Dr. C. S .RAO PG CENTRE

SRI Y.N .COLLEGE (AUTONOMOUS), NARSAPUR, W.G.Dt.,A.P.

NAAC Accredited 'A' Grade College Affiliated to AdikaviNannaya University

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M101:ALGEBRA-I

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss Normal Subgroups and Normal series like Isomorphism theorem and Automorphism.
2. To explain Structure theorem of Groups like Finitely generated Abelian group and Sylow's theorem
3. To discribe Ideal's and Homomorphism, Unique Factorization domain and Euclidean domains.

UNIT-I: Normal subgroups: Normal subgroups and quotient groups-Isomorphism theorem-Automorphisms - Conjugacy and G-sets- Normal series solvable groups- Nilpotentgroups.
(Section 1,2,3 and 4 of Chapter 5, Sections 1,2,3 of Chapter 6)

UNIT-II: Structure theorems of groups: Direct product- Finitely generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p2 ,pq . (Sections 1 to 5 of Chapter 8)

UNIT-III: Ideals and homomorphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma.
(Sections 1 to 6 of Chapter 10)

UNIT-IV: Unique factorization domains - Principal ideal domains- Euclidean domains-

Polynomial rings over UFD.

(Sections 1 to 4 of Chapter 11)

Additional Inputs: Normal subgroups and quotient groups, Isomorphism theorem.

TEXT BOOK: Basic Abstract Algebra, Second Edition by P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M101:ALGEBRA-I

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (a) If N be a normal subgroup of the group G, then prove that the mapping $\emptyset: G \to G/N$ defined by $x \mapsto xN$ is an epimorphism with $Ker\emptyset = N$.
(b) State and prove Burnside theorem.

(OR)

2. (a) Let G be a group. Then the following are true.

i) The set of conjugate classes of G is a partition of G.

ii) $|C(a)| = [G: N(a)]$.
iii) If G is finite, $|G| = \sum[G: N(a)]$.
(b) Define an alternating group A_n . Show that the alternating group A_n
is generated by the set of all 3 – cycles inS_n.

3. (a) State and prove fundamental theorem of finitely generated abliean group.
(b)State and prove Cauchy's theorem.

(OR)

4. (a) A group G is nilpotent if and only if G a normal series $|e| = G_0 \subset G_1 \subset \cdots \subset G_m = G$

Such that G_i / $G_j \subset Z\left({}^{G}/_{G_{i-1}}\right)$ for all i=1,…….m.

(b) State and prove 2^{nd} and 3^{rd}sylow theorems.

5. (a) If a ring R has unity, then every ideal I in the matrix ring R_n is of the form A_n,where A is some ideal in R.

(b)If K is an ideal in a ring R, then show that each ideal in ${}^{R}/_{K}$ is of the form ${}^{A}/_{K}$where A is an ideal in R containing K.

(OR)

6. (a) f be a homomorphism of a ring R into a ring S with kernel N,then prove that ${}^{R}/_{N} \simeq Imf$.

(b) Let $f: R \to S$ be a homomorphism of Ring R into a ring S,then prove $kerf = \{0\}$ if and only if is 1-1.

7. (a) *R* is a non – zero ring with unity and *I* is an ideal in *R* such that *I XR* , then prove that there exists a maximal ideal *M* of *R* such that *I* ≅*M*.

(b) Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime. **(OR)**

8. (a) show that every Euclidean ring is a principal ideal domain
   (b) State and prove Gauss lemma

SECTION – B

9. Answer any THREE questions of the following $\mathbf{3 \times 5 = 15}$

(a) Define the following and give one example for each

(i)Aut(*G*)

(ii)Eisenstein Criteria of irreducibility.

(b) (i) Define invariants of a group.

(ii) Define *p* – group and give an example.

(c) (i) define an ideal and give two examples.

(ii) Define a principal ideal and given an example of a principal ideal ring.

(d) (i) define nilpotent ideal and give an example.

(ii) Show that every nilpotent ideal is nil. What about the converse? Justify?

(e) (i) write Zorn's lemma and give an application.

(ii) Define a Euclidean domain and give an example.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M102:REAL ANALYSIS-I

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To describe the Basic Topology and Numerical Sequences & Series.
2. To explain about Continuity like Continuity and Connectedness & Continuity and Compactness.
3. To Understand some special theorems of Differentiation like Mean value theorems and Taylor's theorem and further more learn about Differentiation of vector valued functions.

UNIT-I: Basic Topology: Finite-Countable- and Uncountable Sets- Metric spaces- Compact sets- Perfect Sets-Connected sets, The Real and Complex Number systems –Ordered sets , Fields.

(Chapter 2 of the text book)

UNIT-II: Numerical Sequences and Series: Convergent Sequences- Subsequences -Cauchy Sequences- Upper and Lower limits- Some Special Sequences- Series- Series of Non-negative Terms- The number e -The Root and Ratio tests- Power series -Summation by parts - Absolute Convergence-Addition and Multiplication of series-Rearrangements.

(Chapter 3 of the text book)

UNIT-III: Continuity: Limits of Functions- Continuous Functions- Continuity and Compactness- Continuity and Connectedness- Discontinuities- Monotonic Functions- Infinite Limits and Limits at Infinity.

(Chapter 4 of the text book)

UNIT-IV: Differentiation: The Derivative of a Real Function -Mean Value Theorems - The Continuity of Derivatives- L' Hospital's Rule- Derivatives of Higher order- Taylor's theorem- Differentiation of Vector- valued Functions.

(Chapter 5 of the text book)

Additional Inputs: The Real and Complex Number systems –Ordered sets , Fields.

TEXT BOOK: Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3rd Edition, 1985.

REFERENCE: Mathematical Analysis by Tom M. Apostal, Narosa Publishing House, 2nd Edition,1985.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M102: REAL ANALYSIS-I

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION - A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (i) Prove that Every infinite subset of a countable set A is countable
   (ii) Define Metric Space. And Prove that "Every neighborhood is an open set".

(OR)

2. Prove that "Every K– cell is compact".

3. Prove that, (i) If $|a_n| \leq c_n \; for \; n \geq n_0 \; where \; N_0$ is some fixed integer , if $\sum c_n$ converges then $\sum a_n$ converges.(ii) If $a_n \geq d_n \; for \; n \geq N_0$ and if $\sum d_n$ diverges then $\sum a_n$ diverges.

(OR)

4. Prove that , $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$.

5. Prove that "a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

(OR)

6. Prove that " if f is a continuous mapping of a compact metric space X into a metricspace Y, then f is uniformly continuous on X".

7. State and Prove, Chain Rule.

(OR)

8. State and Prove, Mean Value Theorem.

SECTION – B

9. Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

(i) Define Cantor Set.

(ii) State and Prove LEIBNITZ Theorem.

(iii) If $\{p_n\}, \{q_n\}$ are Cauchy sequences in a metric space, then show that $(d(p_n, q_n))$ converges

(iv) Prove that every differentiable function is continuous.

(v) Let f be defined on $[a, b]$ if f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists then $f'(x) = 0$.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M103:DIFFERENTIAL EQUATIONS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss the second order linear differential equations and analyze homogeneous equation with constant coefficients, undetermined coefficients and method of variation of parameters.
2. To explain Oscillation theory and Boundary value problems like Eigen values and Eigen functions.
3. To describe Power series solutions and Systems of first order equations.

UNIT-I: Second order linear differential equations: Introduction-general solution of the homogeneous equation - Use of a known solution to find another - Homogeneous equation with constant coefficients - method of undetermined coefficients - method of variation of parameters.

Chapter 3 (Sec 14-19)

UNIT-II: Oscillation theory and boundary value problems: Qualitative properties of solutions – The Sturm comparison theorem - Eigen values, Eigen functions and the vibrating string.
Chapter 4 (Sec 22-24, Appendix A)

UNIT-III: Power series solutions: A review of power series-series solutions of first order equations-second order linear equations - ordinary points-regular singular points-Gauss’s hypergeometric equation.

Chapter 5 (Sec 25-30)

UNIT-IV: Systems of first order equations: Linear systems - Homogeneous linear systems with constant coefficients - Existence and Uniqueness of solutions - successive approximations - Picard’s theorem - Some examples.

Chapter 7 (Sec 36-38) and Chapter 11(Sec 55-56)

Additional Inputs: Gauss’s hypergeometric equation.

TEXT BOOK: George F. Simmons, Differential Equations, Tata McGraw-Hill Publishing Company Limited, New Delhi.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M103:DIFFERENTIAL EQUATIONS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. If $y_1(x)$ *and* $y_2(x)$ are the linearly independent solutions of the homogeneous equation $y'' + p(x)y' + Q(x)y = 0$ on the interval [a,b], then prove that $c_1 y_1(x) + c_2 y(x)$ is a solution of the differential equation. Also prove that Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on [a,b]. **(OR)**

2. (a) Find the general solution of

$y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x$ by the method of

undetermined coefficients.

(b) Find particular solution of $y'' + y = cosec\ x$ by the method of variation of parameters.

3. State and prove Sturm comparison Theorem. **(OR)**

4. Find the Frobenius series solution and the general solution for the differential equation

$4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0.$

5. (a) Find the differential equation satisfied by the function $y(x) = (1+x)^p$ where p is any arbitrary constant, and then solve this equation by power series.

(b) Solve the differential equation $y'' + y = 0$ by using the method of power series.

(OR)

6. Solve the Legendre differential equation.

7. (a) Solve the system

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y.$$

(b) Solve the system

$\frac{dx}{dt} = 3x - 4y$ **(OR)**

8. State and prove Picard's theorem.

<u>SECTION – B</u>

9.Answer any three questions of the following. Each question carries 5 marks.$\mathbf{3 \times 5 = 15}$

(i) Explain the method of undetermined coefficients $y'' + p(x)y' + q(x)y = \sin\ bx$ and

Solve $y'' + 10y' + 25y = 14e^{-5x}$.

(ii) prove that ‘if $q(x) < 0$ and $u(x)$ is a nontrival solution of $u'' + q(x)u = 0$, then $u(x)$

Has at most one zero'?

(iii) Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.

(iv) Write two independent Frobenius series solution for $xy'' + 2y' + xy = 0$.

(v) Solve the linear system

$$\frac{dx}{dt} = x + y - 5t + 2$$

$$\frac{dy}{dt} = 4x - 2y - 8t - 8.$$

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M104:TOPOLOGY

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss the basic concepts of Sets and Functions and also know about Countable Sets & Uncountable Sets, further more about Partitions and equivalence relations.
2. To learn about Metric spaces like open sets, closed sets and continuous mappings and also learn about Euclidean and unitary spaces.
3. To describe Compactness like compact spaces, Product of spaces and discuss about some important theorems like Tychnoff's Theorem and Ascoli's theorem.

UNIT-I: Sets and Functions: Sets and Set inclusion – The algebra of sets – Functions – Products of sets– Partitions and equivalence relations – Countable sets – Uncountable sets – Partially ordered sets and lattices.(Chapter I: Sections 1 to 8.)

UNIT-II: Metric spaces: The definition and some examples – Open sets – Closed sets – Convergence, Completeness and Baire's theorem – Continuous mappings. (Chapter 2: Sections 9 to 13.)

UNIT-III: Metric spaces (Continued): Spaces of continuous functions – Euclidean and unitary spaces.

Topological spaces: The definition and some examples – Elementary concepts – Open bases and open sub bases – Weak topologies – The function algebras C(X, R) and C(X, C).

(Chapter 2: Sections 14,15 and Chapter 3: 16 to20.)

UNIT-IV: Compactness: Compact spaces – Product of Spaces – Tychonoff's theorem and locally Compact spaces – Compactness for metric spaces – Ascoli theorem – Limit point compactness

(Chapter 4: Sections 21 to 25.)

Additional Inputs: limit point compactnesss.

TEXT BOOK-1: Introduction to Topology by G.F.Simmons, Mc.Graw-Hill book company.

TEXT BOOK – 2: Topology by James R. Munkers,Second edition ,Pearson education Asia – Low price edition.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M104:TOPOLOGY

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (i) Let X be a non-empty set. A relation ~ in X is called circular if $x{\sim}y$ & $y{\sim}z \Rightarrow z{\sim}x$ & triangular if $x{\sim}y$ & $x{\sim}z \Rightarrow y{\sim}z$. Then prove that a relation in X is an equivalence if and only if it is reflexive & circular if and only if it is reflexive & triangular.
(ii) State Schroeder Bernstein Theorem.

(OR)

2. Prove that, (i) Any subset of a Countable Set is Countable.
(ii) A Countable union of countable sets is countable.

3. (i) Define metric space. Prove that any metric space X each open sphere is an openset.

(ii) Let X be metric space and let A be a subset of X. If x is a limit point of A. Show that eachopen sphere centered on x contains an infinite number of distinct points ofA.

(OR)

4. Define open set and closed set. Let (X, d) be a metric space. Let A is subset of X. Then thefollowing hold.

(i) b(A) = $\bar{A} \cap \bar{A}^{1}$

(ii) b(A) is a closedset.

(iii) A is closed $\Leftrightarrow$ b (A) $\subseteq$A.

5. (i) State and prove Minkowski's inequality.

(ii) Define Topological space. Let X be a topological space and A is an arbitrary sub set of X.

Then $\bar{A}$ = {x ∈ X | each neighborhood of x intersects A}

$\bar{A}$ = {x ∈ X/G⋂A≠$, for any neighborhood G of x}.
(OR)

6. (i) State and Prove Lindelof'stheorem.

(ii) Show that every separable metric space is second countable.

7. (i) Prove that any closed subspace of a compact space iscompact.

(ii) Prove that a metric space is sequentially compact if it has the Bolzano Weierstrass property.

(OR)

8. State and prove Ascoli'stheorem.

SECTION – B

9. Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

(i) Prove that the set of all positive rational numbers are countable.
(ii) State Cantor Intersection Theorem.
(iii) Prove that a sub set F of metric space is closed iff its's complement F* isopen.
(iv) Prove that any continuous image of compact space iscompact.
(v) Prove that every compact metric space is separable.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M105:DISCRETE MATHEMATICS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss lattices as partially ordered sets, some properties of lattices, lattices as algebraic systems.

2. To acquire the knowledge from Boolean forms and free Boolean algebras, values of Boolean expressions.

3. To Describe representations and minimizations of Boolean functions and to explain finite state machines, Introductory sequential circuits, equivalence of Finite state Machines.

UNIT-I: Relations and ordering: Relations- properties of binary relations in a set-Relation matrix and the graph of a relation, partition and covering of a set, equivalence relations, compatability relation, composition of binary relations- partially ordereing- Partially ordered sets - representation and associated terminology.

[2-3.1 to 2-3.9 of Chapter 2 of the Text Book]

UNIT-II: Lattices: Lattices as partially ordered sets - some properties of Lattices - Lattices as algebraic systems - sub-Lattices - direct product and homomorphism some special Lattices.

[4-1.1 to 4-1.5 of Chapter 4 of the Text Book]

UNIT-III: Boolean Algebra: Sub algebra - direct product and Homorphism - Boolean forms and free Boolean Algebras - values of Boolean expressions and Boolean function.

[4-2.1,4-2.2,4- 3.1, 4-3.2 of Chapter 4 of the Text Book]

UNIT-IV: Representations and minimization of Boolean Function: Representation of Boolean functions – minimization of Boolean functions- Finite State Machines - Introductory Sequential Circuits - Equivalence of Finite-State Machines, Connectives – Negation , Conjunction , Disjunction , Statement formulas and Truth tables.

[4-4.1,4-4.2,4-6.1, 4-6.2 of Chapter 4 of the Text Book]

Additional Inputs: Connectives – Negation , Conjunction , Disjunction , Statement formulasand Truth tables.

Text Book: Discrete Mathematical structures with applications to Computer Science by J.P.Trembly and R. Manohar, Tata McGraw-HillEdition.

ReferenceBooks:1. Discrete Mathematics for Computer Scientists and Mathematicians by J.L.Mott, A.Kandel and T.P. Baker, Prentice-Hall India.

2. Discrete Mathematical Structures by Kolman & Busby & Sharen Ross

3. Applied Abstract Algebra by Rudolf Lidl & Gunter Pilz ,Published by springer Verlag.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER I

(W.e.f. 2020-2021 Admitted Batch)

M105:DISCRETE MATHEMATICS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $4 \times 15 = 60$

1) Let X = {1, 2, 3} if R = {(x, y)/ x ∈ X ∧ y ∈ X ∧ ((x − y) is an integral non-zero multiple of 2)}, S = {(x, y)/ x ∈ X ∧ y ∈ X ∧ ((x − y) is an integral non-zero multiple of3)}
(a) Find R∪S and R∩S.
(b) IfX={1,2,3,….} ,What is R∩S for R and S as defined in(a)

(OR)

2) Let A be a given finite set and q(A) its power set. Let ⊆ be the inclusion relation on the elements of q(A). Draw Hasse diagrams of (q(A), ⊆) for (a) A= {a}; (b) A={a,b}; (c) A= {a, b, c} (d) A= {a, b, c, d}

3) Let (L, ≤) be a lattice. For any a, b, c ∈ L show that the following holds:

$a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

(OR)

4) Show that in a lattice (L,≤), for any a , b, c∈ L, the distributive inequalities hold:

$$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$$

$$a * (b \oplus c) \geq (a * b) \oplus (a * c)$$

5) Write the following Boolean expression in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3(a) $x_1 * x_2$ (b) $x_1 \oplus x_2$(c) $(x_1 \oplus x_2)' * x_3$

(OR)

6) Obtain the values of the Boolean forms $x_1 * (x_1' \oplus x_2)$, $x_1 * x_2$and $x_1 \oplus (x_1 * x_2)$ over the ordered pairs of the two-element Boolean algebra.

7) Prove that if for some integer k ,$p_{k+1} = p_k$, then $p_k = p$andconversely.

(OR)

8) Find the value of the function $f_{\alpha,\beta} : B^n \to B$ $for\ x_1 = a , x_2 = 1, x_3 = b\ where\ a, b,$ 1 are the elements of the Boolean algebra and $\alpha(x_1, x_2, x_3)$ is the expression $\alpha(x_1, x_2, x_3) = (x_1 \oplus x_2)' * x_3$.

<u>SECTION – B</u>

9. Answer any three questions of the following. Each question carries 5 marks.$\mathbf{3 \times 5 = 15}$

(i) DrawHassediagramofthesetx={2,3,6,12,24,36}andtherelation≤besuchthat $x \leq y$ if x divides y

(ii) Write some properties oflattices

(iii) Define subalgebra, Direct product and Homomorphism.

(iv) Obtain the product of sums canonical forms of the Boolean expression $x_1 * x_2$

(v) Write about finite state machines.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M201:ALGEBRA-II

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1.To discuss about Rings of Fractions and Algebraic Extensions of Fields.

2.To explain Normal and Separable extensions like Splitting fields, Multiple roots and Finite fields.

3.To discuss about Galois theory and it's applications to classical problems like cyclic extensions, Roots of Unity and Cyclotomic polynomials.

UNIT-I: Rings of fraction: Rings of fraction-Rings with Ore condition Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion- Adjunction of roots- Algebraic extensions-Algebraically closed fields.
(Sections 1&2of chapter 12, section 1 to 4 of Chapter 15)

UNIT-II: Normal and separable extensions: Splitting fields- Normal extensions- Multiple roots- Finite fields- Separable extensions.

(Sections 1 to 5 of Chapter 16)

UNIT-III: Galois theory: Automorphism groups and fixed fields- Fundamental theorem of Galois theory-Fundamental theorem of Algebra.

(Sections 1 to 3 of Chapter 17)

UNIT-IV: Applications of Galioes theory to classical problems: Roots of unity and cyclotomic polynomials- Cyclic extensions- Polynomials solvable by radicals - Ruler and Compass constructions.

(Sections 1 to 3 and 5 of Chapter 18)

Additional Inputs: Rings of fraction,Rings with Ore condition

TEXT BOOK: Basic Abstract Algebra , Secound Editionby P.B. Bhattacharya, S.K. Jainand S.R. Nagpaul

REFERENCE: Topics in Algrbra By I. N. Herstein.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M201:ALGEBRA-II

Model Question Paper

Time: 3 hours **Max Marks:** 75

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1. (a) (i) state and prove Gauss lemma.
   (ii) let F is contained in E is contained in K be fields. If $[K:E] < \infty\ and\ [E:F] < \infty$ then
   (i) $[K:F] < \infty$
   (ii) $[K:F] = [K:E][E:F]$

(OR)

   (b) Let F be a field. Then show that there exists an algebrically closed field K containing F as a subfield.
2. (a) Let$E = F(u_1, u_2, ... u_r)$be a finitely generated extension of F such that each $u_i, i = 1,2, ... r$ is algebraic over F then prove that each E is finite over F and hence an Algebraic extension of F.
   (b) Let E be a finite extension of a field. Then the following are equivalent.
   i) $E = F(\propto)$ for some $\propto \in$ E.
   ii) There are only a finite number of intermediate fields between F and E.

3. (a) Let F be an field.Then their exist an algebraically closed field K containing F as a subfield.
   (b)State and prove uniqueness of splitting field.

(OR)

4. (a) Let f(x) be an irreducible polynomial over F.Then f(x) multiple root if and only if $f^1(x) = 0$.

(b) If f(x)∈ $f[x]$ is irreduciable over F, roots of f(x) have the same multiplicity.

5. Let H be a finite subgroup of the group of Automorphism of a field E. Then $[E{:}E_h] = |H|$.

(OR)

6.State and prove fundamental theorem of Galois theory.

7.$\emptyset_n(x) = \pi_\omega(x - \omega), \omega$ primitive nth root in C, is an irreducible polynomial of degree $\emptyset(n) is\ Z[x]$.

(OR)

8.a)Let F be a field and let n be a positive integer. Then there exists a primitive nth root of unity in some extension E of F if and only if either $char\ F = 0\ or\ char\ F \nmid n$.

b) If P is a prime number and if a subgroup G of Sp is a transitive group of permutations containing a transposition (a,b) then $G = S_p$.

SECTION – B

9.Answer any three questions of the following. Each question carries 5 marks.$\mathbf{3 \times 5 = 15}$

a)Let $f(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1} + x^n \in Z[x]$ be a monic polynomial. If f(x) has a root $a \in Q$, then $a \in Z\ and\ a|a_0$.

b) the degree of the extension of the splitting field of $x^3 - 2 \in Q[x] is\ 6$.

c) Show that the multiplicative group of non zero elements of a finite field in cyclic.

d) If E is a finite extension of a field F,then prove that $\left|G\left({}^{E}/_{F}\right)\right| \leq [E{:}F]$.

e) Show that if a>0 is constructiable ,then $\sqrt{a}$ is constructiable.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M202:REAL ANALYSIS-II

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To describe the Riemann stieltjes Integral and Sequences and Series of the functions like Uniform convergence, Uniform convergence and Continuity and Integration further more know about Equicontinuous families of functions,the stone wierstrass theorem.
2. To explain the concept of Power Series and Functions of Several Variables like linear transformations ,the contraction principle and the inverse function theorem.
3. To learn about the the Implicit function theorem,the Rank theorem and further more about Derivatives of higher order and Differentiation of integrals.

UNIT-I: Riemann-Stieltjes Integral: Definition and existence of the Riemann Stieltjes Integral,Properties of the Integral, Integration and Differentiation, the fundamental theorem of calculus–Integral of Vector- valued Functions, Rectifiable curves.

(Chapter 6)

UNIT-II: Sequences and Series of the Functions: Discussion on the Main Problem, Uniform Convergence, Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinuous families of Functions, the Stone- Weierstrass Theorem.

(Chapter 7)

UNIT-III: Power Series: (A section in Chapter 8 of the text book) , the exponential and logarithmic functions.Functions of Several Variables: Linear Transformations, Differentiation, The ContractionPrinciple, The Inverse Function theorem.

(First Four sections of chapter 9 of the text book)

UNIT-IV: Functions of several variables Continued: The Implicit Function theorem, The Rank theorem, Determinates, Derivatives of Higher Order, Differentiation of Integrals.

(5 th to 9 th sections of Chapter 9 of the text book)

Additional Inputs: The exponential and logarithmic functions.

TEXT BOOK: Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3 rd Edition, 1985.

REFERENCE: Mathematical Analysis by Tom M. Apostal, Narosa Publishing House,

2 nd Edition, 1985.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M201:REAL ANALYSIS-II

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (i) Prove that , $f \in R(\alpha)\ on\ [a\ , b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

(ii) Prove that ,If f is monotonic on [a , b] and if α is continuous on [a , b] ,then $f \in R(\alpha)$.

(OR)

2. (i)State and prove integration by parts formula.

(ii) Prove that ,If$\gamma^{\,\prime}$ is continuous on [a , b] , then γ is rectifiable , and $\Lambda(\gamma) = \int_a^b \left| \gamma^{\,\prime}(t) \right| dt$.

3. (i) State and prove Cauchy's criterion for uniform convergence of sequence offunction.

(ii) State and prove Weirstrass M- Test.

(OR)

4. (i) State and prove Stone-WeirstrassTheorem.

(ii) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

5. i) State and prove Abel'sTheorem.
(ii) State and prove Taylor's Theorem

(OR)

6. (i) State and prove contractionprinciple.
(ii) Supposethat $\boldsymbol{f}$ mapsaconvexopensetE⊂R^N,$\boldsymbol{f}$ isdifferentiableinE,and thereisarealnumberMsuchthat‖$\boldsymbol{f}'$(x)‖≤Mforevery$x \in E$.

Then |f(b) − f(a)| ≤ M|b− a| for all $a \in E, b \in E$.

7. Prove that ,(i) If I is the identity operator on R^n ,then

$\det[I] = \det(e_1, \ldots, e_n)$.
(ii) If $[A]_1$ is obtained from $[A]$ by interchanging two columns ,then$det[A]_1 = -det[A]$.

(OR)

8. Suppose f is defined in an open set $E \subset R^2$, suppose that $D_1 f, D_{21} f, and\ D_2 f$ exist at every point of E ,and $D_{21} f$ is continuous at some point (a , b) $\in E$.

SECTION – B

9. Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

(i) Suppose $\{f_n\}$ is a sequence of functions defined on E ,and suppose $|f(x)| \leq M_n, x \in E, n = 1,2,3, \ldots \ldots$ Then prove that $\sum f_n$ coverged uniformly on E if $\sum M_n$ converges.

(ii) If $f \in R(\alpha) \& g \epsilon R(\alpha)\ on\ [a, b]$, then prove that $fg \in R(\alpha)$.

(iii) Let B be the uniform closure of an algebra of bounded functions .Then B is a uniformly closed algebra.

(iv) If [A] and [B] are n by n matrices , then prove that $\det([B][A]) = \det[B]\det[A]$.

(v) State contraction principle and Inverse function theorem.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M203:COMPLEX ANALYSIS-I

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss the elementary properties and examples of Analytic functions like Power series and Mobius transformations.
2. To study complex integration and analyze Riemann-Stiltjes integrals, Cauchy theorem and Integral formula.
3. To describe classification of Singularities, Residues and argument principle.

UNIT-I: Elementary properties and examples of analytic functions: Power series- Analytic functions- Analytic functions as mappings, Mobius transformations.

(1,2,3 of chapter-III)

UNIT-II: Complex Integration: Riemann- Stieltjes integrals- Power series representation ofanalyticfunctions- zeros of an analytic functions- The index of a closed curve.

(1,2,3,4 of chapter-IV)

UNIT-III: Cauchy’s theorem and integral formula- the homotopic version of Cauchy’s theorem andsimple connectivity- Counting zeros; the open mapping theorem.

(5,6,7of chapter-IV)

UNIT-IV: Singularities: Classifications of singularities- Residues- The argument principle,

The Maximum Modulus Theorem:The Maximum Principle-Schwarz’s Lemmma

(1,2,3 of chapter-V ,1,2 of chapter-VI)

Additional Inputs: The Maximum Modulus Theorem:The Maximum Principle-Schwarz's Lemmma

TEXT BOOK: Functions of one complex variables by J.B.Conway : Second edition,
Springer International student Edition, Narosa Publishing House, NewDelhi.

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M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M203: COMPLEX ANALYSIS-I

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (a) Let G be an open connected set in C. If f: G → C is differentiable with f'(z) =0 for all z in G, prove that f is constant.

(b) Let $u\ and\ v$ be two real valued continuous functions defined on a region G. If $u\ and\ v$ have continuous first order partial derivatives, satisfying $u_x = v_y$ and $u_y = v_x$, then prove that the function f:G →C defined by $f(z) = u(z) + iv(z)$ is analytic in G.

(OR)

2. (a) Define cross ratio. If z_2, z_3, z_4 are distinct points and T is any Mobius transformation, Prove that $(Z, Z_2, Z_3, Z_4) = (TZ_1, TZ_2, TZ_3, TZ_4)$ for any point Z_1.

(b) State and prove the symmetry principles.

3. (a) State and prove Leibniz 's theorem.

(b) Let r be a rectifiable curve and suppose that f is a function continuous on {r}, prove

That $\left|\int_r \; f\right| \leq \int |f|\,|dz| \leq V(r)\sup\{\left|\frac{f(z)}{z} \in \{r\}\}\right|$.

(OR)

4. State and prove Fundamental theorem of Algebra.

5. State and prove Cauchy's Integral formula second version.

(OR)

6. (a) State and prove the open mapping theorem.

(b) State and prove Morera's Theorem.

7. (a) State and prove Casorarti-Weirstrass theorem.

(b) Show that $\int_0^{\infty} \frac{logx}{1+x^2} = 0$.

(OR)

8. (a) State and prove Rouche's Theorem.

(b) Obtain the Laurent Series Expansion for the function $f(z) = \frac{1}{(z-1)(z-2)}$

following annuli i) a(0;0,1) ii) ann(0;1,2) iii) ann(0;2,∞)

SECTION – B

9.Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

(i) If $u(x, y) = e^x \cos x (x\,, y) \in C$, prove that u is harmonic in C

And find its harmonic conjugate.

(ii) Prove that any Mobius transformation has at most two fixed points

(iii) Define $\gamma: [0\,, 2\pi] \rightarrow C$ by $\gamma(t) = \exp\,(int)$ where n is some integer. Show

That $\int_{\gamma} \frac{dz}{z} = 2\pi in$.

(iv) Find all entire functions such that $f(x) = e^x$ for all $x \in R$.

(v) Evaluate $\int_0^\infty \frac{sinx}{x} dx = \frac{\pi}{2}$.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M204:LINEAR ALGEBRA

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To Explain elementary canonical forms, annihilating polynomials, invariant subspaces.
2. To Describe direct –sum decompositions, invariant direct-sums.
3. To Acquire the knowledge in the Jordan forms, computation of invariant factors, semi simple operators.

UNIT-I: Elementary Canonical Forms : Introduction – Characteristic Values – Annihilating Polynomials –invariant subspaces – Simultaneous Triangulation – Simultaneous Diagonalization.

(Sections 6.1,6.2,6.3,6.4,6.5 of chapter-6)

UNIT-II: Direct – sum Decompositions – invariant direct sums – the primary decomposition theorem –

cyclic subspaces and Annihilators – cyclic decompositions and the rational form.

(Sections 6.6,6.7,6.8 of chapter-6 and Sections 7.1,7.2 of chapter - 7)

UNIT-III: The Jordan Form – Computation of Invariant Factors – Semi Simple Operators.

(Sections 7.3,7.4,7.5 of chapter - 7)

UNIT-IV: Biliear Forms : Bilinear Forms – Symmetric Bilinear Forms – Skew Symmetric Bilinear Forms– Group Preserving Biliear Forms.

(Sections 10.1,10.2,10.3,10.4 of chapter - 10)

TEXT BOOK: Linear Algebra second edition By Kenneth Hoffman and Ray Kunze, PrenticeHall of india Private Limited, New Delhi.

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M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M204:LINEAR ALGEBRA

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. Let T be a linear operator on the finite dimensional space V. Let $c_1, c_2, \dots., c_k$ be the distinct characteristic values of T and let w_i be the If $w = w_1 + w_2 + \cdots + w_k$ then dimension of $w = dim w_1 + dim w_2 + \cdots + dim w_k$infact if β_i is an ordered basic for w_i then $\beta = \beta_{1+}\beta_2 \dots + \beta_k$is an ordered basic for w.

(OR)

2. Let T be a linear operator on an n-dimensional V.S V(F). The characteristic equation and minimal polynomials for T have same roots except for multiplicities.
3. State and prove primary decomposition theorem.

(OR)

4. Let T be a linear operator on the finite dimensional vector space V and $W_1 \dots W_K$ are subspaces of V and $E_{1,\dots}E_k$ are projections corresponding to the linear operator T. Then a necessary and sufficient condition that each subspace W_I be invariant under T is that T is commutes with each of the projections E_I .

5. let Mbe an $m \times m$ matrix with entries in the polynomial algebra $F[x]$. Then M is equivalent to a matrix N which is in normal form.

(OR)

6. Find all rational canonical for with minimal polynomial $m(t) = (t-1)^3$ and characteristic polynomial $\Delta(t) = (t-1)^7$.

7. Let V be a finite dimensional vector space over the field F. Let F be a symmetric bilinear form on V which has a rank r then there is an ordered basis $\mathfrak{B} = \{\beta_1, \beta_2 \dots \beta_n\}$ for V such that

   i) The matrix of F in the ordered basis $\mathfrak{B}$ is diagonal.

   ii) $f(\beta_j, \beta_j) = \begin{cases} 1 \; if \; j = 1,2, \dots r \\ 0 \; if \; j > r \end{cases}$

(OR)

8. Let f be a non-degenerate bilinear form on a finite dimensions vector space V. The set of all linear operators on V which preserves f is a group under the operation of composition.

SECTION – B

9. Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

   (i)Prove that similar matrices have the same characterstic Polynomial.

   (ii)Let $E_{1,}E_k$ be linear operators on the space V suchthat $E_1 + \dots\dots\dots.. + E_k = I$. Thenprovethat if $E_iE_j=0$ fori≠j, then $^2= E_i$ foreachi.

   (iii)Define a Smith Normal form and give an example.

   (iv)Define a bilinear form and give an example.

   (v) Let V be a finite-dimensionalvecor space over the field F and f a symmetric bilinear form on V . For each subspace W of V , let X be the set of all vectors a in V suchthat f (α , β) = 0 for every β in then show that X is asubspace.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M205:PROBABILITY THEORY & STATISTICS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. It gives insight about the concepts, definitions and theorems of Probability and analyze the Probability distributions like Binomial, Poisson and Normal distributions.
2. Describe correlation and regression analysis concepts.
3. Understand the concepts of sampling and explain the large sample test.

UNIT-I:Sample Space & Events, Axioms of Probability, Some theorems on Probability, Boole's inequality, Conditional probability, Multiplication theorem on probability, Independent events, Multiplication theorem on probability for Independent Events, Extension of Multiplication theorem on probability to n events, Pair-wise Independent Events, Baye's theorem

[Section 3.8 to 3.15 , Page no: 3.2 to 3.98 & Section 4.2, Page no: 4.4 to 4.20]

UNIT-II: Distribution function, Discrete Random variables, Continuous random variables, Mathematical Expectation, Expected value of function of a random variable, Properties of Expectation, Properties of variance, Covariance, Moment Generating Function, Characteristic function, Binomial Distribution, Poisson Distribution, Normal Distribution,Uniform Distribution

[Section5.2to5.4,Pageno:5.2 to5.31,Section6.2to6.6,Pageno:6.1to6.22,Section

7.1, 7.3(7.3.1&7.3.2 only), Page no: 7.2 to 7.6, Page no: 7.9 to 7.15, Section 8.4, 8.5,Page

no:8.4to8.47,Section9.2.1to9.2.11and9.2.14,9.3, Pageno:9.2to9.12,9.14to9.28,

and 9.3 to 9.37]

UNIT-III: Correlation: Introduction, meaning of correlation, scatter diagram, Karl Pearson's Coefficient of Correlation, Rank Correlation, Linear and Curvilinear Regression:Introduction, linear regression, curvilinear regression

[Section 10.1 to 10.4(10.4.1, 10.4.2 only & in 10.7 –10.7.1 only), Page no: 10.1 to 10.16

and 10.23 – 10.25, Section 11.1 to 11.3, Page no: 11.1 to 11.19]

UNIT-IV: Large Sampling theory: Introduction, types of sampling, parameters and

statistic, tests of significance, procedure for testing of hypothesis, tests of significance for large samples.

[Section 14.1 – 14.6, Page no: 14.1 to 14.22]

Text Book: Fundamentals of Mathematical Statistics, S.C.Gupta,V.K.Kapoor Eleventh Thoroughly Revised EditionPublished by: Sultan Chand & Sons, NEW-DELHI.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS I YEAR SEMESTER II

(W.e.f. 2020-2021 Admitted Batch)

M205: PROBABILITY THEORY & STATISTICS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (a) State and prove addition theorem of probability.

(b) The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contact is 5/9.If the probability of getting atleast one contract is 4/5,what is the probability that he will get both the contracts?

(OR)

2. (a) State and prove Bayes' theorem.

(b) In a bolt factory, machines A, B, C manufacture respectively 25%, 35%, and 40% .Of the total of their output 5%, 4%, and 2% are known to be defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by i) Machine A

ii) Machine B and

iii) Machine C

3. (a) Derive the moment generating function of Binomial distribution. Hence find the mean and variance of Binomial distribution.

(b) Fit a Poisson distribution to the following data:

Number of mistakes per page	0	1	2	3	4	Total
Number of pages	109	65	22	3	1	200

(OR)

4. (a) The distribution of monthly income of 500 workers may be assumed to be normal with mean of Rs.2000 and a standard deviation of Rs. 200. Estimate the number of workers with incomes

(i) Exceeding Rs.2300 per month

(ii) Between Rs. 1800 and Rs.2300 per month.

(b) If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, then find P(X<0).

5. Calculate the Karl Pearson's correlation coefficient for the following heights of fathers(X) and their sons(Y):

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(OR)

6. (a)Prove that if one of the regression coefficients is greater than unity, the other must be less than unity.

(b) Obtain the equations of two lines of regression for the following data. Also obtain the estimate of X for Y = 70.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

7 . (a) A random sample of 500 pineapples was taken from a large consignment and 65 were found to be bad. Show that the S.E. of the proportion of bad ones in a sample of this size is 0.015 and deduce that the percentage of bad pineapples in the consignment almost certainly lies between 8.5 and 17.5.

(b) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease, is 85% in favour of the hypothesis that it is more, at 5% level.

(OR)

8. A company has the head office at Kolkata and a branch at Mumbai. The personal director wanted to know if the workers at the two places would like the introduction of a new plan of work and a survey was conducted for this purpose. Out of a sample of 500 workers at , 62% favoured the new plan. At Mumbai out of a sample 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level?

SECTION – B

9. Answer any three questions of the following. Each question carries 5 marks. $\mathbf{3 \times 5 = 15}$

(i) From a city population, the probability of selecting (a) a male or a smoker is 7/10, (b) a male smoker is 2/5, and (c) a male, if a smoker is already selected is 2/3. Find the probability of selecting a smoker, if a male is first selected.

(ii) A problem in Statistics is given to three students A, B and C whose chances of solving it are 1/2, 3/4, and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently.

(iii) A random variable X has the following probability function:

Values of X, x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(a) Find k, and (b) Evaluate $P(X < 6)$, $P(X \geq 6)$.

(iv) Prove that correlation coefficient is lies between± 1.

(v) Explain the procedure for testing of hypothesis.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M301: FUNCTIONAL ANALYSIS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss the Banach spaces definition with examples, continuous linear transformation, the Hahn- Banach theorem and the natural imbedding of N in N^{**}.
2. To discuss Hilbert spaces the definition and some simple properties, orthogonal complements and orthonormal sets.
3. To explain Finite- dimensional spectral theory Matrices, determinants and the spectrum of an operator and the spectral theorem.

UNIT-I **Banach spaces**: the definition and some examples, continuous linear transformation, the Hahn-Banach theorem, the natural imbedding of N in N^{**}, The open mapping theorem.

(Sections 46 – 50 of chapter 9)

UNIT-II The conjugate of an operator, **Hilbert spaces:** The definition and some simple properties, orthogonal complements, orthonormal sets.

(Sections 51 of chapter 9 and Sections 52- 54 of chapter 10)

UNIT-III The Conjugate space H^{*}, the ad joint of an operator, Self- ad joint operators, Normal and Unitary operators, Projections.

(Sections 55 - 59 of chapter 10)

UNIT-IV **Finite- dimensional spectral theory:** Matrices, determinants and the spectrum of an operator,the spectral theorem, A survey of the situation.

(Sections 60 - 63 of chapter - 11)

TEXT BOOK: Introduction to Topology and Modern Analysis by G.F.Simmons, McGraw Hill Book Company, Inc-International student ed.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M301: FUNCTIONAL ANALYSIS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. a) Define a Banach Space. Prove that the real linear space R^n is a Banach Space.
   b) In a Banach Space B, prove that the vector addition and scalar multiplication are jointly continuous.
   (OR)
2. a) State and prove Hahn-Banach theorem.
   b)Prove that the mapping $X \to F_x: N \to N^{**}$ where $F_x(f) = f(x)$,$\forall f \in N^*$ is an isometric isomorphism of N into N^{**}.
3. a) State and prove open mapping theorem.
   b) State and prove uniform boundedness theorem.
4. a)If M and N are two closed linear subspaces of a Hilbert space H such that $M \perp N$ then $M + N$ is also a closed linear subspace of H.
   b) State and prove Bessel's Inequality (finite case).
5. a) Prove that the mapping $y \to f_y$ is a norm preserving mapping of H into H^* where $f_y(x) = \langle x, y\rangle$ for all $x \in H$.

   b) If T is an operator on a Hilbert space H for which $\langle Tx, x\rangle = 0$ for all $x \in H$,then prove that T = 0 on H .

6. a)If T is an operator on a Hilbert space H then prove that the following conditions are all equivalent to one another .

   (i) $T^*T = I$

   (ii)$\|Tx\| = \|x\|$

(iii) $(Tx, Ty) = (x, y) \forall x \& y$.

b) Prove that a closed linear subspace M of H is invariant under T an opearator T on H if and Only if $M^{\perp}$ is invariant under T*.

7. a) Prove that two matrices of A_n are similar if and only if they are the matrices of a single opearator on H relative to (possibly) different bases.

   b) Prove that an operator T on H is singular if and only if $\theta \in \sigma(T)$.

8. State and prove Spectral theorem.

9. Answer any **THREE** of the following. Each Question carries 5 marks.
   a) Prove that every normal linear space is a metric space.
   b) State and prove Schwartz inequality.
   c) Define an orthogonal set in a Hilbert space H and given an example.
   d) Define Eigen value and Eigen vector.
   e) Let T be an operator on a Hilbert space H be such that adjoint T* of T is a polynomial in T, then prove that T is Normal.

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M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M302:LEBESGUE THEORY

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss Algebra of sets, Lebesgue measure, Outer measure, Measurable set and Lebesgue measure etc.
2. To discuss the Riemann integral, the Lebesgue integral of a bounded function over a set of finite measures etc.
3. To discuss Differentiation of monotonic functions, functions of bounded variation, differentiation of an integral, absolute continuity, Lp- Spaces the Holder's and Minkowski inequalities etc.

UNIT-I : Algebra of sets, Lebesgue measure, Outer measure, Measurable set and Lebesgue measure, a non-measurable set, measurable function, Little woods's Three principles.(Chapter 3)

UNIT-II : The Riemann integral, the Lebesgue integral of a bounded function over a set of finite measures, the integral of a non-negative function, the general Lebesgue integral convergence in measure. (Chapter 4)

UNIT-III : Differentiation of monotonic functions, functions of bounded variation, differentiation of an integral, absolute continuity. (Chapter 5)

UNIT-IV : Lp- Spaces the Holder's and Minkowski inequalities, convergence and completeness

(Chapter 6)

TEXT BOOK: H.L.Royden, Real Analysis, Macmillan Publishing Company, New York, Third Edition, 1988.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M302: LEBESGUE THEORY

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. Define measurability of a set and prove that the set of all measurable sets m is a σ –algebra.

(OR)

2. Every Borel set is measurable. In particular each open set and each closed set is measurable.
3. State and prove Lebesgue convergence theorem

(OR)

4. State and prove bounded convergence theorem
5. State and prove Fatou's lemma and hence prove the monotone convergence theorem.

(OR)

6. State and prove Vitali Lemma.
7. State and prove Minkowski Inequality in L^P

(OR)

8. State and prove Riesz – Fischer theorem.
9. Answer any **THREE** of the following. Each Question carries 5 marks.

a) Define the outer measure of a set and prove that if E_1 and E_2 are measurable then $E_1 \cup E_2$ is measurable.

b) Describe the invariance property.

c) Define convergence in measure.

d) If f and g are non negative measurable functions, then show that $\int_E cf = c\int_E f$, $c > 0$ and $\int_E (f+g) = \int_E f + \int_E g$.

e) State bounded convergence theorem and monotone convergence theorem.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M303:ANALYTICAL NUMBER THEORY

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss basic concepts of arithemetical functions and dirichlet multiplication like Mobius function μ(n), Euler quotient function φ(n), Mangoldt function $\Lambda(n)$, Multiplicative functions, Liouville's function λ(n), Divisor functions $\sigma_\alpha(n)$- and Generalized convolutions.
2. To explain Averages of arithemetical functions, big oh notation, Euler's summation formula, The average order of d(n), The average order of the divisor functions $\sigma_\alpha(n)$, The average order of φ(n), application to the distribution of lattice points visible from the origin and applications to μ (n) and Λ(n).
3. To discuss some elementary theorems on the distributions of prime numbers chebyshev's function ψ(x) and ϑ(x), Relations connecting ϑ(x) and π(x), Some equivalent forms of the prime number theorem, Shapiro's Tauberian theorem, Definition and basic properties of congruences, Euler-Fermat theorem, Polynomial congruences modulo p. Lagrange's theorem and it's applications, Chinese remainder Theorem and it's applications etc.

UNIT-I:ARITHMETICAL FUNCTIONS AND DIRICHLET MULTPLICATION :-Introduction – The Mobius function μ (n).-The Euler quotient function φ(n)-A relation connecting φ and μ- A product formula for φ(n)-The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The mangoldt function $\Lambda(n)$- multiplicative functions- multiplicative function and Dirichlet multiplication – The inverse of a completely multiplicative function- Liouville's function λ(n)- The divisor functions $\sigma_\alpha(n)$- Generalized convolutions.

(Sections 2.1 – 2.14 of chapter 2)

UNIT-II : AVERAGES OF ARITHMETICAL FUNCTIONS:- Introduction- The big oh notation. Asymptotic equality of functions – Euler's summation formula – Some elementary asymptotic formulas – The average order of d(n)-The average order of the divisor functions $\boldsymbol{\sigma_{\alpha}(n)}$- The average order of φ(n)-An application to the distribution of lattice points visible from the origin– the average order of μ (n) and $\Lambda(\boldsymbol{n})$– The partial sums of a Dirichlet product- Applications to μ (n) and $\Lambda(\boldsymbol{n})$– Another identity for the partial sums of a Dirichlet product.

(Sections 3.1 – 3.12 of chapter 3)

UNIT-III :SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMEBRS:- Introduction – chebyshev's function ψ(x) and $\vartheta(x)$- Relations connecting $\vartheta(x)$ and π(x) – Some equivalent forms of the prime number theorem - Inequalities for π(n) and p_n – Shapiro's Tauberian theorem – Applications of Shapiro's theorem – An asymptotic formula for the partial sums$\sum_{p\leq x}(1/p)$- The partial sums of the Mobius function.

(Sections 4.1 to 4.9 of Chapter 4)

UNIT-IV : CONGRUENCES :- Definition and basic properties of congruences – Residue classes and complete residue systems – linear congruences – Reduced residue systems and the Euler-Fermat theorem – Polynomial congruences modulo p. Lagrange's theorem –Applications of Lagrage's theorem – Simultaneous linear congruences. The Chinese remainder Theorem- Applications of the Chinese remainder Theorem – Polynomial congruences with prime power moduli.

(Sections 5.1 – 5.9 of chapter 5)

TEXT BOOK : Introduction to Analytic Number Theory – By T.M.APOSTOL – Springer Verlag New York, Heidlberg – Berlin – 1976.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M303: ANALYTICAL NUMBER THEORY

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. Define Euler totient function and prove if $n \geq 1$ then $\sum_{d/n} \emptyset(d) = n$ and also prove $\emptyset(p^{\propto}) = p^{\propto} - p^{\propto-1}$ for prime p and $\propto \geq 1$.

(OR)

2. Define Multiplicative function and prove both g and $f * g$ are multiplicative then f is also multiplicative.
3. State and prove Euler summation formula.

(OR)

4. Prove that for $x \geq 2$ we have $\sum_{p \leq x} \left[\frac{x}{p}\right] logp = xlogx + O(x)$ where the sum is extended over all primes $\leq x$.
5. Let p_n denote the nth prime then the following asymptotic relations are logically equivalent
$\lim_{x\to\infty} \frac{\pi(x)logx}{x} = 1$
$\lim_{x\to\infty} \frac{\pi(x)log\pi(x)}{x} = 1$ $\lim_{x\to\infty} \frac{p_n}{nlogn} = 1$

(OR)

6. For $n \geq 1$ the nth prime p_n satisfies the inequalities
$$\frac{1}{6} nlogn < p_n < 12\left(nlogn + nlog\frac{12}{e}\right)$$
7. State and prove Lagrange's theorem.

(OR)

8. State and prove Chinese remainder theorem.
9. Answer any **THREE** of the following. Each Question carries 5 marks.

(a) State and prove Generalized inversion formula.

(b) If $x \geq 1$ then prove that $\sum_{n\leq x}\frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \mathcal{C}(s) + O(x^{-s})$ if $s > 0, s \neq 1$.

(c) For $x > 0$ then prove that $0 \leq \frac{\varphi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(logx)^2}{2\sqrt{x}log2}$.

(d) Solve the congruence $5x \equiv 3(mod24)$.

(e) State and prove Wilson's theorem.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M304:PARTIAL DIFFERENTIAL EQUATIONS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss about basics of partial differential equations and method of solutions of dx/P = dy/Q = dz/R and know about pfaffian differential forms and equations and their solutions in three variables.
2. To describe linear & non-linear equations of the first order and solving the orthogonal surfaces and compatible systems of first order equations further more about some methods like,charpit's method,jacobi's method.
3. 3. To solve the second order partial differential equations and discuss about solutions f linear hyperbolic equations and explaining method of separation of variables,monger's method and further more about Laplace equation and some elementary solutions and solving boundary value problems and more about wave equation and elementary solution in one dimensional form.

UNIT I: Introduction, Methods of Solution of dx/P = dy/Q = dz/R, Orthogonal trajectories of a system of curves on a surface, Pfaffian Differential forms and equations, Solutions of Pfaffian differential equations in three variables, Cauchy's problem for first order partial differential equations.

(Sections 3 to 6 of Chapter 1, Sections 1 to 3 of Chapter 2)

UNIT II: Linear Equations of the first order, Integral surfaces, orthogonal surfaces, non linear partial differential equations of the first order, Cauchy's method of characteristics, Compatible systems of first order equations, Charpit's Method, Special types of first order equations, Jacobi's method.

(Sections 4 to 13 of Chapter 2)

UNIT III: Partial Differential Equations of the second order, Their origin, Linear partial Differential equations with constant and variable coefficients, Solutions of linear hyperbolic equations, Method of separation of variables, Monger's method.

(Sections 1 to 5 and Sections 8,9,11 of Chapter 3)

UNIT IV: Laplace Equation, elementary solutions, families of equipotential surfaces, Boundary value problems, Method of separation of variables of solving Laplace equation, problems with axial symmetry, Kelvin's inversion theorem, The wave equation, Elementary solution in one dimensional form, Riemann-Volterra solution of one dimensional wave equation.

(Sections 1 to 7 of Chapter 4 and Sections 1 to 3 of Chapter 5)

Additional Inputs:

TEXT BOOK: Elements of Partial Differential Equations by I.N.Sneddon, Mc Graw Hill,

International Edition, Mathematics series.

REFERENCE BOOK: Fritz John, Partial Differential Equations, Narosa Publishing House, New Delhi, 1979

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M304: PARTIAL DIFFERENTIAL EQUATIONS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. Find the integral curves of the equations $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$.

(OR)

2. A necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$, not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

3. Find the surface which is orthogonal to the one- parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2$,$z = 0$.

(OR)

4. Show that the equations $xp - yq = x$

$x^2p + q = xz$ are compatible and find their solution.

5. Reduce the equation

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$$ to canonical form,and hence solve it.

(OR)

6. Solve the equation $rq^2 - 2pqs + tp^2 = pt - qs$.

7. Show that the surface $x^2 + y^2 + z^2 = cx^{\frac{2}{3}}$ can form a family of equipotential surfaces, and find

the general form of the corresponding potential function.

(OR)

8. State and prove Kelvin's Inversion theorem.

9. Answer any **THREE** of the following. Each Question carries 5 marks. $3 \times 5 = 15$

(a) Eliminate the arbitrary function f from the equation $z = xy + f(x^2 + y^2)$.

(b) Find the general integral of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.

(c) Prove that a pfaffian differential equation in two variables always possesses an integrating factor.

(d) Explain Charpit's method.

(e) Find the D-Alembert's Solution for one dimensional wave equation.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER III

(W.e.f. 2020-2021 Admitted Batch)

M305:COMMUTATAIVE ALGEBRA

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To explain the concepts of Rings and ring homomorphism, ideals, quotient rings, prime ideals and Maximal ideals, nil radical and Jacobson radical.
2. To study Modules and module homomorphisms, Sub modules and quotient modules, operations on submodules, direct sum and product, finitely generated modules and exact sequences.
3. To discuss the topics Local Properties, Extended and Contracted ideals in rings of fractions and Primary decompostitions.

UNIT-I : Rings and ring homomorphism, ideals, quotient rings, zero divisors, Nilpoent elements, units, prime ideals and Maximal ideals, nil radical and Jacobson radical, operations on ideals, Extensions and contractions.

UNIT-II : Modules and module homomorphisms, Sub modules and quotient modules, operations on submodules, direct sum and product, finitely generated modules, exact sequences, Tensor product of modules, Restriction and extension of scalars, Exactness properties of the tensor product, algebras, tensor product of algebras.

UNIT-III : Local Properties, Extended and Contracted ideals in rings of fractions..

UNIT-IV : Primary decompostitions.(Content and extent of chapters 1 to 4 of the prescribed text book)

TEXT BOOK: Introduction to commutative algebra, M.F.ATIYAH and I.G.MACDONALD,

Addision – Wesley publishing Company, London.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M401:MEASURE THEORY

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss Convergence and Completeness, Measure spaces, Measurable functions, Integration, General convergence Theorems.
2. To discuss Signed Measures, The Raydon-Nikodym Theorem, the Lp spaces, Outer Measure and Measurability, The Extension theorem, The Lebesgue - Stieltjes Integral and Product measures.
3. To discuss Integral Operators, Inner Measure, Extension by sets of measure zero, caratheodory outer measure and Hausdroff Measure.

UNIT- I : Convergence and Completeness, Measure spaces, Measurable functions, Integration, General convergence Theorems.

[Section 3 of Chapter 6, Section 1 to 4 of Chapter 11 of the text book]

UNIT- II : Signed Measures, The Raydon-Nikodym Theorem, the Lp spaces.

[Sections 5 to 7 of Chapter 11 of the text book]

UNIT- III : Outer Measure and Measurability, The Extension theorem, The Lebesgue - Stieltjes Integral, Product measures.

[Sections 1 to 4 of Chapter 12 of the text book]

UNIT- IV : Integral Operators, Inner Measure, Extension by sets of measure zero, caratheodory outer measure, Hausdroff Measure.

[Sections 5 to 9 of Chapter 12 of the text book]

TEXT BOOK: Real Analysis by H. L. Royden, Macmillan Publishing Co. Inc. 3 rd Edition, New York, 1988.

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M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M401: MEASURE THEORY

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. a) Prove that a normed linear space X is complete if and only if every absolutely summable series is summable.

   b) Let (X,) be measurable space and f be an extended real valued function defined on X then prove that the following statements are equivalent.

   (i) {x / f(x) < α} € $\mathfrak{B}$ for each α.

   (ii) {x / f(x) ≤ α} € $\mathfrak{B}$ for each α.

   (iii) {x / f(x) > α} € $\mathfrak{B}$ for each α.

   (iv) {x / f(x) ≥ α} € $\mathfrak{B}$ for each α.

(OR)

2. a) State and prove Fatous lemma.

   b) If (X,$\mathfrak{B}$,u) is any measure space then there exist a complete measure space $(X, \mathcal{B}_0, \mu_0)$ such that

   i) $\mathfrak{B} \subseteq \mathfrak{B}_0$

   ii) If B € $\mathfrak{B}$ then u (B) = u_0(B)

   iii) E € $\mathfrak{B}_0$ iff E = AU B then B € $\mathfrak{B}$ and A ⊆ C for some C € $\mathfrak{B}$ with u (C) = 0.

3. a) Let E be a measurable set such that $0 < \gamma(E) < \infty$. Then prove that there is a positive set A contained in E with $\gamma(A) > 0$.

   b) State and prove Hahn Decomposition theorem.

(OR)

4. State and prove Riesz representation theorem.

5. a) Prove that the class $\mathfrak{B}$ of u^{*} measurable sets is a σ- algebra.

b) Let u be a measure on an algebra $\mathcal{G}$ and μ^{*} be a outer measure induced by u and E is any set then for any $\in > 0$ $\exists$ a set $A \in \mathcal{G}_{\sigma}$ with E ⊆ A and $\mu^{*}(A) \leq \mu^{*}(E) + \in$ also there is a set $\mathcal{B} \in \mathcal{G}_{\sigma\delta}$ with $E \subseteq \mathcal{B}$ and $\mu^{*}(E) = \mu^{*}(\mathcal{B})$.

(OR)

6. state and prove Tonelli theorem.

7. a) Let E and F be two disjoint sets. Then prove that

$$\mu_{*}(E) + \mu_{*}(F) \leq \mu_{*}(E \cup F) \leq \mu_{*}(E) + \mu^{*}(F)$$
$$\leq \mu^{*}(E \cup F) \leq \mu^{*}(E) + \mu^{*}(F).$$

b) Suppose that $\mu^{*}(E) < \infty$. Then prove that E is measurable if and only if $\mu_{*}(E) = \mu^{*}(E)$.

(OR)

8. If u^{*} is a caratheodory outer measure with respect to Γ then prove that every function in Γ is u^{*} - measurable.

9. Answer any three of the following. Each question carries 5 marks.

a). Prove that every σ - finite measure is saturated.

b). Show that the Hahn decomposition need not be unique.

c). Show that if γ is a signed measure such that γ is mutually singular with respect to u and $\gamma << u$ then $\gamma = 0$.

d) .Let B be a u^{*} measurable set with $\mu^{*}(B) < \infty$. Prove that u_{*} (B) $= u^{*}$ (B).

e). Give an example for an algebra of sets which is not an σ - algebra of sets .Justify.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M402:NUMERICAL ANALYSIS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To perform the iterations for the smallest root by using Bisection method, Regulafalsi method, Newton-Raphson method, Mullers method, Chebyshev method, Multipoint iterative method and secant method.

2. To discuss the iterations for the smallest root by using Guass elimination method, Triangularization method, Cholesky method, Lagrange and Newton's divided difference interpolation, sterling and Bessel interpolation, Hermite interpolation, piecewise and Spline Interpolation method.

3. To find the smallest root by using Gauss Legendre Integration method, Euler's method, Taylor series method, Runge kutte second and forth order methods.

UNIT I : Transcendental and polynomial equations: Introduction, Bisection method, Iteration methods based on first degree equation; Secant method, Regulafalsi method, Newton- Raphson method, Iteration method based on second degree equation; Mullers method, Chebyshev method, Multipoint iterative method, Rate of convergence of secant method, Newton Raphson method,

(Section 1 of the Text Book pages 1 to 52 above specified methods only)

UNITII : System of linear algebraic equation:Direct methods, Guass elimination method, Triangularization method, Cholesky method, Partition method, Iteration method: Gauss seidel Iterative method, OR method.

(Section 2 of the Text Book pages 53 to 169 above specified methods only)

UNIT III : Interpolation and Approximation: Introduction, Lagrange and Newton's divided difference interpolation, Finite difference operators, sterling and Bessel interpolation, Hermite interpolation, piecewise and Spline Interpolation, least square approximation.

(Section 3 of the Text Book pages 210 to 300 above specified methods only)

UNIT IV : Numerical Differentiation: methods based on Interpolation, methods based on Finite difference operators Numerical Integration: methods based on Interpolation, Newton's cotes methods, methods based on Undetermined coefficients, Gauss Legendre Integration method, Numerical methods ODE: Single step methods: Euler's method, Taylor series method, Runge kutte second and forth order methods, Multistep methods: Adam Bash forth method, Adam Moulton methods, Milne-Simpson method.

(Section 4 of the Text Book pages 320 to 495 above specified methods only)

Text Book: [1] Numerical Methods for Scientific and Engineering computation by M.K. Jain, S.R.K. Iyengar, R.K. Jain, New Age Int. Ltd., New Delhi.

Reference: [1] Introduction to Numerical Analysis, by S.S. Sastry,Prentice Hall Flied.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M402: NUMERICAL ANALYSIS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

(OR)

2. Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation $x^4 - x - 10 = 0$.

3. Solve the system of equations $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$ using the Cholesky method.

   Also determine A^{-1}.

(OR)

4. Find the inverse of the matrix

   A= $\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}$ using partition method. Hence, solve the system of equation $AX = b$,

   where $b = [-10,8,7,-5]^T$

   .

5. Construct the Hermite interpolation polynomial that fits the data

x	f(x)	$f^1(x)$
2	29	50
3	105	105

Interpolate f(x) at $x = 2.5$.

(OR)

6. Obtain the cubic spline approximation for the function defined by the data

x	0	1	2	3
f(x)	1	2	33	244

with $M(0) = 0$, $M(3) = 0$. Hence find an estimate of f(2.5).

7. Evaluate the integral $= \int_1^2 \frac{2x dx}{1+x^4}$, using the Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. Compare with the exact solution $I = tan^{-1}(4) - (\frac{\pi}{4})$.

(OR)

8. Given the initial value problem $u' = -2tu^2$, $u(0) = 1$ estimate $u(0.4)$ using
   (i) modified Euler-cauchy method, and
   (ii) Heun method with $h = 0.2$. compare the results with the exact solution $u(t) = \frac{1}{1+t^2}$.

9. Answer any Three of the following. Each question carries 5 marks.
   a) Perform two iterations of the Chebyshev method to find an appropriated value of 1/7 . Take the initial approximation as $x_0 = 0.1$
   b) Solve the equations $10x_1 - x_2 + 2x_3 = 4$
      $x_1 + 10x_2 - x_3 = 3$
      $2x_1 + 3x_2 + 20x_3 = 7$
      Using the Gauss elimination method.
   c) Show that (i) $\delta = \nabla(1 - \nabla)^{-1/2}$ (ii) $\mu = \left(\frac{1+\delta^2}{4}\right)^{1/2}$
   d) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule. Obtain a bound for the error. The exact value of $I = ln2 = 0.693147$ correct to six decimal places.
   e) For the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$, find all the eigen values.

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M403:GRAPH THEORY

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To understand the basic concepts of graphs and about trees & fundamental circuits.
2. To discuss about cutsets & cut vertices and planar & dual graphs, further more ,to learn about the matrix representation of graphs like, Incidence, Circuit,Path and Adjacency matrices of graphs.
3. To explain about the concepts of coloring , covering and partitioning of graphs and at athe end of the chapter we learn about graph theory in operation research.

UNIT-I: Basic concepts, Paths and Circuits, Trees andFundamental Circuits.

[Chapters 1, 2, 3 of the text book]

UNIT-II: Cut Sets and Cut Vertices: Cut sets, Some properties of a cut set, All cut sets in a graph, Fundamental circuits and cut sets, Connectivity and Separability, Network Flows, 1- Isimorphsm, 2-Isomorphsm; Planar and Dual Graphs: Combinatorial Vs Geometric graphs, Planar graphs, Kuratowski's Two graphs, Different Representations of Planar Graphs, Detection of Planarity, Geometric Dual, Combinational Duals of a Graph,

[Chapter 4 and Sections 5.1 to 5.7 of Chapter 5 of the text book]

UNIT-III: Matrix Representation of graphs: Incident matrix of a Graph, Sub Matrices of A(G), Circuit Matrix, Fundamental Circuit Matrix and Rank of B, An Applications to a Switching Network, Cut set matrix, Relationship among A_f, B_f and $C_{f,}$, Path Matrix and Adjacency Matrix.

[Chapter 7 of the text book]

UNIT-IV: Coloring, Covering and Partitioning: Chromatic Number, Chromatic Partitioning, Chromatic Polynomial, Matchings, Coverings, The four color Problem; Graph Theory in Operation Research: Transport networks, Extensions of Max-flow Min cut theorem, Minimal cost flows.

[Chapter 8 and Sections 14.1 to 14.3 of Chapter 14 of the text book]

Additional Inputs:

TEXTBOOK: Graph Theory with applications to Engineering and computer Science by Narsingh Deo; Prentice-Hall of India.

REFERENCES: **1.** Graph Theory with applications by Bond JA and Murthy USR, North Holland, NewYork.

2. Introduction to Graph Theory by Donglas B. West. Prentice Hall of India.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M403: GRAPH THEORY

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. a) Explain Konigsberg Bridge Problem.

   b) (i) Define Euler Graph

   (ii) Prove that the number of vertices of odd degree in a graph is always even.

(OR)

2. a) Prove that in a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian circuits, if n is odd number ≥ 3.

   b) (i) Define Binary Tree

   (ii) Prove that a tree with n vertices has $n-1$ edges.

3. a) (i) Define cut set and give an example

   (ii) Prove that every circuit has an even number of edges in common with any cut set.

   b) Define 1-Isomorphism.

(OR)

4. a) Prove that a graph has a dual if and only if it is planar.

b) In any simple connected planar graph with f regions ,n vertices and e edges($e > 2$) then prove that (i) $e \geq \frac{3}{2} f$

(ii) $e \leq 3n - 6$.

5. a) (i) Define Incidence Matrix.

(ii) Define fundamental circuit matrix.

b) Let B and A be, respectively, the circuit matrix and the incidence matrix (of a self - loop free graph) whose columns are arranged using the same order of edges. Then prove that, every row of B is orthogonal to every row A, that is, $AB^T = BA^T = 0 \ \ (mod\ 2)$.

(OR)

6. a) prove that, (i) $B_f = [I_\mu | A_c^T . A_T^{-1^T}]$

(ii) $C_f = A_t^{-1}. A_f$.

b) Prove that in a connected graph, the distance between two vertices v_i and v_j (for $i \neq j$) is k, if and only if k is the smallest integer for which the i, jth entry in X^k is non-zero. That is, $[X^k]_{ij} \neq 0$.

7. a) Prove that a graph with atleast one edge is 2-chromatic if and only if it has no circuit of odd length.

b) Prove that the vertices of every planar graph can be properly colored with five colors.

(OR)

8. a) Let a and b be two nonadjacent vertices in a graph G. Let G' be a graph obtained by adding an edge between $a$and b. Let G'' be a simple graph obtained from G by fusing the vertices a and b together and replacing sets of parallel edges with single edges. Then prove that,

$$P_n(\lambda)\text{of G} = P_n(\lambda)\text{of}G' + P_{n-1}(\lambda)\text{ of}G''.$$

b) State and Prove Max Flow Min Cut Theorem.

9. Answer any three of the following. Each question carries 5 marks.

a) Prove that a graph G with n vertices,$n - 1$ edges, and no circuits is connected.

b) Prove that the complete graph of five vertices is non planar.

c) Define Circuit matrix and give an example.

d) Prove that every tree with two or more vertices is 2-chromatic.

e) Explain Multi Commodity Flow.

Dr. C. S .RAO PG CENTRE

SRI Y.N .COLLEGE (AUTONOMOUS), NARSAPUR, W.G.Dt.,A.P.

NAAC Accredited 'A' Grade College Affiliated to Adikavi Nannaya University

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M404: LINEAR PROGRAMMING

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To study the concepts of formulation of Linear Programming problems, Graphical solution, General formulation and Simplex Method.

2. To solve the problems by using the methods two-phase, Big-M, degeneracy, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

3. Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Mathematical formulation of Transportation problem, North West corner rule, Lowest cost entry method, Vogel's approximation methods,

UNIT I : Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method.

[Sections 3.1 to 3.7 of Chapter 3 and Section 5.4 of Chapter 5 of the text book]

UNIT II : Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions. Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

[Sections 5.5, 5.7, 5.8, 5.12, 5.13 of Chapter 5 and Sections 7.1, 7.7 of Chapter 7]

UNIT III : Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.

[Sections 12.1 to 12.4, 12.9 of Chapter 12 of the text book]

UNIT IV : Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

[Sections 11.1 to 11.5 and 11.8 to 11.12 of Chapter 11 of the text book]

TEXT BOOKS:

[1] S. D. Sharma, Operations Research.

REFERENCE BOOKS:

[1] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.

[2] H. A. Taha, Operations Research – An Introduction.

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M404: LINEAR PROGRAMMING

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

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DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M405: DISCRETE DYNAMICAL SYSTEMS

Course Outcomes: The study of M.Sc. degree programme will enable the students:

1. To discuss about Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's theorem, Differentiability and its Implications.
2. To explain Parameterized Families of Functions and Bifurcations; The Logistic Function Part I, The Logistic Function Part II Topological Conjugacy, The Logistic Function Part III and Newton's method.
3. Numerical solutions of Differential Equations, The Dynamics of Complex functions, Quadratic Family and Mandelbrot Set

UNIT I : Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's theorem, Differentiability and its Implications [Hyperbolic, Attractive and Repelling Periodic Points]

[Chapters 1,4,5,6]

UNIT II : Parameterized Families of Functions and Bifurcations; The Logistic Function Part I [Cantor Sets], Symbolic Dynamics and Chaos.

[Chapters 7,8,9]

UNIT III : The Logistic Function Part II Topological Conjugacy, The Logistic Function Part III [Period Doubling Cascade], newton's Method

[Chapters 10,11,12]

UNIT IV : Numerical solutions of Differential Equations, The Dynamics of Complex functions [newton's Method in Complex Plane], the Quadratic Family and Mandelbrot Set

[Chapters 13, 15 and Sections 14.3, 14.5]

TEXT BOOK : Richard M. Holmgren, A First Course in Discrete Dynamical Systems,Springer Verlag

DEPARTMENT OF MATHEMATICS

M.Sc.MATHS II YEAR SEMESTER IV

(W.e.f. 2020-2021 Admitted Batch)

M405: DISCRETE DYNAMICAL SYSTEMS

Model Question Paper

Time: 3 hours **Max Marks:** 75

SECTION – A

Answer **ALL** questions. Each question carries 15 marks. $\mathbf{4 \times 15 = 60}$

1. (a) Define fixed point and prove that if $I = [a, b]$ be a closed interval and $f: I \rightarrow I$ be a continuous function then f has a fixed point in I. (b) Define stable set and prove that the stable set of distinct periodic points do not intersect.

(OR)

2. State and prove Sarkovaskii's Ordering Theorem.
3. (a) Define Saddlenode Bifurcation with suitable example. (b) Define Cantor set and prove that the Cantor Middle-α Set is a Cantor set.

(OR)

4. Let $f: X \rightarrow X$ be a topologically transitive and suppose that periodic points of f are dense in X. If X is infinite then f exhibits sensitive dependence on initial conditions.
5. Let D and E be metric spaces, $f: D \rightarrow D$, $g: E \rightarrow E$ and $\tau: D \rightarrow E$ be a topological conjugacy of f and g then

   (a)$\tau^{-1}: E \rightarrow D$ is a topological conjugacy.

   (b) The periodic points of f are dense in D if and only if the periodic points of g are dense in E.

   (c) f is chaotic on D if and only if g is chaotic on E.

(OR)

6. (a) Let $f(x) = ax^2 + bx + c$ and $q(x) = x^2 - A$ where $A = (b^2 - 4ac)$ then $\tau(x) = 2ax + b$ is a topological conjugacy from $N_f(x)$ to $N_q(x)$.

(b) Prove that the function $\Psi: \Lambda \longrightarrow \Sigma_2$ is topological conjugacy.

7. (a)Let f be a differentiable complex function and p be a fixed point of f. If $|f'(p)| < 1$, then the stable set of p contains a neighbourhood of p.

   (b) Let $f(z) = az$ where 'a' is complex number whose module is not 1 and note that 0 is the only fixed point of f and investigate the orbit of z_0 when $z_0 \neq 0$.

(OR)

8. Let $f(z) = e^{i\theta} z$ and z_0 is a nonzero complex number. Show that

   (a) z_0 is a periodic point of f if θ is a rational multiple of π.

   (b) If θ is a rational multiple of π then z_0 is not a periodic point of f and its orbit is dense in the circle containing z_0.

9. Answer any **Three** of the following. Each question carries 5 marks.

   (a) Define Discrete Dynamical system and give three examples.

   (b) Define Shift map and prove that shift map is continuous

   (c) If D and E be metric spaces, $f: D \to D$, $g: E \to E$ and $\tau: D \to E$ be a topological conjugacy of f and g prove that f is topological transitive on D iff g is topological transitive on E.

   (d) Prove that the orbit of a complex number under iteration of a complex quadratic polynomial is either bounded or the number is the stable set of ∞

   (e) Define topological conjugacy and prove that the periodicity and period of a periodic point is preserved by topological conjugacy.