleus within the nucleus. The total angular momentum of a nucleus is given by $L_N = \sqrt{l_N\,(l_N+1)}\,\frac{h}{2\pi}$. This total angular momentum is called nuclear spin.

vi) Magnetic dipole moment of nuclei (μ) : Proton has a positive elementary charge (e). Due to its spin, it has magnetic dipole moment μ_N. According to Dirac's theory magnetic dipole moment, $\mu_N = \dfrac{e(h/2\pi)}{2m_p}$, where m_p is the proton mass. Here μ_N is called as nuclear magneton.

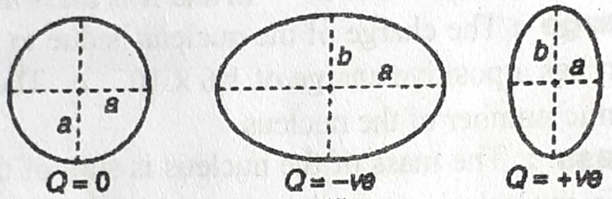
Neutron is a neutral particle. It is found that neutron has a magnetic moment $\mu_n = -1.9128\,\mu_N$.

The magnetic moments of proton and neutron can be understood on the basis of meson theory.

vii) Electric quadrupole moment (Q) : In addition to its magnetic moment, a nucleus may have an electric qudrupole moment.

In general, the shape of the nucleus is not spherical but it is an ellipsoid of revolution. Indeed, most nuclei do assume approximately such a shape. The deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.

Considerering the symmetry as ellipsoid of revolution, let the diameter along the axis be $2a$ and the diameter in a perpendicular direction be $2b$ as shown in fig (ii) and (iii).



(i) (ii) (iii)

The electric quadrupole moment is given by $Q = \frac{2}{5} Ze\,[b^2 - a^2]$ where Z is atomic number an Ze is the total charge on the nucleus.

2. What is mass defect and nuclear binding energy ? Draw a binding energy curve. What information do we get from such a curve ? **[AU 18; BRAU O18; KU O18; SVU O17; VSU O18, S17; YVU N15]**

A. **Mass defect :** The difference of mass between the actual mass of the nucleus and masses of the nucleons, is called mass defect.

Binding energy :

1) The equivalent energy of mass defect is called Binding energy.
2) When the Z protons and N neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into an amount of energy $\Delta E = (\Delta m)\,c^2$. This is called the binding energy of the nucleus.
3) To disrupt a stable nucleus into its constituent protons and neutrons, the minimum energy is required is the binding energy.
4) The magnitude of the B.E of a nucleus determines its stability against disintegration.
5) If the B.E is large, the nucleus is stable.
6) A nucleus having the least possible energy, equal to the B.E, is said to be in the ground state.
7) If the nucleus has an energy $E > E_{Min}$, it is said to be in the excited state.

8) The case $E = 0$ corresponds to dissociation of the nucleus into its constituent nucleons.

9) If M is the experimentally determined mass of a nuclide having Z protons and N neutrons.

B.E= $\{(Z m_p + N m_n) - M\}c^2$

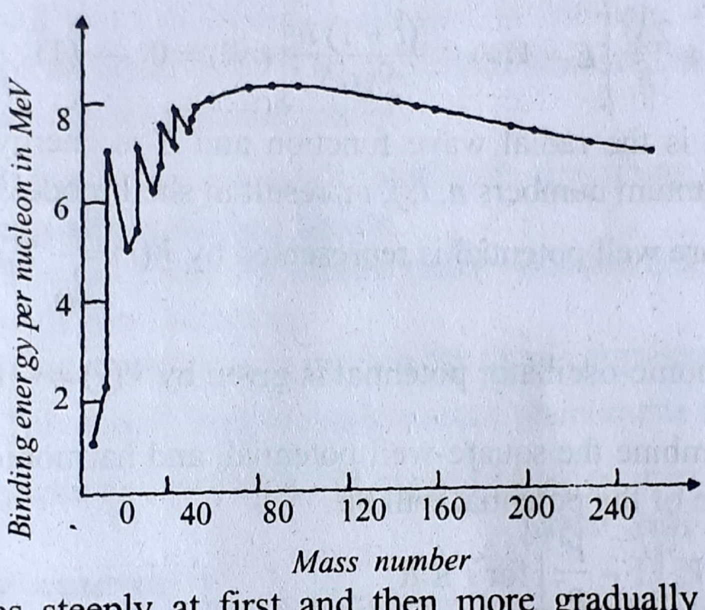
10) If B.E > 0, the nucleus is stable and energy must be supplied from outside to disrupt it into its constituents.

11) If B.E < 0, the nucleus is unstable and it will disintegrate by itself.

Stability of nucleus and binding energy :

$$\text{B.E per nucleon} = \frac{\text{Total B.E of a nucleus}}{\text{The number of nucleons}}$$

The B.E per nucleon is plotted as a function of mass number A is shown in figure.



1) The curve rises steeply at first and then more gradually until it reaches a maximum of 8·79 Mev at $A = 56$ corresponding to $_{26}Fe^{56}$.
2) The curve then drops slowly to about 7·6 Mev at the highest mass numbers.
3) Evidently nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons.
4) This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be split into lighter ones or if light nuclei can somehow be joined to form heavier ones. The former process is known as nuclear fission and the latter as nuclear fusion. Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

3. Explain how the shell model of nucleus accounts for the existence of magic numbers.

[AdNU N17; BRAU O17; KU M15; RU O17; SKU O18; VSU S17]

A. 1) According to shell model, the nucleus consists of a series of protons and neutrons placed in certain discrete levels or shells.
2) According to Pauli's exclusion principle two protons with opposite spins and two neutrons having opposite spins are accommodated in a particular shell.
3) In this way the first shell accommodates two protons and two neutrons and is more tightly bound than other shells.

4) The nuclei containing protons and neutrons number 2, 8, 20, 50, 82, 126 etc., known as magic numbers or shell numbers. The nuclei for which Z and $A - Z$ are 2 and 8 are more stable than their neighbours.

5) The electric quadrupole moments of magic number nuclei are very low (nearly zero) compared with those other nuclei. This shows that there nuclei have almost spherical charge distribution. This is expected for more stable nuclei.

6) The shell model is based on the following 2 assumptions.

i) Each nucleon moves freely in a force field described by the potential, which is a function of radial distance from the centre of the system.

ii) The energy levels or shells are filled according to pauli exclusion principle.

7) Consider a nucleon of mass M with angular momentum $\sqrt{l(l+1)}\,\hbar$ is moving in a potential $V(r)$. Assuming that $V(r)$ is independent of θ and ϕ, the scroedinger wave equation can be written as

$$\frac{d^2}{dr^2}(rR) + \frac{2M}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2Mr^2}\right](rR) = 0 \rightarrow (1)$$

where R is the radial wave function and E is energy eigen value. Here the same quantum numbers n, l, j, m_l result in shell model is an atomic model.

8) The square well potential is represented by $V(r) = -V_0$ for $r \le R$

$$= 0 \quad \text{for } r > R$$

9) The harmonic-oscillator potential is given by $V(r) = -V_0 + \frac{1}{2}kr^2$

10) If we combine the square-well potential, and harmonic oscillator potential, the new form of the potential will be

$$V(r) = -V_0\left(1 - \frac{r^2}{R^2}\right) \text{ for } r \le R$$

$$= 0 \qquad r > R$$

11) Using the above potential equation (1) and solving it, we get all the nuclear magic numbers (shell structure) except magic number 28.

12) In this way the shell model of the nucleus in an attempt to account for the excistence of magic numbers and certain other nuclear properties in terms of nucleon behaviour in a common force field.

4. Describe about liquid drop model. **[ANU O18, J18, J16, 15, M15; AdNU O18, N17; AU 18; KU O18M O17; RU O18, O17; SKU O17; SVU O17; VSU S17; YVU O18, N17]**

A. **Liquid drop Model :** This was one of the earliest models for the nucleus and was proposed by Niels Bohr. This model is explained successfully the nuclear fission. In this model, the nucleus of an atom is similar to a drop of liquid in many ways such as the close packing of the nucleons, constant density and short range forces etc. The similarities between the nucleus and a liquid drop are the following.

i) The nucleus is supposed to be spherical in shape in the stable state, just as a liquid drop is spherical due to the symmetrical surface tension forces.

ii) The force of surface tension acts on th surface of the liquid drop. Similarly, there is a potential barrier at the surface of the nucleus.

iii) The density of a liquid drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

iv) The intermolecular forces in a liquid are short range forces. The molecules in a liquid drop interact only with their immediate neighbours. Similarly, the nuclear forces are short range forces. Nucleons in the nucleus also interact only with their immediate neighbours. This leads to the saturation in the nuclear forces and a constant binding energy per nucleon.

v) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

vi) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into smaller nuclei.

Merits : 1) It has been successfully applied in describing nuclear reactions and explaining nuclear fission.

2) The calculation of atomic masses and binding energies can be done with good accuracy.

Demerits : 1) This model fails to explain the magic numbers.

2) This model fails to explain some phenomena involving nucleons.

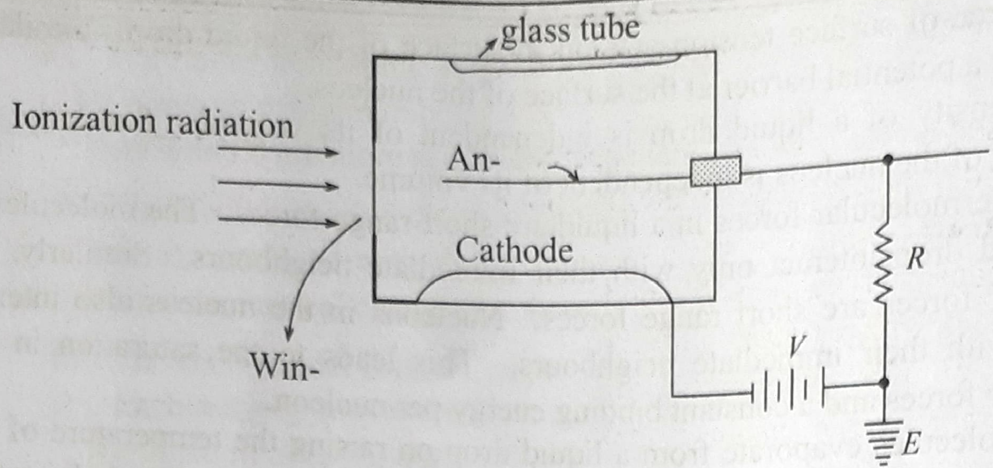
5. Explain the principle and working of Geiger-Muller counter and discuss its merits and demerits. **[ANU July 2007, March 2007]**

A. **Geiger-Muller counter :**

Principle : " When an ionizing particle (say α-particle) enters the counter, positive and negative ions are produced. The flow of electrons moves towards central wire and passing through the resistance causing a small current pulse. As each incoming particle into counter produces a pulse, hence the number of incoming particles can be counted.

Description : It is a device used to detect and measure ionization radiations. It consists a hollow metal cylinder containing a mixture of 90% argon at 10 cm pressure and 10% ethyl alcohol vapour at 1 cm pressure. A fine wire like tungsten (Anode) is placed along the metal cylinder (cathode) electrode enclosed in a thin glass tube. At one end of the tube, a very thin mica window is provided through which the ionizing particles or radiations may enter the tube. A d.c potential of about 1200 volts is applied between the cathode and the wire which acts as an anode.

The value of the voltage is adjusted to be some what below the break down voltage of the gaseous mixture. A high resistance R is connected in series with battery.



Working : When a charged particle passes through the counter, it ionizes the gas molecules. The central wire attracts the electrons while the cylindrical electrode attracts the positive ion. This causes an ionization current which depends upon the applied voltage. At sufficiently high voltages, the electrons gain high kinetic energy and cause further ionization of argon atoms. Thus, the larger number of secondary electrons are produced. The number of secondary electrons is independent of the number of primary ions produced by incoming particle due to the following reasons.

i) The production of secondary electrons is not combined to the region near the primary electrons but it takes place all along the length of the wire as their number is extremely large ($\simeq 10^8$)

ii) The production of secondary electrons at one point effects the production at other point.

The incoming particles serves the purpose of triggering the release of an avalanche of secondary electrons. The electrons quickly reach the anode and cause ionization current. The positive ions move more slowly away from anode and they form a sheath around the anode for a short while. They reduce the potential difference between the electrodes to a very low value because ion sheath depresses electric field near anode. The current therefore, stops. In this way brief pulse of current flows through resistance R. This current creates a potential difference across R. The pulse is amplified and fed to counter circuit. As each incoming particle produces a pulse hence the number of incoming particles can be counted.

Advantages :

1) It is very useful for counting β-particles

2) It can also be used for measuring γ-ray intensities.

Disadvantages :

1) It cannot be used for counting α-particles due to their low energy as the window cannot be made thin enough the pass them.

2) It cannot give other informations as charge, momentum, energy of the particle.

6. Explain the construction and working of Wilsion's cloud chamber. Write its advantages and disadvantages. What are the limitations of this apparatus ?

[ANU July 2010, Aug 2009, March 2009, 2008]

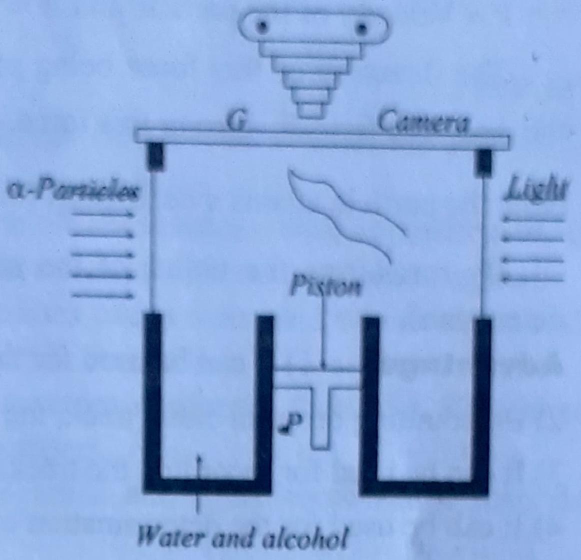
A. Wilson cloud chamber is used to detect and record the paths of charged particles.

Principle :

It is based on the principle that supercooled vapour condensed only on charged particles, and if the charged particles are not present they remain in vapour phase .

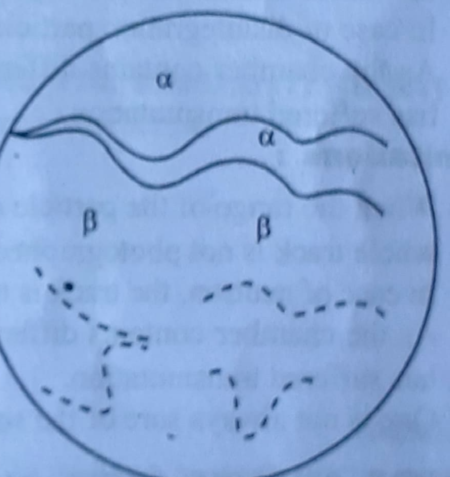
Construction :

Expansion type Wilson cloud chamber is shown in figure. It consists of an air tight cylinder *C* provided with a movable piston *P* and upper end lowered with a glass plate *G*. The chamber contains a mixture of alcohol vapour and air. A small amount of water and alcohol is kept in a trough at the lower end of the chamber. The chamber is illuminated by mercury vapour lamp *L* whose light enters in the chamber by a window. α or β-rays emitted by a radioactive substance are also allowed to enter the chamber by a side window as shown in the figure. A photographic camera is adjusted on the upper side of the chamber.



Working :

The volume of the chamber is suddenly increased by pulling back the piston. The adiabatic expansion of the gases takes place and air cooling is produced. The air in the chamber now becomes super-saturated with the result that alcohol vapour condenses into drops which form fog in the chamber . When α or β-rays allowed to pass in the chamber, they ionise the gas. Now negative and positive ions are formed all along the path of these rays. If at the same time expansion takes place, fog drops will be formed on newly created ions. A clear track in the form of drop is thus formed. The drops are visible when the chamber is illuminated by light. The tracks can be photographed with the help of camera. Different particles produce different types of tracks shown in figure. Heavy slow and alpha particles produce short, broad densely packed straight line tracks. β particles produce thin, beaded and tortuous tracks.



When the particle enters the chamber, the piston drops down, the shutter of camera opens and the track of the particle is photographed.

Measurement of energy :

The wilson cloud chamber is placed in strong magnetic field in such a way that magnetic field acts perpendicular to the direction of motion of the particle. The particle experiences a force F is given by

$F = qVB$, where q = charge on the particle

V = Velocity of the particle and B = magnetic field strength

The direction of this force being perpendicular to the direction of motion and the magnetic field B. Due to this force, the particle describes a circular path.

Now the particle attains a centripetal force $\frac{mV^2}{r}$. Thus $\frac{mV^2}{r} = qVB$ or $mV = rqB$

By measuring the radius of the path, the momentum of the particle can be determined.

Advantages : 1) It can be used for the study of radioactive radiations.

2) By counting drops in cloud track, the specific ionization can be determined.

3) It can be used for recording the tracks of ionizing particles.

4) It cab be used for the determination of energy of various particles.

Disadvantages :

1) When the range of the particle exceeds, the dimensions of the chamber then the whole track is not photographed.
2) In case of disintegration particle neutron, the track is not registered.
3) As the chamber contains different gases, it is difficult to know which of them has suffered transmutation.

Limitations :

1) When the range of the particle exceeds the dimensions of the chamber then the whole track is not photographed.
2) In case of neutron, the track is not registered.
3) As the chamber contains different gases, it is difficult to know which of them has suffered transmutation.
4) One is not always sure of the sense of the track photographed.

SHORT ANSWER TYPE QUESTIONS

1. Write a short note on semi empirical formula.

A. **Semi-emperical formula :** **[ANU 018]**

1) The atomic mass of a nucleus can be expressed in terms of masses of protons, neutrons and series of correction terms of binding energy. The modified formula for the mass is called semi emperical formula.
2) Consider a nucleus of atomic number Z and atomic number A with atomic mass ${}_ZM^A$. If m_p and m_n are the mass of proton and neutron, ${}_ZM^A = \{[Zm_p + (A - Z)\, m_n] - B\}$ where B is the binding energy expressed in mass units.
3) B is calculated emperically as the sum of some correction terms as $B = E_V + E_S + E_C + E_r + E_P$

where E_V = Volume energy correction

E_S = Surface energy correction

E_C = Coulomb energy correction

E_r = Asymmetry energy correction

E_P = Pairing energy correction

2. What are magic numbers? Explain. **[RU O18]**

A. 1) According to shell model, the nucleus consists of a series of protons and neutrons placed in certain discrete levels or shells.

2) According to Pauli's exclusion principle two protons with opposite spins and two neutrons having opposite spins are accommodated in a particular shell.

3) In this way the first shell accommodates two protons and two neutrons and is more tightly bound than other shells.

4) The nuclei containing protons and neutrons number 2, 8, 20, 50, 82, 126 etc., known as magic numbers or shell numbers.

5) The nuclei for which Z and $A-Z$ are 2 and 8 are more stable than their neighbours.

6) The electric quadrupole moments of magic number nuclei are very low (nearly zero) compared with those other nuclei.

7) This shows that there nuclei have almost spherical charge distribution. This is expected for more stable nuclei.

3. Mention the basic properties of nucleus. **[ANU J16; BRAU O17; KU M16, J15]**

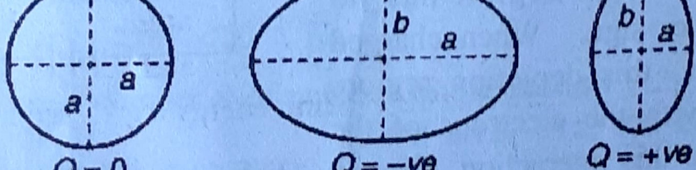
A. The basic properties of nucleus of an atom are given below.

i) Nucleus size
ii) Nuclear charge
iii) Nuclear mass
iv) Nuclear spin
v) Magnetic dipole moment of nuclei
vi) Electric quadrupole moment

4. Write about quadrupole moment of a nucleus. **[AdNU O18]**

A. **Electric quadrupole moment (Q) :** In addition to its magnetic moment, a nucleus may have an electric qudrupole moment.

In general, the shape of the nucleus is not spherical but it is an ellipsoid of revolution. Indeed, most nuclei do assume approximately such a shape. The deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.



$Q=0$ (i) $Q=-ve$ (ii) $Q=+ve$ (iii)

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By measuring the radius of the path, the momentum of the particle can be determined.

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6) The electric quadrupole moments of magic number nuclei are very low (nearly zero) compared with those other nuclei.

7) This shows that there nuclei have almost spherical charge distribution. This is expected for more stable nuclei.

3. Mention the basic properties of nucleus. **[ANU J16; BRAU O17; KU M16, J15]**

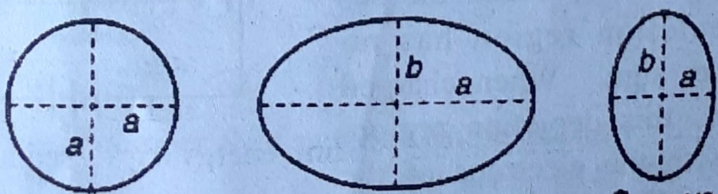
A. The basic properties of nucleus of an atom are given below.

i) Nucleus size
ii) Nuclear charge
iii) Nuclear mass
iv) Nuclear spin
v) Magnetic dipole moment of nuclei
vi) Electric quadrupole moment

4. Write about quadrupole moment of a nucleus. **[AdNU O18]**

A. **Electric quadrupole moment (Q) :** In addition to its magnetic moment, a nucleus may have an electric qudrupole moment.

In general, the shape of the nucleus is not spherical but it is an ellipsoid of revolution. Indeed, most nuclei do assume approximately such a shape. The deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.



$Q = 0$ (i) $Q = -ve$ (ii) $Q = +ve$ (iii)

The Considerering the symmetry as ellipsoid of revolution, let the diameter along the axis be $2a$ and the diameter in a perpendicular direction be $2b$ as shown in fig (ii) and (iii).

electric quadrupole moment is given by $Q = \frac{2}{5} Ze\,[b^2 - a^2]$ where Z is atomic number an Ze is the total charge on the nucleus.

5. Write a short notes on magnetic dipolement.

A. **Magnetic dipole moment of nuclei (μ) :** Proton has a positive elementary charge (e). Due to its spin, it has magnetic dipole moment μ_N. According to Dirac's theory magnetic dipole moment, $\mu_N = \frac{e(h/2\pi)}{2m_p}$, where m_p is the proton mass. Here μ_N is called as nuclear magneton.

Neutron is a neutral particle. It is found that neutron has a magnetic moment $\mu_n = -1{\cdot}9128\ \mu_N$.

The magnetic moments of proton and neutron can be understood on the basis of meson theory.

6. What is massdefect and binding energy of nucleus.

A. **Mass defect :** The difference of mass between the actual mass of the nucleus and masses of the nucleons, is called mass defect.

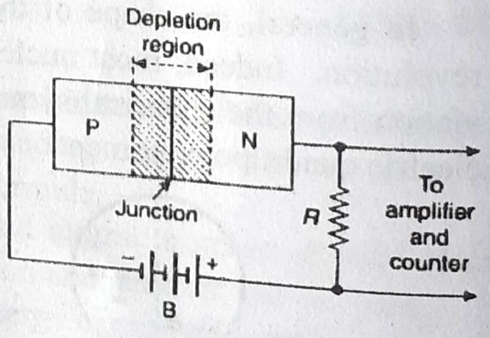
Binding energy :

1) The equivalent energy of mass defect is called Binding energy.
2) When the Z protons and N neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into an amount of energy $\Delta E = (\Delta m)\, c^2$. This is called the binding energy of the nucleus.
3) If the B.E is large, the nucleus is stable.
4) If M is the experimentally determined mass of a nuclide having Z protons and N neutrons.
   $\text{B.E} = \{(Z\,m_p + N\,m_n) - M\}\, c^2.$

7. Briefly explain the working of solid state detector.

A. A semiconductor junction device used as a particle detector is shown in fig.

It consists of a $P-N$ junction formed between P – type and N - type silicon. When a battery is connected across the junction with its – ve terminal connected P - side and + ve terminal to N - side (i.e., $P-N$ junction is reversed biased), a depletion region is formed within the device. The depletion region has no carriers of either sign. When charged particles enters this depletion region, they interact with the electrons of the crystal. Due to this interaction, electron - hole pairs are produced. The charge carriers (electron and holes) are then quickly swept away by the electric field.



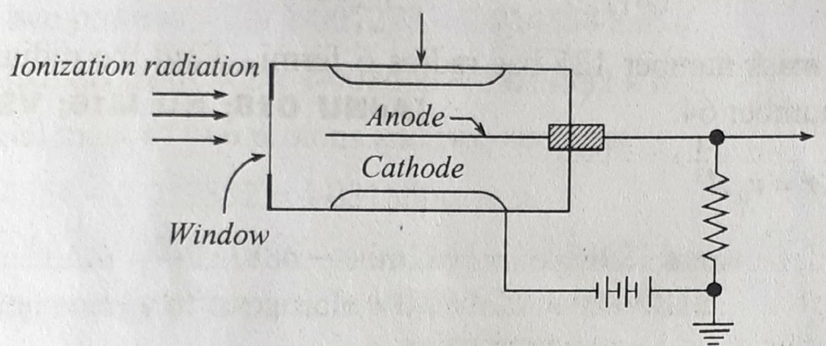
Thus they produce a voltage pulse across the resistor R. This output pulse depends upon the number of carriers collected. The incoming particles must lose all of their kinetic energy within the depletion region. The output pulse is then amplified and is either measured or counted.

8. Describe Geiger Muller counter and explain its operation

[ANU Revised Model Paper]

A. **Geiger-Muller counter :**

Geiger-Muller counter is a device used to detect and measure ionization radiations. It consists a hollow metal cylinder containing a mixture of 90% argon at 10cm pressure and 10% ethyl alcohol vapour at 1cm pressure. A fine wire like tungsten (Anode) is placed along the metal cylinder (cathode) electrode enclosed in a thin glass tube. At one end of the tube, a very thin mica window is provided.



A d.c potential of about 1200 volts is applied between the cathode and anode. A large electric field is produced near the surface of the wire. When an ionizing particle passes through the window into the tube, the free electrons produced by the ionization are attracted towards the anode. These electrons cause further ionization and hence an avalanche effect is produced. A current pulse starts through the high resistance and a p.d. produced across it is amplified and passed into a counter. Thus the counter registers the number of ionizing particles passed into the tube.

PROBLEMS

1. Calculate in kg the total mass of fundamental particles present in $_3Li^7$ nucleus. Mass of proton is $1{\cdot}6725\ 10^{-27}$ Kg and mass of neutron is $1{\cdot}6748 \times 10^{-27}$ Kg.

Sol : $_3Li^7$ has 3 protons and 4 neutrons.

Mass of 3 protons $= 3 \times 1{\cdot}6725 \times 10^{-27} = 5{\cdot}0175 \times 10^{-27}$ Kg

Mass of 4 neutrons $= 4 \times 1{\cdot}6748 \times 10^{-27} = 6{\cdot}6992 \times 10^{-27}$ Kg

$\therefore$ Total mass $= (5{\cdot}0175 \times 10^{-27} + 6{\cdot}6992 \times 10^{-27})$ Kg

$= 11{\cdot}7167 \times 10^{-27}$ Kg.

2. What is the mass number A of nucleus whose radius $r = 2{\cdot}71$ fermi ? Given that $r_0 = 1{\cdot}3 \times 10^{-15}$ m. **[ANU M15, J12; KU O18, J16; SVU O18]**

Sol : $r = 2{\cdot}71$ fermi $= 2{\cdot}71 \times 10^{-15}$ m

We know that $r = r_0 A^{1/3}$

$\therefore\ 2 \cdot 71 \times 10^{-15} = 1 \cdot 3 \times 10^{-15} A^{1/3}$ or $A^{1/3} = \frac{2 \cdot 71}{1 \cdot 3} = 2 \cdot 08$

$\therefore\ A = (2 \cdot 08)^3 = 9.$

3. A nucleus of mass number 165 has radius 7·731 fermi. Calculate the radius of nucleus of mass number 4.

Sol : Let r_1 and A_1 and r_2 and A_2 be the radius and mass number of Ho^{165} and He^4 respectively. Then

$r_1 = r_0 A^{1/3}$ and $r_2 = r_0 A^{1/3}$

$$\therefore\ \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \text{ or } r_2 = r_1 \left(\frac{A_2}{A_1}\right)^{1/3} \text{ or } r_2 = 7 \cdot 731 \left(\frac{4}{165}\right)^{1/3} = 2 \cdot 238 \text{ fermi.}$$

4. A nucleus of mass number 125 has radius 6 fermi. Find the radius of a nucleus having mass number 64. **[AdNU O18; KU M16; VSU O18, S17]**

Sol : We know that $r = r_0 A^{\frac{1}{3}}$

$$\frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Given that $A_1 = 125$, $A_2 = 64$, $r_1 = 6$ fermi

6×10^{-15}m; $r_2 = ?$

$$\therefore\ \frac{6 \times 10^{-15}}{r_2} = \left(\frac{125}{64}\right)^{1/3} = \left[\frac{(5)^3}{(4)^3}\right]^{1/3} = \frac{5}{4}$$

$$\text{or } r_2 = \frac{(6 \times 10^{-15}) \times 4}{5} = 4 \cdot 8 \times 10^{-15} = 4 \cdot 8 \text{ fermi.}$$

5. Estimate the density of nuclear matter in kg/m^3, given that the nuclear radius $r = 1 \cdot 3 \times A^{1/3}$ fermi. Take the mass of the nucleus as A amu where A is the mass number and 1 amu = $1 \cdot 66 \times 10^{-27}$ Kg and 1 fermi = 10^{-15} metre.

Sol : $M = A$ a.m.u. $= A \times 1 \cdot 66 \times 10^{-27}$ Kg

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (1 \cdot 3 \times A^{1/3} \times 10^{-15})^3 = \frac{4}{3}\pi \times \{(1 \cdot 3)^3 \times A \times 10^{-45}\}$$

$$\text{Now } \rho = \frac{M}{V} = \frac{A \times 1 \cdot 66 \times 10^{-27}}{\frac{4}{3}\pi\ \{(1 \cdot 3)^3 \times A \times 10^{-45}\}} = 1 \cdot 8 \times 10^{17} \text{ Kg/m}^3.$$

6. Calculate the electrostatic potential energy between two fragments of equal nuclei produced by the fission of U^{235} just before separation. The nuclear radius $r_0 = 1 \cdot 3 \times 10^{-15} A^{1/3}$, where A is the mass number.

Sol : At the moment of fission, let r be the radius of each nucleus.

$$r = (1 \cdot 3 \times 10^{-15}) \left(\frac{1}{2} \times 235\right)^{1/3} = 1 \cdot 3 \times 4 \cdot 88 \times 10^{-15}$$

The electrostatic potential energy U between the nuclei is given by

$$V = \frac{\left(\frac{1}{2}Ze\right)\left(\frac{1}{2}Ze\right)}{4\pi\epsilon_0 \times 2r} = \frac{(46)^2 \times (1\cdot6 \times 10^{-19})^2}{4\pi\,(8\cdot65 \times 10^{-12}) \times 2\,(1\cdot3 \times 4\cdot88 \times 10^{-15})}$$

$$= \frac{2116 \times (1\cdot6 \times 10^{-19})}{4\pi\,(8\cdot65 \times 10^{-12}) \times 2(1\cdot3 \times 4\cdot88 \times 10^{-15})}\ \text{eV} = 240 \times 10^6\ \text{eV}.$$

7. Find the binding energy of an α-particle from the following data :
Mass of the helium nucleus = 4·001265 a.m.u.

Mass of proton = 1·007277 a.m.u ; Mass of neutron = 1·008666 a.m.u.

Take 1 a.m.u = 931·4812 MeV

Sol : Mass of two protons = 2 × 1·007277 = 2·014554 a.m.u.

Mass of two neutrons = 2 × 1·008666 = 2·017332 a.m.u.

Total initial mass of two protons and two neutrons

= 2·014554 + 2·017332 = 4·031886 a.m.u.

Mass defect ΔM = 4·031886 – 4·001265 = 0·030621 a.m.u.

∴ Binding energy of α-particle = 0·030621 × 931·4812

= 28·5221 MeV

Binding energy per nucleon $= \frac{28\cdot5221}{4} = 7\cdot10525$ MeV.

8. In a thermonuclear reaction $1\cdot00 \times 10^{-3}$ kg hydrogen is converted into $0\cdot993 \times 10^{-3}$ kg helium.
i) Calculate the energy released in joule.
ii) If the efficiency of the generator be 5%.
Calculate the electrical energy in kilowatt hours.

Sol : i) Mass defect $\Delta M = (1\cdot000 - 0\cdot993) \times 10^{-3}$

$= 0\cdot007 \times 10^{-3}$ Kg

∴ The energy released is given by energy-mass relation

$E = mc^2 = 0\cdot007 \times 10^{-3} \times (3 \times 10^8)^2 = 63\cdot0 \times 10^{10}$ J

ii) Electrical energy $= \frac{5}{100} \times E = \frac{5}{100} \times 63\cdot0 \times 10^{10}$

$= 0\cdot05 \times 63\cdot0 \times 10^{10}\ \text{J} = \frac{0\cdot05 \times 63 \times 10^{10}}{36 \times 10^5} = 8\cdot75 \times 10^3$ kilowatt hour.

9. A neutron breaks into a proton and an electron. Calculate the energy produced in this reaction in MeV. Mass of an electron $= 9 \times 10^{-31}$ Kg. Mass of proton $= 1\cdot6725 \times 10^{-27}$ Kg. Mass of neutron $= 1\cdot6747 \times 10^{-27}$ Kg. Speed of light $= 3 \times 10^8$ m/sec.

Sol : Mass defect ΔM = Mass of neutron – [(Mass of proton + Mass of electron)]

$= 1\cdot6747 \times 10^{-27} - (1\cdot6725 \times 10^{-27} + 9 \times 10^{-31}) = 0\cdot0013 \times 10^{-27}$Kg

$\therefore$ Emergy released $E = \Delta M \cdot c^2 = 0{\cdot}0013 \times 10^{-27} \times (3 \times 10^8)^2$

$$= 1{\cdot}17 \times 10^{-13}\text{ J} = \frac{1{\cdot}17 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}}\text{ MeV} = 0{\cdot}73\text{ MeV.}$$

10. The binding energy of ${}_{17}Cl^{35}$ nucleus is 298 MeV. Find its atomic mass. The mass of hydrogen atom (${}_1H^1$) is 1·007825 a.m.u. and that of a neutron is 1·008665 a.m.u. Given 1 a.m.u. = 931·5 MeV

Sol : The ${}_{17}Cl^{35}$ atom has 17 protons and 18 neutrons in its nucleus.

Mass of 17 protons = 17 × 1·007825 = 17·133025 a.m.u.

Mass of 18 neutrons = 18 × 1·008665 = 18·155970 a.m.u.

Total = 35·288995 a.m.u.

The mass equivalent of the binding energy of ${}_{17}Cl^{35}$ nucleus is

$$\Delta m = \frac{298}{931{\cdot}5} = 0{\cdot}319914\text{ a.m.u.}$$

$\therefore$ Atomic mass of ${}_{17}Cl^{35}$ = 35·288995 – 0·319914 = 34·96908 a.m.u.

11. If 3·6 g of uranium be completely converted into energy, how many kilowatts of energy will be obtained from it ?

Sol : We know that $E = mc^2$

$m = 3{\cdot}6\text{ g} = 3{\cdot}6 \times 10^{-3}$ Kg and $c = 3 \times 10^8$ m/sec

$\therefore\ E = 3{\cdot}6 \times 10^{-3} \times (3 \times 10^8)^2 = 32{\cdot}4 \times 10^{13}$ J

$$= \frac{32{\cdot}4 \times 10^{13}}{3600}\text{ watt – hours} = \frac{32{\cdot}4 \times 10^{13}}{3600 \times 10^3}\text{ kwh} = 9 \times 10^7\text{ kwh.}$$

12. A neutron breaks into a proton and an electron. Calculate the energy produced in this reaction in MeV. Mass of an electron = 9×10^{-31} Kg. Mass of proton = $1{\cdot}6725 \times 10^{-27}$ Kg. Mass of proton = $1{\cdot}6747 \times 10^{-27}$ Kg. Speed of light = 3×10^8 m/s.

Sol : Mass defect Δm = mass of neutron – mass of (proton + electron)

$$= 1{\cdot}6747 \times 10^{-27} - (1{\cdot}6725 \times 10^{-27} + 9 \times 10^{-31}) = 0{\cdot}0013 \times 10^{-27}\text{ Kg}$$

Energy produced $E = mc^2 = 0{\cdot}0013 \times 10^{-27} \times (3 \times 10^8)^2$ J

$$= \frac{0{\cdot}0013 \times 10^{-27} \times (3 \times 10^8)^2}{1{\cdot}6 \times 10^{-13}}\text{ MeV} = 0{\cdot}73\text{ MeV.}$$

13. Calculate the amount of energy released in Joules when 10 micrograms of uranium undergoes fission.

Avogadro's number = $6{\cdot}023 \times 10^{23}$

Energy released per fission = 200 MeV

Sol : We know that 235 g of uranium contains $6{\cdot}023 \times 10^{23}$ atoms. Hence, one gram of uranium contains

$$= \frac{6{\cdot}023 \times 10^{23}}{235}\text{ atoms}$$

$\therefore$ 20 μg contains $= \dfrac{6.023 \times 10^{23}}{235} \times (10 \times 10^{-6})$ atoms $= 2.563$ atoms

Energy released $= 200 \times (2.563 \times 10^{16}) = 5.126 \times 10^{18}$ MeV

$\therefore$ Energy released by fission of 1 atom

$= 200 \text{ MeV} = 5.126 \times 10^{18} \times 10^{6}$ eV

$= 5.126 \times 10^{18} \times 10^{6} \times (1.6 \times 10^{-19})$ J $= 8.2 \times 10^{5}$ Joule

14. Find the amount of energy produced in Joules due to fission of 1g of uranium assuming that 0·1 percent of mass is transformed into energy.

Take 1 a.m.u. $= 1.66 \times 10^{-27}$ Kg $= 931$ MeV

Mass of uranium $= 235$ a.m.u.

Avogadro number $= 6.02 \times 10^{23}$

Sol : Energy released per atom of uranium $= \dfrac{0.1}{100} \times 235$ a.m.u. $= 0.235$ a.m.u.

$= 0.235 \times 931 \times 10^{6} \times 1.6 \times 10^{-19}$ J $= 3.5 \times 10^{-11}$ Joule

Number of atoms present in one gram of uranium $= \dfrac{6.02 \times 10^{23}}{235} = 2.562 \times 10^{21}$ atom

$\therefore$ Energy released by one gram of uranium

$= (2.562 \times 10^{21})(3.5 \times 10^{11})$ Joule $= 8.967 \times 10^{10}$ Joule

15. Calculate the energy released by the fission of 2g of ${}_{92}U^{235}$ in kwh. Given that the energy released per fission is 200 MeV.

Sol : The number of atoms in 2g of ${}_{92}U^{235} = \dfrac{2 \times 6.025 \times 10^{23}}{235}$ atoms

Energy released by 2g of ${}_{92}U^{235}$

$$= \frac{2 \times 6.025 \times 10^{23} \times 3.2 \times 10^{-11}}{235} \text{ J} = \frac{2 \times 6.025 \times 10^{23}}{235} \times \frac{3.2 \times 10^{-11}}{3600 \times 10^{3}} \text{ kwh}$$

$= 4.55 \times 10^{4}$ kwh

16. Calculate the mass number of the nucleus of radius 3.9×10^{-15} m. (given $R_0 = 1.3 \times 10^{-15}$ m).

Sol : $R = 3.9 \times 10^{-15}$ m ; $R_0 = 1.3 \times 10^{-15}$ m

$A = ?$

$R = R_0 A^{1/3}$

$$A = \left[\frac{R}{R_0}\right]^3 = \left[\frac{3.9 \times 10^{-15}}{1.3 \times 10^{-15}}\right] = 9$$

17. If 7.2 g of uranium is completely converted into energy, how many KWH energy is obtained.

Sol : We know that $E = mc^2$

Given that $m = 7 \cdot 2 \text{ g} = 7 \cdot 2 \times 10^{-3}\ Kg$ and $c = 3 \times 10^{8}$ m/s

$\therefore\ E = mc^2 = 7 \cdot 2 \times 10^{-3} \times (3 \times 10^8)^2 = 64 \cdot 8 \times 10^{13}$ J

$$= \frac{64 \cdot 8 \times 10^{13}}{3600} \text{ watt - hours} = \frac{64 \cdot 8 \times 10^{13}}{3600} \text{ kwh} = 18 \times 10^7 \text{ kwh.}$$

18. Determine the density of a nucleus. Given that mass of the proton / neutron $1{\cdot}67 \times 10^{-27}$ Kg. Radius of the nucleus is $1{\cdot}5 \times 10^{-15} A^{1/3}$. A = Mass number

Sol : $r = 1{\cdot}5 \times 10^{-15} A^{1/3}$

$m = 1{\cdot}67 \times 10^{-27}$ Kg

$M = A \text{ amu} = A \times 1{\cdot}67 \times 10^{-27}$ Kg

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (1{\cdot}5\ 10^{-15} A^{1/3})^3$$

$$V = \frac{4}{3}\pi \times (1{\cdot}5)^3 \times A \times 10^{-45}$$

$$\text{Density } (\rho) = \frac{M}{V} = \frac{A \times 1{\cdot}67 \times 10^{-27}}{\frac{4}{3}\pi (1{\cdot}5)^3 \times A \times 10^{-45}}$$

$$= \frac{4{\cdot}98}{42{\cdot}39} \times 10^{18} = 0{\cdot}1174 \times 10^{18} = 1{\cdot}174 \times 10^{17} \text{ Kg/m}^3$$

19. How many KWH of Energy will be released by the complete fission of 1 gm of $U-235$? **[ANU J15]**

Sol : $m = 1 \text{ gm} = 10^{-3}$ Kg, $C = 3 \times 10^8$ m/s

$E = mc^2$

$E = 10^{-3} \times (3 \times 10^8)^2$; $E = 9 \times 10^{13}$ J

$$E = \frac{9 \times 10^{13}}{36 \times 10^5} = 2{\cdot}5 \times 10^7 \text{ KWH}$$

20. Calculate the binding energy per nucleon ${}_6C^{12}$, $M_p = 1.007276$, $M_n = 1.008665$ and $M_e = 0.00055$ amu mass of ${}_6C^{12}$ atom is 12.00000 amu. **[ANU J16]**

Sol : Given, $M_p = 1.007276$ amu

$M_n = 1.008665$ amu

$M_e = 0.00055$ amu; $M = 12.00000$ amu

$Z = 6$; $A - Z = 12 - 6 = 6$

Mass defect, $\Delta M = [Z(M_p + M_e) + (A - Z)M_n - M]$

$= [6(1.007276 + 0.00055) + 6 \times 1.008665 - 12.00000]$ amu $= 0{\cdot}098946$ amu

$$\text{Binding energy per nucleon} = \frac{\Delta M}{A} \times 931.5\text{Mev}$$

$$= \frac{0.098946}{12} \times 931.5\text{Mev} = 7.680\text{Mev.}$$

21. Find the energy needed to remove a neutron from the nucleus of the calcium isotope ${}_{20}Ca^{41}$ is 40.962278 amu. **[ANU M16]**

Sol : Energy needed to remove neutron,

$$E = (41{\cdot}958622 - 40{\cdot}962278) \text{ amu}$$

$$= 0{\cdot}996344 \text{ amu} = 0{\cdot}996344 \times 931{\cdot}5 \text{ Mev} = 928{\cdot}09 \text{ MeV}.$$

22. What is the mass number of a nucleus whose radius is 3·6 fermi? (given $r_0 = 1{\cdot}2$ femi)

Sol : $R = 3{\cdot}6$ fermi $= 3{\cdot}6 \times 10^{-15}$m.

$r_0 = 1{\cdot}2$ fermi $= 1{\cdot}2 \times 10^{-15}$m.

$$R = r_0 A^{1/3}$$

$$A = \left[\frac{R}{r_o}\right]^3 = \left[\frac{3{\cdot}6 \times 10^{-15}}{1{\cdot}2 \times 10^{-15}}\right]^3 \quad \therefore A = 27.$$

23. Calculate the energy released when two protons and two neutrons combine to form α-particle. [Given mass of α particle = 4·00389 a.m.u, mass of proton = 1·00813 a.m.u, mass of neutron = 1·00893a.m.u. and 1amu = 931 MeV]

Sol : Mass of two protons = 2 × 1·00813 amu = 2·01626 amu

Mass of two neutrons = 2 × 1·008993 amu = 2·01786 amu

Total mass of nucleons in α-particle = (2·01626 + 2·01786) amu = 4·03412 amu

Mass of α- particle Nucleus, $M_N = 4{\cdot}00389$ amu.

Mass defect, $\Delta M = (4{\cdot}03412 - 4{\cdot}00389)$ amu= 0·03023 amu

$\therefore$ Energy released, $E = \Delta M \times C^2 = \Delta M \times 931 \text{ MeV} = 0{\cdot}03023 \times 931 = 28{\cdot}14413$ MeV.

24. Calculate the approximate radius of Al^{27}. (Given $r_0 = 1{\cdot}2$ fermi).

Sol : $A = 27$; $r_0 = 1{\cdot}2$ fermi; $R = ?$

$$R = r_0 A^{\frac{1}{3}} = 1{\cdot}2 \times [27]^{\frac{1}{3}} \quad \therefore R = 3{\cdot}6 \text{ fermi}.$$

25. Calculate the mass number A of a nucleus whose radius (R) is 2·17 fermi? ($r_0 = 1{\cdot}3 \times 10^{-10}$ metre). **[SVU 18]**

Sol : $R = 2{\cdot}71$ fermi $= 2{\cdot}71 \times 10^{-15}\, m$

$r_0 = 1{\cdot}3 \times 10^{-10}\, m$

We know that $R = r_0 A^{1/3}$

$$2{\cdot}71 \times 10^{-15} = 1{\cdot}3 \times 10^{-10} \times A^{1/3}$$

$$A^{1/3} = \frac{2{\cdot}71 \times 10^{-15}}{1{\cdot}3 \times 10^{-10}} = \frac{2{\cdot}71}{1{\cdot}3} \times 10^{-5} = 2{\cdot}08 \times 10^{-5}$$

$$\Rightarrow A = \left[2{\cdot}08 \times 10^{-5}\right]^{\frac{1}{3}} = \left[20{\cdot}8 \times 10^{-6}\right]^{\frac{1}{3}} \quad \therefore A = (20{\cdot}8)^{\frac{1}{3}} \times 10^{-2}.$$

26. Calculate energy equal to 1 amu.

Sol : 1 amu $= 1.66 \times 10^{-27}$ Kg

$C = 3 \times 10^8$ m/s^2

$$E = mc^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.66 \times 10^{-27} \times 9 \times 10^{16} \text{ J}$$

$$= 1.66 \times 10^{-11} \times 9 \times 1 \frac{Mev}{1.6 \times 10^{-13}} \quad [\because MeV = 1.6 \times 10^{-13} J]$$

$$= \frac{1.66 \times 9 \times 10^2}{1.6} \quad \therefore\ E = 933.75\ MeV.$$

27. Calculate the energy released by the fission of 2gm of U-235 in KWH. Energy released per fission 200 MeV.

Sol : Mass of uranium = $2g$

Energy per fission = 200 MeV = $200 \times 10^6 \times 1.6 \times 10^{-19}\ J$

No. of atoms in $2g$ of uranium, $n = \dfrac{2 \times 6.023 \times 10^{23}}{235}$

Total energy released $E^1 = nE = \dfrac{2 \times 6.023 \times 10^{23}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19}\ J$

$$= \frac{1640 \times 3 \times 10^8}{36 \times 10^5} Kwh$$

$$\therefore\ E^1 = 45.56 \times 10^3\ Kwh.$$

28. Calculate the mass number of nucleus whose radius is 3.9×10^{-15} m. ($R_0 = 1.3 \times 10^{-15} m$).

Sol : $R = 3.9 \times 10^{-15}$ m ; $R_0 = 1.3 \times 10^{-15}$ m ; $A = ?$

$R = R_0 A^{1/3}$

$$\therefore\ A = \left[\frac{R}{R_0}\right]^3 = \left[\frac{3.9 \times 10^{-15}}{1.3 \times 10^{-15}}\right]^3 = 27$$

29. Calculate the Binding Energy (BE) per nucleon of α - particle, $m_n = 1.008665$ amu, $m_p = 1.0087276$ amu, $m_\alpha = 4.00150$ amu, 1 amu = 931 MeV. **[ANU O18, J18]**

Sol : Mass of two protons $= 2 \times 1.007276$ a.m.u $= 2 \cdot 014552$ a.m.u

Mass of two neutrons $= 2 \times 1.008665$ a.m.u $= 2.01733$ a.m.u

Total mass of two protons and two neutrons $= 2.014552 + 2.01733 = 4.031882$ a.m.u

Mass defect, $\Delta M = (4.031882 - 4.00150)$ a.m.u $= 0.030382$ a.m.u

$\therefore$ Binding energy of α - particle, (B.E) $= 0.030382 \times 931$ Mev $= 28.285642$ a.m.u

Binding energy of an α - particle per nucleon $= \dfrac{B \cdot E}{A} = \dfrac{28.285642 \text{ Mev}}{4} = 7.0714$Mev

30. A nucleus of mass number 125 has radius 0.8 femi. Find the radius of nucleus having mass number 64.

Sol : We know that $r = r_0 A^{\frac{1}{3}}$

$$\frac{r_2}{r_1}=\left(\frac{A_2}{A_1}\right)^{\frac{1}{3}}$$

Here $r_1 = 0{\cdot}8$ fermi, $A_1 = 125$; $A_2 = 64$

$$\frac{r_2}{0{\cdot}8}=\left[\frac{64}{125}\right]^{\frac{1}{3}} \Rightarrow r_2 = 0{\cdot}8\times\frac{4}{5}=\frac{3{\cdot}2}{5}=0{\cdot}64 \text{ fermi}$$

31. Calculate the binding energy per nucleon of α - particle, $m_n = 1{\cdot}00893$ amu, $m_p = 1{\cdot}00813$ amu, $m_\alpha = 4{\cdot}00389$ amu, 1 amu = 931 MeV. **[ANU J18]**

Sol : Mass of two protons $= 2 \times 1{\cdot}00813$ a.m.u $= 2\cdot01626$ a.m.u

Mass of two neutrons $= 2 \times 1{\cdot}00893$ a.m.u $= 2{\cdot}01786$ a.m.u

Total mass of two protons and two neutrons $= 2{\cdot}01626 + 2{\cdot}01786 = 4{\cdot}031882$ a.m.u

Mass defect, $\Delta M = (4{\cdot}18046 - 4{\cdot}00389)$ a.m.u $= 0{\cdot}17677$ a.m.u

$\therefore$ Binding energy of α - particle, (B.E) $= 0{\cdot}17677 \times 931$ Mev

Binding energy of an α - particle per nucleon $=\dfrac{B{\cdot}E}{A}=\dfrac{164{\cdot}57 \text{ Mev}}{4}=41{\cdot}14$ MeV.

32. Determine the binding energy of deuteron given that the mass = 2·0 13553 a.m.u. **[BRAU 18]**

Sol : The mass of deuteron$\left({}_1^2H\right) = 2{\cdot}013553$ amu

$m_p = 1{\cdot}007276$ amu ; $m_n = 1{\cdot}008665$ amu

The total mass of 1 proton and one neutron $= m_p + m_n$

$= (1{\cdot}007276 + 1{\cdot}008665)$ amu

$= 2{\cdot}015941$ amu

Mass defect $\Delta M = (2{\cdot}015941 - 2{\cdot}013553)$ amu

$= 0{\cdot}002388$ amu

$= 0{\cdot}002388 \times (1{\cdot}67 \times 10^{-27})$

$= 3{\cdot}9654 \times 10^{-30}$ kg

Energy released $E - (\Delta M)C^2$

$= (3{\cdot}9654 \times 10^{-30})\,(3 \times 10^8)^2$

$\Rightarrow E = 35{\cdot}6886 \times 10^{-14} = 3{\cdot}56886 \times 10^{-13}\, J$

$\therefore\ E = \dfrac{3{\cdot}56886}{1{\cdot}6}\, eV = 2{\cdot}2305\, eV.$

33. A sample of uranium emitting α-particle of 4·18 MeV is placed near ionization chamber. Assuming that only 10 particles per second enter the chamber. Calculate the current produced. Given that ion pair requires energy of 35 eV ; $e = 1{\cdot}6 \times 10^{-19}$ C.

Sol : Energy emitted by α-particle $= 4{\cdot}18$ MeV

Number of ion-pairs produced by one α-particle $= \dfrac{4.18 \times 10^6}{35} = 1.19 \times 10^5$ pairs

Number of ion-pairs produced per second

$= 1.19 \times 10^5 \times 10 = 1.19 \times 10^6$ ion – pair/sec ($\because$ 10 particles enter in one second)

Current = number of ion-pair produced per second × charge on each ion

$= (1.19 \times 10^6) \times (1.6 \times 10^{-19}) = 1.904 \times 10^{-13}$ Å.

34. An ionisation chamber is connected to an electrometer of capacity 0·5 PF and voltage sensitivity of 4 divisions per volt. A beam of α- particles produced a deflection of 0·8 division. Calculate the number of ion pairs required and the energy of α- particles. Given that 1 ion pair required energy of 35 eV and $e = 1.6 \times 10^{-19}$ C.

Sol : The voltage required to produce a deflection of 0·8 division $= \dfrac{0.8}{4} = 0.2$ Volt

($\because$ Voltage sensitivity of electrometer is 4 div/volt)

Now $Q = CV = (0.5 \times 10^{-2})(0.2) = 10^{-3}$ C

$\therefore$ Number of ion-pairs required $= \dfrac{10^{-3}}{1.6 \times 10^{-19}} = 6.25 \times 10^5$

Now total energy required $= (6.25 \times 10^5) \times 35 = 21.88 \times 10^6$ eV= 21·88 MeV

($\because$ one pair required 35 eV energy)

35. A G.M counter wire collects 10^8 electrons per discharge. When the counting rate is 400 counts/minute, what will be the average current in the circuit?

[ANU Model Paper 2010]

Sol : Counting rate = 400 counts/min

The wire collects 10^8 electrons per discharge

$\therefore$ The total number of electrons collected in one min $n = 400 \times 10^8 = 4 \times 10^{10}$

Charge/min $= ne = (4 \times 10^{10})(1.6 \times 10^{-10})$ coul/min

Charge/second $= \dfrac{(4 \times 10^{10})(1.6 \times 10^{-19})}{60} = 1.066 \times 10^{-10}$ A

$\therefore$ Average current $= 1.066 \times 10^{-10}$ A.

36. A self Quenched G.M counter operates at 1000 volts and has a wire of diameter 0·2 mm. The radius of the cathode is 2 cm and the tube has a guaranteed life time of 10^9 counts. What is the maximum radial field and how long will the counter last if it is used on an average for 30 hours per week at 3000 counts per mintue? Assume 50 weeks to a year.

Sol : The radial field at the centre is given by

$$E_{max} = \frac{V}{r \log_e\left(\dfrac{b}{a}\right)} = \frac{1000}{0.02 \times \left[2.3 \log_{10} \dfrac{2 \times 10^{-2}}{10^{-4}}\right]} = 1.89 \times 10^6 \text{ V/m}$$

Let the life time of the tube be N years. Now the total number of counts recorded will be

$N \times 40 \times 30 \times 60 \times 3000 = 2{\cdot}7 \times 10^8$ N

According to given problem $2{\cdot}7 \times 10^8$ N $= 10^9$

$$N = \frac{10^9}{2{\cdot}7 \times 10^8} = 3{\cdot}7 \text{ years.}$$

37. A G.M counter wire collects 10^{10} electrons per discharge. When the counting rate is 1000 counts/minute, what will be the average current in the circuit ?

[ANU March 2010]

Sol : Count rate per second = 1000/60

Number of electrons collected per second $= \left(\frac{1000}{60}\right) \times 10^{10}$

$\therefore$ Charge passing per second $= \frac{1000}{60} \times 10^{10} \times (1{\cdot}6 \times 10^{-19}) = 2{\cdot}66 \times 10^{-8}$

$\therefore$ Average current = charge passing per second $= 2{\cdot}66 \times 10^{-8} A$.

38. α-particle of energy 5 MeV pass through an ionisation chamber at the rate of 10 per second. Assuming all the energy is used in producing ion-pairs, calculate the current produced. (35eV is required for producing an ion pair and $e = 1{\cdot}6 \times 10^{-19}$C)

Sol : Energy of α-particle $= 5 \times 10^6$ eV

Number of ion-pairs produced by one α-particle

$$= \frac{\text{Energy}}{\text{Energy required produce ion– pair}} = \frac{5 \times 10^6}{35} = 1{\cdot}429 \times 10^5$$

Number of ion-pairs produced per second

= Number of pairs × Number of α- particles entering per second

$= (1{\cdot}429 \times 10^5) \times 10$

$\therefore$ Current = ion-pairs produced × Charge on one ion

$= (1{\cdot}429 \times 10^5) \times 10 \times (1{\cdot}6 \times 10^{-10}) = 2{\cdot}287 \times 10^{-13}$ A.

39. A GM counter wire collects 10^9 electrons per discharge. When the count rate is 500/minute, what will be the average current in the circuit. (charge of an electron is $1{\cdot}6 \times 10^{-19} c$).

[ANU July 2010]

Sol : Count rate per second $= \frac{500}{\text{minute}} = \frac{500}{60}$/sec

Number of electrons collected per second $= \frac{500}{60} \times 10^9$

$\therefore$ Charge passing per second $= \frac{500}{60} \times 10^9 \times (1{\cdot}6 \times 10^{-19})$

$= 1{\cdot}33 \times 10^{-9} A$

$\therefore$ Average current = Charge passing per second $= 1{\cdot}33 \times 10^{-9} A$.

✣ ✦ ✣

UNIT - V

5 NANOMATERIALS

LONG ANSWER TYPE QUESTIONS

1. What are nanomaterials ? Discuss briefly different types of nanoparticles and discuss their structures.

A. **Nanomaterials :** Nano means 10^{-9}. Nanoparticles are very small particles. A group of 10^6 (or) less number of atoms (or) molecules bounded together to form a cluster with redius less than 100 nm form a nanoparticle.

The size of nano particle lies between molecular size and bulk material size. The properties of nanoparticls depend on size of the particle. Basically there are two factors which make nanoparticles to exhibit different properties from those at bulk size. They are

1) Increase in surface area to volume ratio.

2) Quantum effects at nanoscale.

1) **Increase in surface area to volume ratio :** Nanoparticles have large surface to volume ratio than their bulk form.

E.g : 1) consider a spherical particle of radius r. Let s be its surface area and V be its volume.

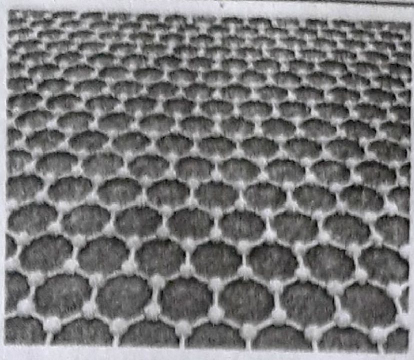
Then the ratio of surface area to volume is, $\frac{s}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$

This ratio is inversely proportional to radius of the particle.

2) **Quantum confinement effects :** We know that when atoms are isolated, the energy levels are discrete. Nanoparticles (or) nanomaterials represent an intermediate stage. When the material is in bulk form, the difference between the energy levels is very small and can be viewed as almost continuous. When the size of that material reduces to nanoscale, the energy levels are no longer continuous and they can be treated as discrete. The quantum confinement of particle can affect the optical, electrical, magnetic properties of materials.

2. Write about the structure and properties of graphene.

A. Graphene is a single layer of carbon atoms, tightly bound in a haxagonal honey comb lathice. It is an allotrope of carbon in the form of a plame of spz-bonded atoms with molecular bond length of 0·142 nanometers.. Layers of graphene stacked on top of each other from graphite, with an interplanar spacing of 0·335 nanometers. The separate layers of graphene in graphite are held together by



vander waals forces, which can be overcome during exboliation of graphene from graphite.

Graphene is the thinnest compound known to man at one atom thick, the lightest material known, the strongest compound discovered, the best conductor of heat at room temperature and also the best conductor of electricity known. Other notable properties of graphene are its uniform absorption of light across the visible and mear infrared parts of the spectrum.

Properties : The properties of graphene are unique due to its all carbon structure and nanoscale geometry.

Electronic Properties : Graphene has delocalilzed Ri-electron system across the entiraty of its surface, the moment of electrons is very fluid.

The graphene system also exhibits no bond gap, due to over lapped Ri-electrons, allowing for an easymovement of electrons without the need to input energy into the system. The electronic mobility of graphene is very high and the electrons act like photons, with respect to their movement capabilities.

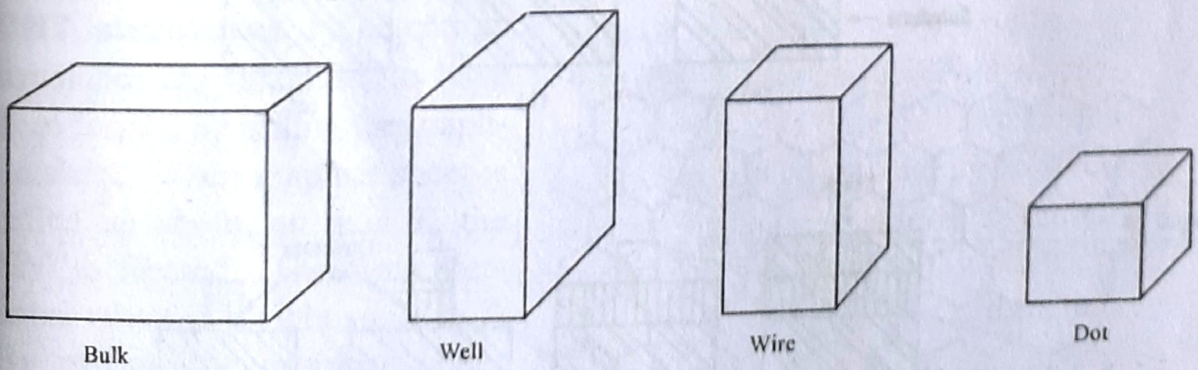
Thermal Properties : The repeating structure of graphene makes it an ideal material to conduct heat in plane. The regular structur allows the movement of phonons through the surface. Graphene can exhibit two types of thermal conductivity-in-plane and inter-plame.

Mechanical Strength : Graphene is one of the strongest materials ever discovered with a tensile strength of $1{\cdot}3 \times 10^{11}$ pa.

Elasticity : The repeating SPZ hybridized backbone of graphene molecules allow for flexibility, as there is rotation around some of the bonds, whilst still providing enough regidity and stability that the molecule can withstand changes in confirmation and support of other ions.

3. What do you mean by quantum well, quantum wire and quantum dot ? How quantum dots are synthesised.

A. **Quantum well :** If one dimension is reduced to the nanorange while other dimensions remain large, then the structure so formed is known as quantum well.



Quantum well confine electrons (or) holes in dimension and allow free propagation in two dimensions. It forces quantum well to occupy a planar region. The effects of quantum confinement take place, when the quantum well thickness becomes comparable at the de Broglie wave length of the electrons and holes, leading to energy values.

Quantum wells are widely used in diode lasers. They are also used to make High electron mobility transistors (HEMT).

Quantum wire : If the two dimensions are reduced to nanorange while the third remains the same, them the structure so formed is known as quantum wire.

A quantum wire is an electrically conducting wire, in which quantum effects are affecting transport properties. Quantum wires confine electrons (or) holes in two spatial dimensions and allow free propagation in the third. Due to the confinement of conduction electrons in the transverse direction of the wire, their transverse energy is quantized into a series of discrete values. Quantum wire has 10 times better conductivity than copper at one-sixth the weight. It would be made with carbon nanotubes.

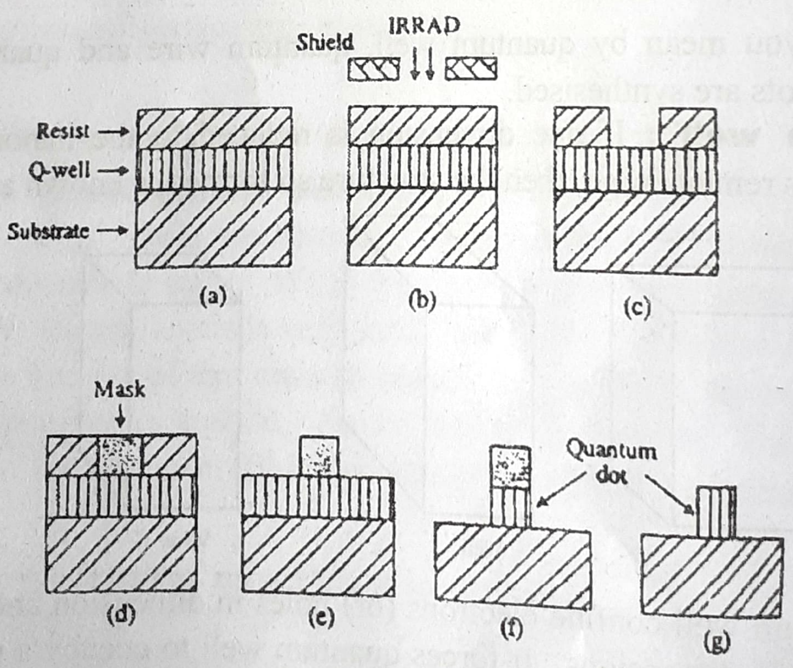
Quantum dot : When all the three dimensions of the material are reduced to nano range, then it is called as quantum dot.

The quantum dot contains 100 to 100,000 atoms with. in the quantum dot volume with a diameter of 10 to 50 atoms. This corresponds to about 2 to 10 nm in diameter. Quantum dot confines electrons (or) holes in three dimensions. Due to smaller size of the crystal, band gap is larger and the greater is the difference in energy between the highest valence band and the lowest conduction band. There fore more energy is needed to excite the dot and more energy is released when the dot returns to its ground state.

In a quantum dot semiconductor excitations are confined to potential well in all three spatial dimensions.

Quantum dots are used as light emitting diodes and quantum dots reduce the cost of photovoltaic cells.

Electron-Beam Lithography (Fabrication of Quantumdot) :



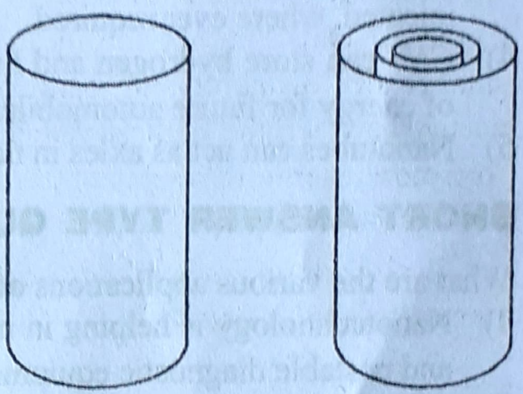
In electron-beam lithography, a thin film (quantum well) of the sample material is supported by its substrate. The quantum well is coated with a radiation-sensitive resist like polymethyl methacrylate. The region on the surface of the resist where the nanostructure is designed is irradiated by electron beam through the mask. Other wise the scanning electron beam that strikes the surface only in the desired region can also be used. The radiation chemically modifies the exposed parts of resist, so that it becomes soluble in a developer. This part of photo resist is removed by dissolving it into developer (fig c).

The unexposed part of resist should be removed, for this purpose an etching mask is planted into the groove (fig d). The remaining part of the resist is now taken out (fig e) and the region of the quantum well uncovered by the etching mask is chemically etched away (fig f), Now the quantum structure is covered by etching mask. Finally the etching mask is removed (fig g). The resulting structure is a quantum dot.

4. Describe in brief how carbon nanotubes are formed. Discuss their structure, properties and applications.

A. Carbon nanotubes have 10 to 20 nm in diameter and 100 *um* in length. The carbon nanotubes (CNT) form as.

1) Multi wall carbon nanotubes (MWCNT) with walls separated by small distance of 0·3 nm and.
2) Single wall carbon nanotubes (SWCNT). Under specific conditions SWCNT are fabricated by mixing traces of some catalyst cobalt, nickel (or) iron with the graphite target.

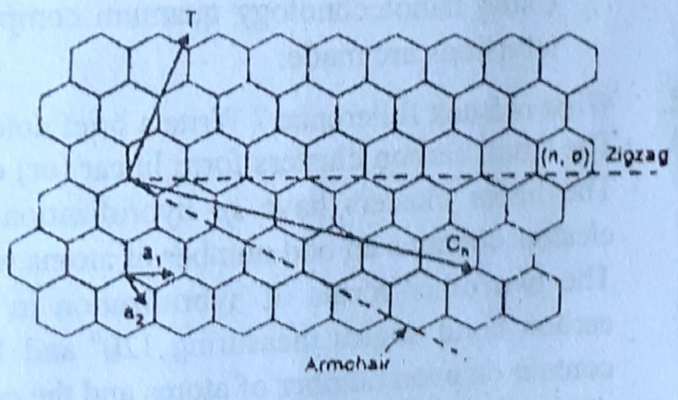


Single wall (SW) Multi wall (MW)

Carbon nanotubes are synthesizing in three methods.

1) Laser evaporation method
2) Carbon arc method
3) Chemical deposition method.

CNT structures : The carbon nanotubes are imagined to have been formed by rolling the graphite sheet. When graphite sheet is rolled up about, an axis *T*, the *CNT* is formed. The circumferential vector is at right angle to *T*. By rolling the graphite sheet about the *T* vector at different orientations, then different CNT structure can be obtained. The vector T at different orientations armchair structure, zigzag structure and the chiral structure are formed.



Properties of CNT

1) The semiconducting *CNT* electronic structure is studied using scanning tunneling microscope (STM).
2) All nanotubes are good thermal conductors. The temperature stability is estimated to be up to 2500°C in vacuum and 750°C in air.
3) The $CNT^{'s}$ are mechanically robust.
4) The molecular symmetry of *CNT* classifies their vibrations into normal modes by A_{1g} and E_{2g}. The A_{1g} mode represents the in and out oscillations of the diameter of the tube. In the other mode (E_{2g}), the tube's cross-section oscillates between the circular and elliptical shapes.

Applications of CNT :

1) A *CNT* used as connector in computer chips.
2) Long filaments of nanotubes are used in fiber-reinforced plastics. They have less weight, so they are used in aeroplanes, space ships.
3) *CNT* are thin, so they can penetrate the skin with out pain. Medicine can be injected where ever required.
4) *CNT* can store hydrogen and helium. So it can be used as an alternate source of energy for future automobiles.
5) Nanotubes can act as axles in nano mechanics

SHORT ANSWER TYPE QUESTIONS

1. What are the various applications of nanotechnology?

A. 1) Nanotechnology is helping in medical diagnostics by providing faster, cheaper and portable diagnostic equipments.
2) Nanotechnology will provide new methods to effectively utilize our current energy resources.
3) Nanotechnology will provide efficient water purification techniques. Water from the oceans can also be converted into drinking water.
4) Computers can be made more powerful and smaller using nanotechnology.
5) Sensors based on nanotechnology are more sensitive and hence more effective.
6) Using nanotechnology bullet-proof clothing, sports materials, resistant clothes are made.
7) Using nanotechnology quantum computers, spin FET, spin LED. Flat panel telvisions are made.

2. What is Buck fullerence ? Write a brief note on it.

A. The small carbon clusters form linear (or) closed non-planer nonocyclic structures. The linear clusters have *sp* hybridization with carbon bond angle 180°. These elesters contains an odd number of atoms with 3, 11, 15, 19 and 23 are more stable. The two other forms of hybridization in carbon molecule are sp^2 and sp^3 with carbon bond angles measuring 120° and 109°23^1. Clusters with closed structure contain on even number of atoms and the carbon bond angles are different from the three standard hybridization values. One such cluster highest stability in mass spectrum is composed of 60 atoms. The C_{60} molecule has three fold coordinates carbon atoms bounded to form a symmetrical array form a molecular ball composed of 12 pentagons and 20 hexagons. C_{60} molecule was named "Buck fullerence". The fullerence is a molecule composed entirely of carbon, in the form of a hollow sphere, ellipse (or) tube.

3. Write the distinct properties of nano materials.

A. 1. Opaque substances, like copper, become transperent in nano scale.

2. Inert materials, like platinum, become catalyst.

3. Stable materials can turn combustible. E.g. aluminium.

4. Solid can turn into liquids at room temperature. E.g. gold.

5. Insulatory can become conductors. E.g. silicon.

The most striking property of nano particles made of semi conductor elements is change in their optical properties compared to those of bulk materials.

6 SUPERCONDUCTIVITY

LONG ANSWER TYPE QUESTIONS

1. What is super conductivity? Explain Type - I and Type - II super conductors ?
[ANU O18; AdNU O18, N17; AU 18, 17; BRAU O17; KU O18, O17; RU O18, O17; SKU O18, O17; SVU O18; VSU O18, S17; YVU O18]

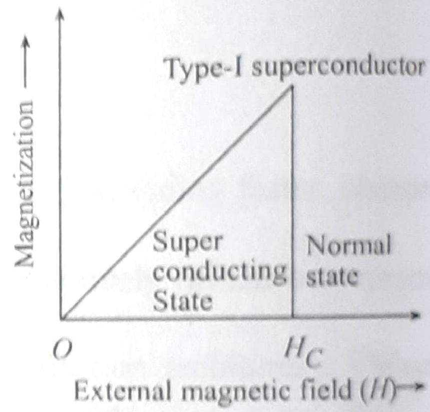
A. **Super Conductivity :-** When the substance is cooled below a certian temperature, its resistivity suddenly drops to zero and behaves like super conductor. This phenomena is called super conductivity.

The temperature at which the resistance disappears is called the transition temperature (or) critical temperature.

Based on magnetic behaviour, the superconductors are classified into two categories. They are

1) Type - I super conductors or soft superconductors and
2) Type - II or hard superconductors.

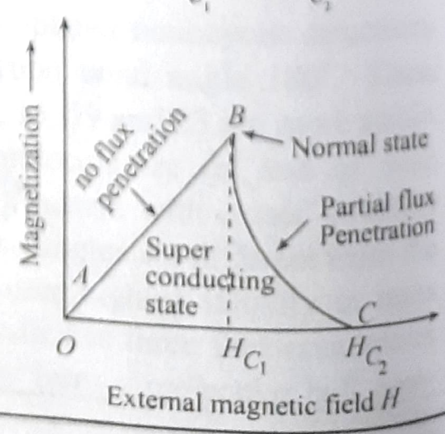
Type - I super conductors :



1) Type - I supertconductor is one in which the transition from superconducting state to normal state in presence of magnetic field occurs sharply at the critical value H_C.
2) In presence of an external magnetic field $H < H_C$, type - I superconductor in super conducting state is a perfect diamagnet.
3) When H exceeds H_C, the super conductor enters the normal state, i.e., it loses its diamagnetic property completely.
4) In this state, the magnetic flux penetrates through out the superconductor.
5) The critical field value H_C for type - I super conductor is found to be very low.
6) E.g. Aluminimum, lead and indium

Type - II super conductors :

1) The magnetization curve of type - II superconductor is shown in fig.
2) Type - II superconductors has two critical magnetic fields H_{C_1} and H_{C_2}.
3) For the field strength below H_{C_1}, the superconductor expells the magnetic field from its body completely and behaves as a perfect diamagnet. H_{C_1} is called the lower critical field. The curve is represented by AB.
4) As the H increases from H_{C_1}, the magnetic field lines begin to penetrate the material. The penetration increases until H_{C_2} is called the upper critical field.



5) At H_{C_2}, the magnetization vanishes completely, i.e., the external field has completely penetrated into super conductor and destroyed the superconductivity.
6) In region from H_{C_1} to H_{C_2}, the specimen assumes a complicated mixed structure of normal and superconducting states. The superconductor is said to be in a mixed state which is commonly known as vortex state. In the region H_{C_1} and H_{C_2}, the material is in a magnetically mixed state but electrically it is a superconductor.
7) After H_{C_2}, the material turns to normal state. So, type - II superconductor is one which is characterised by two critical fields H_{C_1} and H_{C_2} and transition to normal state takes place gradually as magnetic field is increased from H_{C_1} and H_{C_2}.
8) Between H_{C_1} and H_{C_2}, the state is called as mixed state or vortex state. In this state, though there is a flux penetration, yet the material retains its zero resistance property and it is still a super conductor.

2. Explain the terms critical temperature, critical field and Meissner effect on superconductors.

(Or)

What is super conductivity ? Discuss the thermodynamic properties of super conductors.

[ANU 18; VSU O/N18]

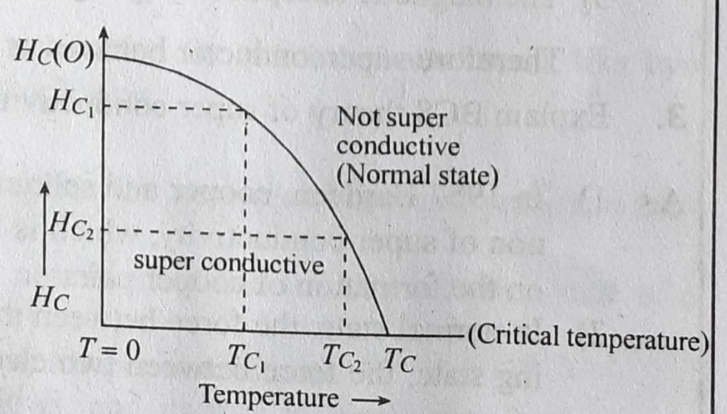
A. **Super Conductivity :** When the substance is cooled below a certian temperature, its resistivity suddenly drops to zero and behaves like super conductor. This phenomena is called super conductivity.

Critical temperature (T) :

1) The temperature at which the conductor resistance disappers is called the transition temperature or critical temperature.
2) The transition temperature T_C is different for different isotopes of an element. It decreases with increasing atomic weight of the isotopes. The thermal properties [Entropy, heat capacity, thermal conductivity etc.] of a metal changes sharply at transition temperature.

Critical magnetic field $H_C(O)$:

1) Superconductivity is destroyed if a sufficient strong magnetic field is applied. The superconducting material restores its normal resistance when a strong magnetic field is applied. The minimum magnetic field which is necessary to regain the normal resistivity is called critical magnetic field $H_C(o)$.
2) The minimum value of applied magnetic field when the superconductor loses its superconductivity is called the critical magnetic field.



3) If the applied magnetic field exceeds the critical value $H_C(O)$, the superconducting state is destroyed. The variation of critical magnetic field with temperature is shown in fig.

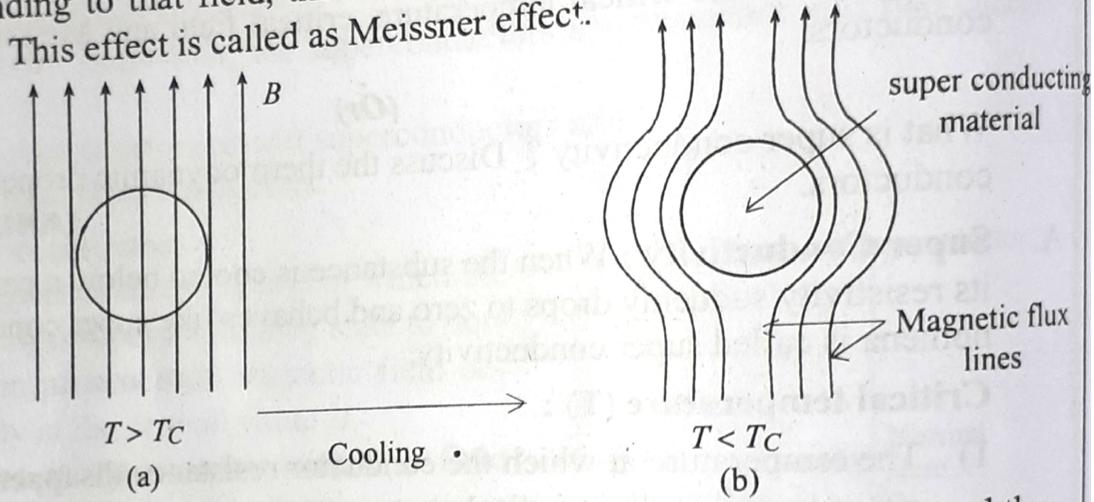
4) It is observed that the normal conducting state of the material is restored if the magnetic field is greater than the critical value or the temperature of the specimen is raised above critical temperature T_C.

We have $H_C(T) = H_C(O)\left[1 - \frac{T^2}{T_C^2}\right]$.

Where $H_C(T)$ is the maximum critical field strength at temperature T, $H_C(O)$ is the maximum Critical field strength occuring at absolute zero and T_C is the critical temperature.

Meissner effect :

1) If a superconductor is cooled in a magneticfield, below critical temperature corresponding to that field, then the lines of induction are expelled from the material. This effect is called as Meissner effect.



2) If $T > T_C$ as shown in figure (a) the superconductor is in normal state, and the magnetic lines of force pass through it.

3) If $T < T_C$ as shown in figure (b) the superconductor is diamagnetic and the magnetic lines of force are expelled out of it.

4) In case of superconductors.

$B = \mu_o(H + M) = 0 \Rightarrow M = -H$ where H = magnetising field intensity and M = Intensity of magnetization.

5) The magnetic suceptibility χ is given by $\chi = \frac{M}{H} = \frac{-H}{H} = -1$

Therefore superconductor behaves as a perfect diamagnet.

3. Explain BCS theory of super conductivity and write its applications.

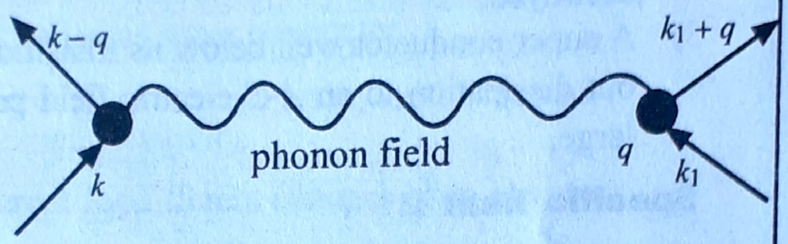
[ANU O17; BRAU O18, O17; M16]

A. 1) In 1957 Bardeen, cooper and schrieffer gave a theory to explain the phenomenon of super conductivity, which is known as BCS theory. The theory is bases on the formation of cooper pairs.

2) In normal state, the force between the electrons is repulsive. In super conducting state, the force between two electrons become attractive due to formation of cooper pair.

3) When a current flows through a super conductor and an electron comes near the positive ion core of the lattice, then the electron experiences an attractive force. Due to interaction between electron and ion core, the ion core is slightly displaced. This is known as Lattice distortion. The distortion in the lattice then travels away as a mechanical wave (phonons). Now another electron comes near the distorted lattice. The phonon interacts with the second electron and hence there is a force of attraction between second electron and phonon. In this way, two electrons interact with each other through the lattice vibration. This process is called as electron-lattice-electron interaction via phonon field (Mechanical wave)

4) When an electron with vector K distors the lattice, the lattice gains momentum. As a result, the momentum of electron decreases. So, a phonon of wave vector q is emitted. When another electron with wave vector K_1 absorbs the energy from phonon it gains momentum. There fore, due to interaction, we have two electrons with wave vector $K-q$ and K_1+q. The pair of electrons is called a cooper pairs.



5) "Cooper pair is a bound pair of electrons formed by the interaction between the electrons in a phonon field". The two electrons which pair up have opposite momenta and spin.

6) Cooper pair of electrons moves on without suffering any deviation either by impurities (or) thermal vibrations. Hence there is no exchange of energy between pair of electrons and lattice ions. If an electric field is established inside the substance, the electrons gain additional $K{\cdot}E$ and give rise to a current. The main important point is that the pair of electrons does not transfer any energy to the lattice. There, they do not get slowed down. As a consequence of this, the substance does not possess any electrical resistivity and the conduction is large.

7) When a pair of electrons flow in the form of cooper pair, the resistance factor vanishes, i.e., conductivity becomes infinity which is named as super conductivity.

When current flows in a super conductor, the pair of electrons are like two dances who move in step without collision.

Applications of BCS theory :

1) The theory could successfully explain the zero resistance and persistent currents in a super conductor.
2) The theory successfully explain normal state and super conducting state of a super conductor.
3) It could explain the specific heat of a super conductor.
4) It could explain the critical field B_c and its role in the behavior of a super conductor.

5) It confirms the existence of an energy gap in the excitation spectrum of a super conductor.

4. Explain the persistent currents, specific heat, entropy and transition temperatures in connection with super conductors.

A. **Persistent currents :** When a super conductor is at a temperature below the critical temperature T_c $(T < T_c)$, in a super conductor currents can flow with no appreciable loss of energy. This super current persists for ever with out any need for an external deriving electromotive force.

Limitations for these persistent currents are

1) Super conductivity is destroyed by application of a sufficiently large magnetic field.
2) If the current exceeds a critical current, the super conducting state will be destroyed.
3) A super conductor well below its transition temperature will also respond with out dissipation to an $A \cdot C$ electric field provided that the frequency is not too large.

Specific heat :

1) At low temperatures the specific heat C_n of a normal metal (Below T_c) is given by $C_n = AT + BT^3 \rightarrow (1)$ (Debye's theory).

   The linear term AT is due to the electronic excitations and BT^3 is due to the lattice vibrations.

2) When the metal is in the super conducting state (Below T_c) this behavior changes drastically. As the temperature drops below T_c, the specific heat C_s will jump to a higher value initially at T_c and then will slowly decrease and eventually fall of the well below the value for normal state.

Entropy of super conductors :

1) The Gibbs free energy for the normal and super conducting states of a metal are related by $G_n - G_s = \dfrac{B_c^2}{2\mu_0} \rightarrow (1)$

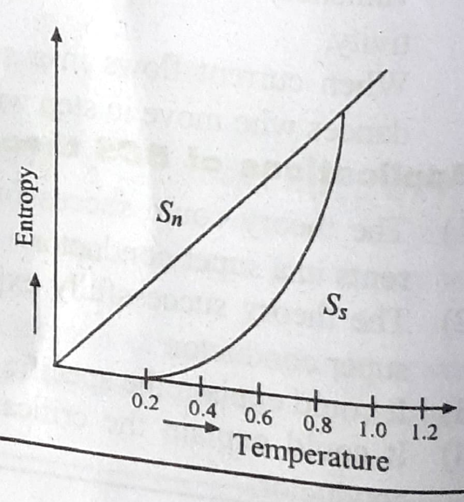
2) Entropy S and G are related by $S = -\left(\dfrac{\partial G}{\partial T}\right) \rightarrow (2)$ and hence

   $$S_n - S_G = -\frac{B_c}{\mu_0}\left(\frac{dB_c}{dT}\right) \rightarrow (3)$$

3) The slope of the critical field curve, $\dfrac{dB_c}{dT}$ is negative as can be seen in figure.

   The normal state entropy S_n will be greater than or equal to the super conducting state entropy S_s.

   $S_n \geq S_s \rightarrow (4)$



4) From figure, the slope $\frac{dB_c}{dT}$ tends to zero at absolute zero.

At $A \to 0$ we have $S_n \to S_s$.

Transition temperature : The temperature at which the transition takes place from the state of normal conductivity to that of super conductivity is called transition temperature.

SHORT ANSWERS TYPE QUESTIONS

1. What is super conductivity ? Mention the properties of super conductivity.

[ANU O18, J18, M16; AdNU N17; AU 18; RU O18; YVU O18]

A. The resistivity of a substance drop suddenly to zero, when its specimen is cooled below a certain temperature the phenomenon is known as superconductivity. The substance showing this property is called as super conductor.

Properties :

1) At room temperature, they have high resistivity.

2) Critical temperature (T_c) is different for different isotopes of an element.

3) Due to transition, the thermal expansion and elastic properties do not change.

4) All thermoelectric effects disappear in superconductors.

5) Superconductors are perfectly diamagnetic.

6) T_c decreases with increasing atomic weight of isotopes.

7) When a strong magnetic field is applied to a superconductor below its transition temperature, the super conducting property is destroyed.

2. Mention the applications of superconductors.

[ANU O18, J18; AdNU N17; BRAU O17; RU O18; SVU O17; YVU N17]

A. 1) **Power cables :** Superconducting materials if used for power cables will enable transmission of power over very long distances without much power loss.

2) **Electromagnets :** Superconducting solenoids which do not produce any heat during operation have been produced.

3) In the construction of very sensitive electrical measuring instruments like galvanometers.

4) They are employed in switching devices.

5) Superconductors are used for amplifying very small direct current and voltages.

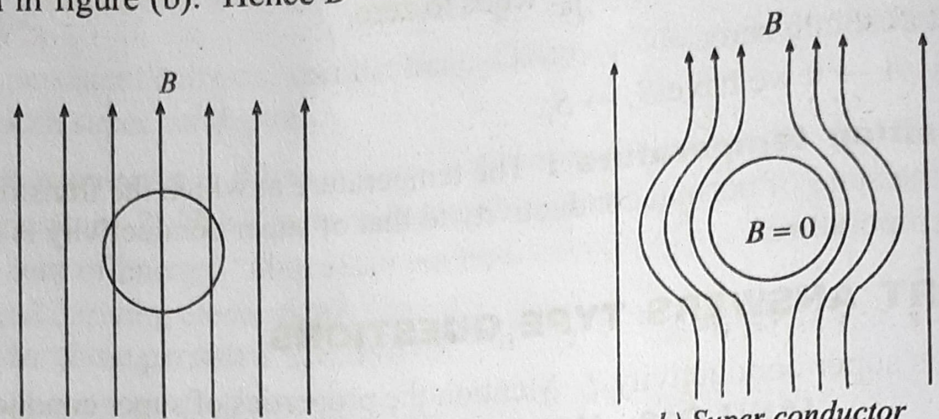
3. Briefly explain Meissner effect.

[ANU O17, J16, M15; AdNU O/N 17; AU 18; BRAU O18; KU 17; RU O18; SKU O18; SVU O17; VSU S17; YVU O18]

A. 1) When a magnetic field is applied to a perfect conductor and it is cooled to low temperature such that it is below its transition temperature and becomes a perfect conductor, the resistivity decreases to zero but the flux distribution does not change as shown in figure (a).

2) In case of super conductor the lines of magnetic induction are pushed out as

shown in figure (b). Hence $B = 0$ inside it. This effect is called the Meissner effect.



a) Perfect conductor

b) Super conductor

4. Explain zero resistance with respect to super conductors?

A. 1) In super conducting state, the electrons scattered in pairs rather than individual. This gives rise to an exchange force between electrons. This force is similar to forces between nucleons in a nucleus.

2) When the electrons have opposite spins and momenta, there is very strong attractive force between them.

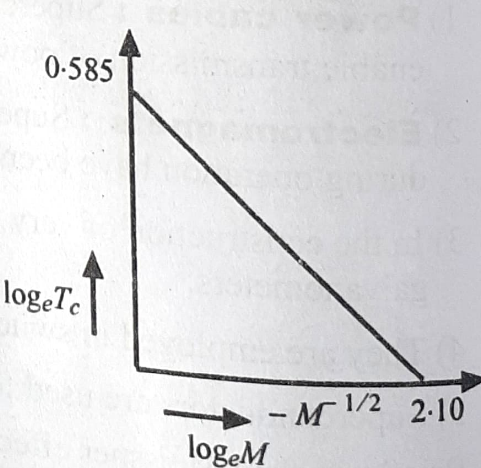
3) In super conducting state, the force of attraction is greater the electrostatic forces of repulsion. Thus all conduction electrons become a bound system. Now no transfer of energy takes place from this system to lattice ions.

4) When an electric field is applied to a substance in super conducting state, the pairs of electrons gains additional kinetic energy. Thus they give rise to a current.

5) As these electrons do not transfer any energy to the lattice hence they do not get slowed down and as a result of this the substance does not possess any electrical resistivity. It means that the resistance of super conductor is zero.

5. Explain isotope effect with respect to super conductors.

A. 1) It was discovered that the critical temperature T_c of the isotopes of a super conducting element decrease with increasing atomic mass of the isotope.



2) The T_c for the isotope ^{199}Hg is 4·161 K and for the isotope ^{204}Hg is 4·126 K.

3) The isotopic effect in its generalized form can be stated as $T_c M^{\frac{1}{2}} =$ constant.
Where $M =$ Isotopic mass.

4) This isotopic effect suggests that the current carrying electrons in a superconductor do not move independently of ion lattice, instead of some low interacting with the lattice.

PROBLEMS

1. A super conducting material has a critical temperature of 3·7 K in zero magnetic field of 0·306 tesla at OK. Find the critical field at $2K$.

Sol : $B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$; $T_c = 3{\cdot}7$ K, $B_c(0) = 0{\cdot}306T$, $T = 2$ K

$$B_c(T) = 0{\cdot}0306\left[1-\left(\frac{2}{3{\cdot}7}\right)^2\right] = 0{\cdot}0306\,[1-0{\cdot}29002] = 0{\cdot}0217 \text{ Tesla.}$$

2. At what temperature is $H_c(T) = 0{\cdot}1\, H_c(0)$ for lead (pb) having $T_c = 7{\cdot}2$ K?

Sol : $H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$; $0{\cdot}1\, H_c(0) = H_c(0)\left[1-\frac{T^2}{(7{\cdot}2)^2}\right]$

$$0{\cdot}1 = 1-\frac{T^2}{(7{\cdot}2)^2}\ ;\ T^2 = (7{\cdot}2)^2\,[1-0{\cdot}1]\ ;\ T^2 = (7{\cdot}2)^2 \times 0{\cdot}9\ ;\ T = 6{\cdot}83 \text{ K.}$$

3. The critical field for niobium is 1×10^4 A/m at 8K and 2×10^5 A/m at OK. Calculate the transition temperature of the element

Sol : $T_c = \frac{T}{\left[1-\frac{B_c(T)}{B_c(0)}\right]^{\frac{1}{2}}} = \frac{8}{\left[1-\frac{1\times10^4}{2\times10^5}\right]^{\frac{1}{2}}} = \frac{8}{\left[1-\frac{1}{20}\right]^{\frac{1}{2}}} = \frac{8}{\left(\frac{19}{20}\right)^{\frac{1}{2}}} = 7{\cdot}08$ K.

4. The transition temperature for lead is 7·26 K. The maximum critical field for the material is 8×10^5 A/m. Lead has to used as a super conductor subjected to a magnetic field of 4×10^4 A/m. What precaution will have to be taken?

Sol : $T = T_c\left[1-\frac{B_c(T)}{B_c(0)}\right]^{\frac{1}{2}} = 7{\cdot}26\left[1-\frac{4\times10^4}{8\times10^5}\right]^{\frac{1}{2}} = 7{\cdot}08$ K.

5. The transition temperature for pb is 7·2 K. However it loses the super conducting property if subjected to a magnetic field of $3{\cdot}3\times10^4$ A/m. Find the value of $B_c(0)$ which will allow the metal to retain its superconductivity at OK.

Sol : $B_{(T)} = B_c(0)\left[1-\frac{T^2}{T_c^2}\right]$; $B_c(0) = \frac{B_c(T)}{\left[1-\frac{T^2}{T_c^2}\right]}$

$$B_c(0) = \frac{3{\cdot}3\times10^4}{\left[1-\frac{(5)^2}{(7{\cdot}2)^2}\right]} = \frac{3{\cdot}3\times10^4}{\left[1-\frac{25}{51{\cdot}28}\right]}\ ;\ B_c(0) = 6{\cdot}37\times10^4 \text{ A/m.}$$

6. For a specimen of super conductor, the critical fields are $1{\cdot}4\times10^5$ and $4{\cdot}2\times10^5$ A/m respectively for temperature 14 K and 13 K respectively. Calculate the transition temperature and critical fields at OK and 4·2 K.

Sol : $(B_c)_1 = B_0\left[1-\left(\frac{14}{T_c}\right)^2\right] = 1{\cdot}40\times10^5 \rightarrow (1)$

$$(B_c)_2 = B_0\left[1-\left(\frac{13}{T_c}\right)^2\right] = 4\cdot2\times10^5 \rightarrow (2)$$

Dividing (2) by eq (1) , we get $\dfrac{(B_c)_2}{(B_c)_1} = \dfrac{T_c^2-(13)^2}{T_c^2-(14)^2} = \dfrac{4\cdot2}{1\cdot4} \rightarrow (3)$

Solving eq (3) for T_c, we get $T_c = 14{\cdot}5$ K.

Substituting T_c value in eq (1) $B_0\left[1-\left(\frac{14}{14\cdot5}\right)^2\right] = 1{\cdot}40\times10^5$

$$B_0 \frac{1\cdot4\times10^5}{1-\left(\frac{14}{14\cdot5}\right)^2} = 20\cdot67\times10^5 \text{ A/m}$$

Now $(B_c)_{4\cdot2} = B_0\left[1-\left(\frac{4\cdot2}{14\cdot5}\right)^2\right] = 20{\cdot}67\times10^5\times0{\cdot}916 = 18{\cdot}9\times10^5$ A/m.

7. Calculate the critical current which can flow through a long thin super conducting wire of diameter 10^{-3} m. Given $H_c = 7{\cdot}9\times10^3$ A/m. **[J15, M14]**

Sol : According to silsbee's rule, $I_c = 2\pi rH_c = 2\times3{\cdot}14\times\left(\frac{10^{-3}}{2}\right)\times7{\cdot}9\times10^3$

$\therefore\ I_c = 24{\cdot}81A.$

8. The critical temperature T_C for mercury is $4{\cdot}185\ K$ and isotopic mass is 199·5. If the isotopic mass changes to 205·4. Calculate its critical temperature. **[ANU 18]**

Sol : $T_C M^{\frac{1}{2}}$ = constant $\Rightarrow T_1M_1^{\frac{1}{2}} = T_2M_2^{\frac{1}{2}}$ Here $T_1 = 4{\cdot}185\ K$; $M_1 = 199{\cdot}5$ amu

$M_2 = 205{\cdot}4$ amu ; $T_2 = ?$

$$4\cdot185\times(199\cdot5)^{\frac{1}{2}} = T_2\times(205\cdot4)^{\frac{1}{2}}$$

$$\therefore\ T_2 = 4\cdot185\times\left(\frac{199\cdot5}{205\cdot4}\right)^{\frac{1}{2}} = 4\cdot185\times0\cdot9865 = 4\cdot1285\ K.$$

9. A super conductor ten has a critical temperature of $4{\cdot}7\ K$ at zero magnetic field and a critical field of 0·0106 Tesla at OK. Find the critical field at 3 K.

Sol : $B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$

$T_c = 4{\cdot}7\ K$; $B_c(0) = 0{\cdot}0106$; $T = 3\ K$

$$B_c(T) = 0\cdot0106\left[1-\left(\frac{3}{4\cdot7}\right)^2\right] = 0\cdot0106\,[1-0\cdot4074] = 0\cdot9957 \text{ Tesla.}$$