ALCMS

Depts of Mathematics, Statistics& Computer Science
Andhra Loyola College (Autonomous), Vijayawada, INDIA
in colloboration with
International Multidisciplinary Research Foundation (IMRF)
Institute of Higher Education & Research, INDIA

INTERNATIONAL CONFERENCE ON ADVANCES & APPLICATIONS IN MATHEMATICS , STATISTICS & COMPUTER SCIENCE - 2019

Govt of Inida Approved Conference

PROCEEDINGS

LOYOLA ANDHRA COLLEGE
FAITH DUTY

INTERNATIONAL MULTIDISCIPLINARY RESEARCH FOUNDATION
IMRF
CHOOSE TO BE HUGE

Nov 01 - 02, 2019
Andhra Loyola College (Autonomous)

Conference Director:
Rev.Fr.G.M.Victor Emmanuel, S.J.

Conference Chairman:
Prof.Dr.Ratnakar D Bala, IMRF

Organising Secretaries : Dr.N.Srinivasa Rao, Mr.P.Venu Gopala Rao, Mr.T.Kamalakar Raju

PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON ADVANCES & APPLICATIONS IN MATHEMATICS, STATISTICS & COMPUTER SCIENCE – ANDHRA LOYOLA 2019

Nov 01-02, 2019

ISBN 978-93-86435-93-4

Govt of India Approved Conference
(Govt of India Approved: MHA Vide : F.No 42180123/CC-1285 ; MEA : F.No. AA/162/01/2019-1488)

Organized by

Andhra Loyola College (Autonomous) Vijayawada, Andhra Pradesh, India
IMRF Institute of Higher Education & Research, India

TABULAR CUSUM CHARTS WITH STATISTICAL PROCESS CONTROL

DR. N. VISWAM, K.SRINIVASA RAO, T. RAGHAVAIAH, G. RANGABABU

Abstract: Control charts are the most popular tool of statistical process control for monitoring variety of processes. In this paper, I explained cumulative sum (CUSUM) control scheme in brief, present numerical example and it is verified that the CUSUM is an efficient alternative to Shewhart procedures. It is shown that CUSUM scheme is more efficient in detecting small shifts in the mean of a process. The comparison shows the overall good detection performance of our scheme for a span of shifts in the mean. The CUSUM Control Chart platform creates a CUSUM chart with decision limits, similar to a Shewhart chart.

Keywords: CUSUM Control Chart, $\overline{X}$ -Control Chart, Simulation, Statistical Process Control.

***

Dr. N. Viswam,
HOD & Vice-Principal, Dept. of Statistics, Hindu College, Guntur, Guntur Dist., AP
Email: nandigamaviswam@gmail.com
K.Srinivasa Rao,
Assoc. Prof., Bhimavaram Institute of Engg. & Tech.,Bhimvaram, W.G.Dt., AP.
Email: kasaganasrinivas@gmail.com
T. Raghavaiah,
Asst. Prof, Guntur Engineering College, Guntur, Guntur Dist., A.P.
Email: thatiraghavaiah@gmail.com
G. Rangababu,
Asst. Prof, Dr. C.S.Rao PG Centre, Sri YN College, Narasapuram, W.G.Dt., A.P.
Email: rangababugedda@gmail.com

TABULAR CUSUM CHARTS WITH STATISTICAL PROCESS CONTROL

Dr. N. Viswam
HOD & Vice-Principal, Dept. of Statistics, Hindu College, Guntur, Guntur Dist., AP
Email: nandigamaviswam@gmail.com

K.Srinivasa Rao
Assoc. Prof., Bhimavaram Institute of Engg. & Tech.,Bhimvaram, W.G.Dt., AP.
Email: kasaganasrinivas@gmail.com

T. Raghavaiah
Asst. Prof, Guntur Engineering College, Guntur, Guntur Dist., A.P.
Email: thatiraghavaiah@gmail.com

G. Rangababu
Asst. Prof, Dr. C.S.Rao PG Centre, Sri YN College, Narasapuram, W.G.Dt., A.P.
Email: rangababugedda@gmail.com

Received: Sep. 2019 Accepted: Oct. 2019 Published: Nov. 2019

Abstract: Control charts are the most popular tool of statistical process control for monitoring variety of processes. In this paper, I explained cumulative sum (CUSUM) control scheme in brief, present numerical example and it is verified that the CUSUM is an efficient alternative to Shewhart procedures. It is shown that CUSUM scheme is more efficient in detecting small shifts in the mean of a process. The comparison shows the overall good detection performance of our scheme for a span of shifts in the mean. The CUSUM Control Chart platform creates a CUSUM chart with decision limits, similar to a Shewhart chart.

Keywords: CUSUM Control Chart, $\overline{X}$ -Control Chart, Simulation, Statistical Process Control.

Introduction: The control chart is the widely applied technique for controlling the industrial processes. The pioneer work on Statistical Process Control (SPC) was done by Walter A. Shewhart in the earlier 1920's.Shewhart developed the basis for the control chart and the state of statistical control. Shewhart's charts effective for detecting large changes in process parameters; however Shewhart chart may take a longtime to detect a small persistent shift in the process parameter. The ability to detect smaller parameter Shifts can be improved by using a chart based on a statistic that corporate information from past samples in addition to current samples. One such chart is Cumulative Sum (CUSUM) control chart developed by Page (1954).This chart plots the cumulative sums of deviations of the sample values of a quality characteristic from a target value against time. It is noted that Shewhart's control chart for mean is very effective if the magnitude of the shift is 1.5 -sigma or larger (Montgomery 2001). Some authors namely Duncan (1974), Lucas (1976), Hawkins (1981), Lucas and Saccucci (1982a, 1990) stated that the CUSUM control chart is much more efficient than the usual $\overline{X}$ control chart for detecting smaller variations in the average. There are two ways to represent a CUSUM, the tabular or algorithmic cusum and the V- mask CUSUM. Of these two forms the tabular form of the cusum is practiced more. So we consider the construction and use of the tabular CUSUM in this paper. An excellent discussion of cusum control scheme is given by Hawkins and Olwell (1998). The rest of the paper is organized as follows.

1. View on the cusum control chart with principles of CUSUM control scheme.
2. An example is given for Shewhart and CUSUM charts are plotted and compared.
3. Shift detection properties of the CUSUM are verified.
4. Final conclusion of the paper by a brief summary of results and discussion.

What is a CUSUM Chart?

Like control charts, CUSUM charts are used to plot data in time-series. The charts are meant to alert users to significant changes in process performance. CUSUM charts do not plot raw data values, averages, ranges or standard deviations. Instead, plot points on a CUSUM chart are data values roughly representative of the cumulatively summed, subgroup-to-subgroup deviations from a specified target or production mean. The primary advantage of the tabular CUSUM chart is that the chart is more sensitive to small changes in the mean, especially when compared with $\overline{X}$ and R control charts.

Cumulative Sum (CUSUM) Control Chart: CUSUM may be constructed for individual observations as well as rational subgroups. Here only case of individual observations is considered.

The Tabular CUSUM for Monitoring the Process Mean: Let x_i be the ith observation on the process. Xi has normal distribution with mean μ_o and standard deviation σ (known or estimable), when the process is under control. Sometimes, μ_o is taken to be the target value for the quality characteristic X. The tabular CUSUM works by accumulating the deviations from μ_o that are above target with one statistic C+ and that are below target with another statistic C -. The statistics C+ and C - are called one sided upperand lower CUSUM respectively.

How are CUSUM Charts Created?

When compared with traditional control charts, the tabular CUSUM chart is unique, and certain parameters and statistics must be used in its creation.

Table 1

Parameter	Description
$C_i^+ = max[0, x_i - (\mu_0 + K_u) + C_{i-1}^+]$	The C_i^+ plot point is used exclusively for plotting the upper line on the tabular CUSUM chart. The upper line is used for identifying changes in the process above the stated target value, μ_0.
$C_i^- = min[0, x_i - (\mu_0 - K_l) - C_{i-1}^-]$	The C_i^- point is used exclusively for plotting the lower line on the tabular CUSUM chart. The lower line is used for identifying changes in the process below the chosen target value, μ_0.
μ_0	Process Mean or Target value.
$\hat{\sigma}_{LT} = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{(n-1)}}$	Sample standard deviation, or Long Term (LT) standard deviation.
msz_u	The positive shift in Process Mean that the CUSUM chart is meant to detect, in standard deviation units. Typically between 0.5σ and 1.5σ.
msz_l	The negative shift in Process Mean that the CUSUM chart is meant to detect, in standard deviation units. Typically between 0.5σ and 1.5σ.
$K_u = \left(\frac{msz_u \times \hat{\sigma}_{LT}}{2}\right)$	The "Upper Reference" value specified for plotted *Positive CUSUM* line. Positive CUSUM values are accumulated only when the deviation from the target value exceeds

	specified K value. K value is half the magnitude of the mean shift value to detect.
$K_l = \left(\frac{msz_l \times \hat{\sigma}_{LT}}{2}\right)$	The "Lower Reference" value specified for plotted *Negative CUSUM* line. Negative CUSUM values are accumulated only when the deviation from the target value exceeds specified K value. K value is half the magnitude of the mean shift value to detect.
h	Decision Parameter. Factor for determining Decision Interval, H. Generally, h is defined as 5, although sometimes 4 is utilized.
$H = h\sigma$	H is the "Decision Interval," which acts as a control limit.
$FIR = \frac{1}{2}H$	Optional. Fast Initial Response or "head start" is typically half of H and is treated as the initial CUSUM value when no data exists.

Example: A quality control inspector at the cocoa fizz soft drink company as taken 25 samples with four observations each of the volume of the bottles filled. The data and the computed means are shown in the table. If the standard deviation of bottling operation is 0.14 ounces. Using this information to develop control limits of three standard deviation for the bottle operations.

Table 2

Sample Number	Observations				Mean
	1	2	3	4	
1	15.85	16.02	15.83	15.93	15.91
2	16.12	16	15.85	16.01	15.99
3	16	15.91	15.94	15.83	15.92
4	16.2	15.85	15.74	15.93	15.93
5	15.74	15.86	16.21	16.1	15.98
6	15.94	16.01	16.14	16.03	16.03
7	15.75	16.21	16.01	15.86	15.98
8	15.82	15.94	16.02	15.94	15.93
9	16.04	15.98	15.83	15.98	15.96
10	15.64	15.86	15.94	15.89	15.83
11	16.11	16	16.01	15.82	15.99
12	15.72	15.85	16.12	16.15	15.96
13	15.85	15.75	15.74	15.98	15.83
14	15.73	15.84	15.96	16.1	15.91
15	16.2	16.01	16.1	15.89	16.05
16	16.12	16.08	15.83	15.94	15.99
17	16.01	15.93	15.81	15.68	15.86
18	15.78	16.04	16.11	16.12	16.01
19	15.84	15.92	16.05	16.12	15.98
20	15.92	16.09	16.12	15.93	16.02
21	16.11	16.02	16	15.88	16
22	15.98	15.82	15.89	15.89	15.9
23	16.05	15.73	15.73	15.93	15.86
24	16.01	16.01	15.89	15.86	15.94
25	16.08	15.78	15.92	15.98	15.94

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{k} \bar{X}_i}{k}$$

$$\bar{\bar{X}} = \frac{398.7}{25}$$

$$\bar{\bar{X}} = 15.94$$

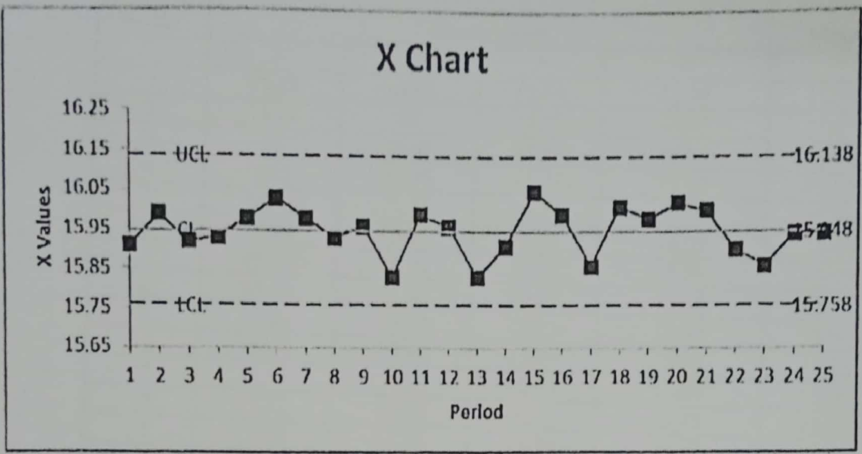


Figure 1

Comment: All the sample observations are lies between Upper and Lower control limits, so the process is under control according to $\bar{X}$ -Chart.

Creating the Tabular CUSUM Chart: These are several steps for creating a tabular CUSUM chart:

1. The data must be converted and plotted into two different CUSUM plot points:
   a) C_i^- values indicating the cumulative summation of values below the target
   b) b. C_i^+ values indicating the cumulative summation of values above the target
1. Identification of the target, μ_0
2. Selection of the positive shift in the mean to detect, msz_u
3. Selection of the negative shift in the mean to detect, msz_l
4. Selection of Decision parameter, H

First, the CUSUM plot point values for the steel I-beam example are calculated and found

Plotted Points

Following are the assumptions and calculations necessary for completing the CUSUM chart for the example:

1. μ_0 = Target Value = Actual Process mean = 15.95
2. $\hat{\sigma}$ = Estimated Standard Deviation = 0.0604
3. msz_u = Positive mean shift to detect (z) = 1.0σ
4. msz_l = Negative mean shift to detect (z) = 1.0σ

$$K_u = \text{Upper Reference} = \left(\frac{msz_u \times \hat{\sigma}_{LT}}{2}\right) = \left(\frac{1 \times 0.0604}{2}\right) = 0.0302$$

$$K_l = \text{Lower Reference} = \left(\frac{msz_l \times \hat{\sigma}_{LT}}{2}\right) = \left(\frac{1 \times 0.0604}{2}\right) = 0.0302$$

5. h = Decision Parameter = 5

6. H = Decision Interval = $h\sigma = 5 \times \sigma = 5 \times 0.0604 = 0.302$

Begin creating the CUSUM chart by calculating the Upper and Lower CUSUM plot points using this equation

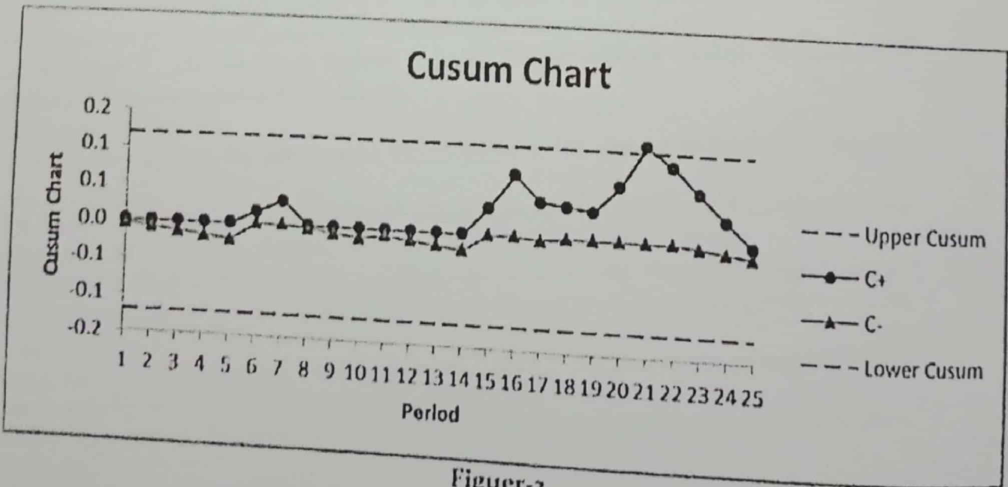
$C_i^+ = \max[0, x_i - (\mu_0 + K_0) + C_{i-1}^+]$

$C_i^- = \min[0, x_i - (\mu_0 - K_1) - C_{i-1}^-]$

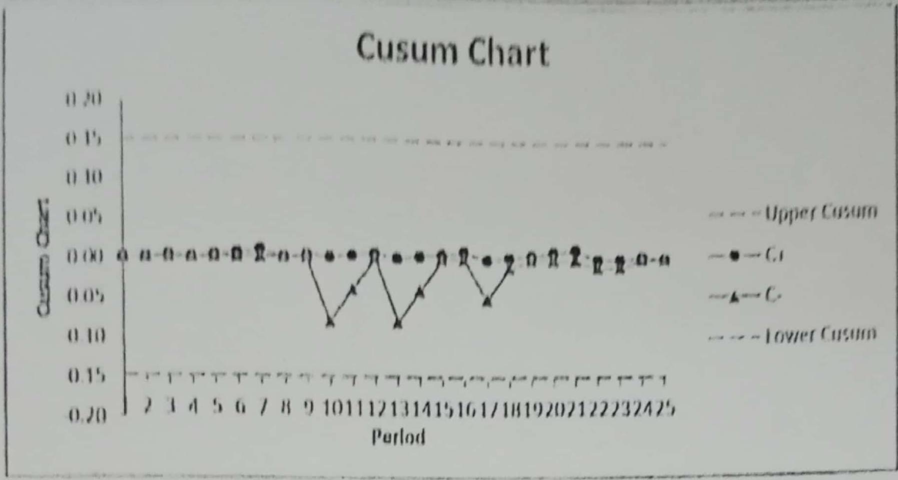
Once both the Upper and Lower CUSUM values have been calculated, they are plotted on the same chart

Table 3

Sample Number	Observations				Mean	C_i^+	C_i^-
	1	2	3	4			
1	15.85	16.02	15.83	15.93	15.91	0	-0.04
2	16.12	16	15.85	16.01	15.99	0	-0.0302
3	16	15.91	15.94	15.83	15.92	0	0
4	16.2	15.85	15.74	15.93	15.93	0	-0.02
5	15.74	15.86	16.21	16.1	15.98	0	0
6	15.94	16.01	16.14	16.03	16.03	0.05	0
7	15.75	16.21	16.01	15.86	15.98	0.0498	0
8	15.82	15.94	16.02	15.94	15.93	0	0
9	16.04	15.98	15.83	15.98	15.96	0	-0.02
10	15.64	15.86	15.94	15.89	15.83	0	0
11	16.11	16	16.01	15.82	15.99	0	-0.12
12	15.72	15.85	16.12	16.15	15.96	0.0098	0
13	15.85	15.75	15.74	15.98	15.83	0	0
14	15.73	15.84	15.96	16.1	15.91	0	-0.12
15	16.2	16.01	16.1	15.89	16.05	0	0
16	16.12	16.08	15.83	15.94	15.99	0.0698	0
17	16.01	15.93	15.81	15.68	15.86	0.0796	0
18	15.78	16.04	16.11	16.12	16.01	0	-0.09
19	15.84	15.92	16.05	16.12	15.98	0.0298	0
20	15.92	16.09	16.12	15.93	16.02	0.0296	0
21	16.11	16.02	16	15.88	16	0.0694	0
22	15.98	15.82	15.89	15.89	15.9	0.0892	0
23	16.05	15.73	15.73	15.93	15.86	0.009	-0.05
24	16.01	16.01	15.89	15.86	15.94	0	-0.04
25	16.08	15.78	15.92	15.98	15.94	0	0
						0	-0.01



Figuer-2



Figuer-3

Conclusion: One sided upper CUSUM C_i^+ verses sample number are plotted as blue color points and lower CUSUM C_i^- are plotted as light blue color points. The parameter values are k=0.5 and H= 5. This control scheme (for individual observations) gives an out of control signal at 20th observation and thus is very effective for small shifts. This control scheme gives an out of control signal for the 25 observations, indicating that the Shewhart control scheme is slow for detecting smaller shifts. Hence CUSUM control charts are more effective than Shewhart control charts.

References

1. Page ES (1954). Continuous Inspection Schemes. Biometrika 41(1-2) 100-115.
2. Montgomery DC (2001). Introduction to Statistical Quality Control, 3rdedition, Wiley, New York.
3. Duncan AJ (1974). Quality Control and Industrial Statistics, 4th Ed, Richard D. Irwin, Inc.,Hornewood, Illinois.
4. Hawkins DM (1981). A CUSUM for a Scale Parameter. *Journal of Quality Technology* 13(4) 228-231.
5. Hawkins DM and Olwell DH (1998). Cumulative sum chart and Charting for Quality Improvement. In Statistics for engineering and physical science Springer-Verlag Inc., New York, USA 16 pp. 247
6. Lucas JM and Crosier RB (1982a). Fast Initial Response for CUSUM Quality Control Schemes: Give your CUSUM A Head Start. Technometrics 24 (3) 199-205.
7. Lucas JM and Saccucci MS (1990). Exponentially Weighted Moving Average Control Schemes Properties and Enhancements. Technometrics 32 (1) 1-12.
8. De leval, Marc R., Francois K, Bull C, Brawn W.B and Spiegelhalter D. (1994). Analysis of a cluter of surgical failures. *The Journal of Thoracic and Cardiovascular Surgery*, 104, 914-924.
9. Gallus G., Mandelli C., Marchi M and Radaell G. (1986). On surveillance methods for congenitial malformations. *Statistics in Medicine*, 5, 565-571,
10. DOI: 10.1002/sim.4780050603
11. Grigg O, Farewell V.(2004). An overview of risk adjusted charts. *Journal of the Royal Statistical Socity*, 167, 523-539,
12. Grigg OA, Farewell VT, Spiegelhalter DJ. (2003). Use of risk-adjusted CUSUM and RSPRT charts for monitoring in medical contexts. *Statistical Methods in Medical Research*, 12,147-170.
13. Lim T,O, Soraya A, Ding L.M and Morad Z. (2002). Assessing doctor;s competence: application of CUSUM technique in monitoring doctors' performance. *International Journal for Quality in Health Care*, 14, 251-258.
14. Lovegrove J, Valencia O, Treasure T, Sherlaw-Jhonson C, Gallivian S. (1997). Monitoring the results of cardiac surgery by variable life display. *Lancet*, 316,1697-1700.
15. Montgomery D.C. (2009). Introduction to Statistical Quality Control.6th edn. New York: Jhon Wiley and Sons.