II BSC FORTH SEMESTER

PAPER-IV

PHYSICS STUDY MATERIAL

ELECTRICITY, MAGNETISM & ELECTRONICS

(w.e.f 2020-2021 Batch)

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Department of Physics
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Narsapur

II B.Sc.
FOURTH SEMESTER

PHYSICS

PAPER - 4

ELECTRICITY, MAGNETISM AND ELECTRONICS

(FOR MATHS COMBINATION)

QUESTION BANK

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UNIT I

1. ELECTROSTATICS

LONG ANSWER TYPE QUESTIONS

1. State and prove Gauss theorem in electrostatics.

[ANU 19, 18; AdNU 17; AU 18, 17; BRAU 18, 17; KU 18; RU 18; SKU 17; SVU 18; VSU 18; YVU 18, 17]

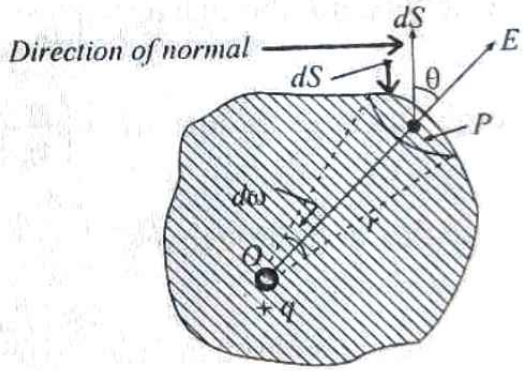
A. **Statement :** The total normal electric flux (ϕ_E) over a closed surface is $\left(\frac{1}{\epsilon_0}\right)$ times the total charge q enclosed within the surface.

Mathematically, $\phi_E = \oint \vec{E} \cdot d\vec{s} = \oint E ds \cos\theta = \left(\frac{1}{\epsilon_0}\right) q$

where ϵ_0 is the permittivity of the free space.

Proof : i) When the charge is within the surface :

1) Consider a closed surface of irregular shape with isolated charge q at the point 'O' as shown in figure.



2) Consider a small element of the surface of area ds at point p at a distance r.

3) The normal to the surface ds is represented by a vector $d\vec{s}$ which makes an angle θ with the direction of electric intensity $\vec{E}$ along op.

4) The electric flux through the area ds is $d\phi_E = \vec{E} \cdot d\vec{s} = E ds \cos\theta \rightarrow (1)$

where θ is angle between $\vec{E}$ and $d\vec{s}$.

5) From coulombs law, the electric intensity at P due to q is $E = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \rightarrow (2)$

6) Putting (2) in (1), we get $d\phi_E = \frac{q}{4\pi\epsilon_0} \frac{ds \cos\theta}{r^2}$

But $\frac{ds \cos\theta}{r^2}$ is the solid angle $d\omega$ subtended by ds at 'O'.

Hence $d\phi_E = \frac{q}{4\pi\epsilon_0} d\omega$.

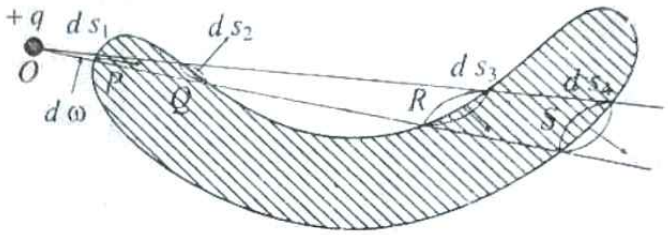
7) The total flux (ϕ_E) over a closed surface is $\phi_E = \oint d\phi_E = \oint \frac{q d\omega}{4\pi\epsilon_0}$

$$\phi_E = \frac{q}{4\pi\epsilon_0} \phi\, d\omega = \frac{q}{4\pi\epsilon_0} \times 4\pi \; [\because \oint d\omega = 4\pi]$$

$$\therefore \; \phi_E = \frac{q}{\epsilon_0}$$

Hence Gauss law is proved.

ii) **When the charge is outside the surface :**



1) Let a point charge $+q$ be placed at O outside the closed surface as shown in figure.
2) Now a cone of solid angle $d\omega$ from O cuts the surface area ds_1, ds_2, ds_3, ds_4 at P, Q, R and S respectively.
3) The electric flux at P through $ds_1 = \left(\frac{-q}{4\pi\epsilon_0}\right) d\omega$

   The electric flux at Q through $ds_2 = \left(\frac{+q}{4\pi\epsilon_0}\right) d\omega$

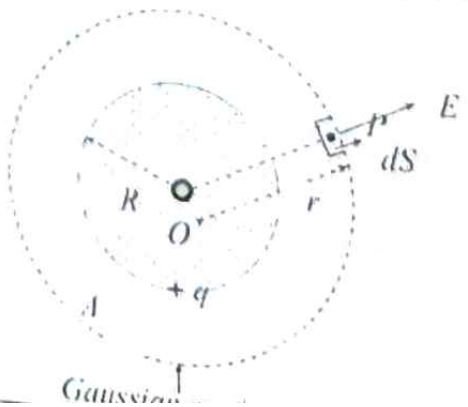
   The electric flux at R through $ds_3 = \left(\frac{-q}{4\pi\epsilon_0}\right) d\omega$

   The electric flux at S through $ds_4 = \left(\frac{+q}{4\pi\epsilon_0}\right) d\omega$

   [$\because$ electrix flux for outward normal is positive, and inward normal is negative]
4) $\therefore$ Total electric flux $= \frac{-q}{4\pi\epsilon_0} d\omega + \frac{q}{4\pi\epsilon_0} d\omega - \frac{q}{4\pi\epsilon_0} d\omega + \frac{q}{4\pi\epsilon_0} d\omega = 0$
5) Therefore total electric flux over the whole surface is zero when the charge is outside the surface.

2. Derive an expression for the electric field due to uniformly charged Non-conducting sphere. **[AdNU 17; AU 18, 17; RU 18, 17; SKU 18; SVU 17; YVU 17]**

A. Case (i) : **Electric field at a point outside a charged sphere :**



1) Consider a sphere A of radius R with centre 'O' as shown in figure.
2) Let a charge q be uniformly distributed over it.

3) Let P be an external point at a distance r from the centre of a charged sphere.
4) Let the electric field at P be E. Draw an imaginary sphere of radius r.
5) The point P lies on this Gaussian Surface.
6) According to Gauss's law, $\oint \vec{E} \cdot \vec{d}s = \frac{q}{\epsilon_0}$;$\Rightarrow \oint E\, ds \cos\theta = \frac{q}{\epsilon_0}$

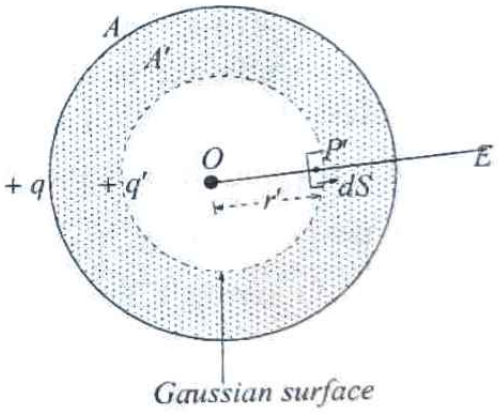
$$\Rightarrow E\oint ds = \frac{q}{\epsilon_0} \quad [\because \theta = 0]$$

$$\Rightarrow E4\pi r^2 = \frac{q}{\epsilon_0} \quad \left[\because \oint ds = A = 4\pi r^2\right] \quad \therefore\ E = \frac{q}{4\pi\,\epsilon_0\, r^2}\ \text{N/C}$$

Case (ii) : **Electric field at a point on the surface :** If the point P lies on the sphere, then $r = R$. Hence field intensity $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$ N/C

Case (iii) : **Electric field at a point inside the charged sphere :**

1) Let E be electric field intensity at P^1 which is inside the charged sphere at a distance r^1 from centre 'O'.
2) Draw the Gassian Surface with radius r^1 is shown in figure.
3) According to Gass's law $\oint E \cdot ds = \frac{q^1}{\epsilon_0}$



Charge enclosed with in the gaussian surface is given by $q^1 =$ volume enclosed by it × charge per unit volume.

$$q^1 = \frac{4}{3}\pi r^{1^3} \times p$$

$$\text{But } p = \frac{\text{Total charge}}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^3}$$

$$\therefore\ q^1 = \frac{4}{3}\pi r^{1^3} \times \frac{3q}{4\pi R^3}$$

$$q^1 = q\left(\frac{r^1}{R}\right)^3$$

$$\text{From Gauss law } E(4\pi r^{1^2}) = \frac{1}{\epsilon_0} q\left(\frac{r^1}{R}\right)^3$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qr^1}{R^3}$$

3. Define electric potential. Derive an expression for the potential due to a charged spherical conductor. **[ANU 18]**

or

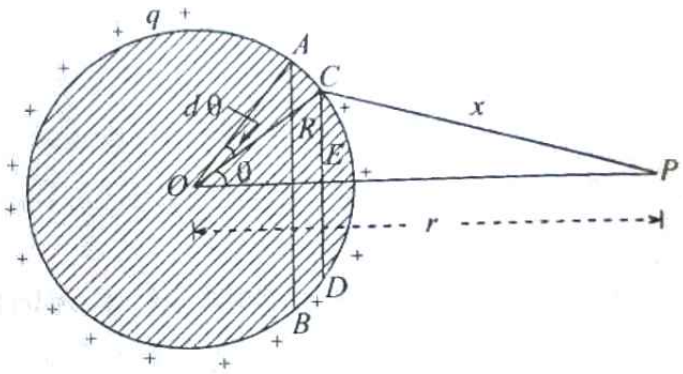
Derive an expression for the electric potential due to a uniformly charged spherical conductor at a point

a) Outside the sphere b) on the surface and c) inside the sphere

[ANU 17; AdNU 18; BRAU 18, 17; RU 18; VSU 17]

A. **Definition of Electric potential :** The workdone by a unit positive test charge from infinite to that point in an electric field is called electric potential.

Case (i) : **When point *P* lies outside the shell :**



1) Consider a charged spherical shell of radius R with centre 'O'. Let σ be the surface charge density.
2) Let the total charge q is distributed uniformly on the surface of sphere.
3) Draw two planes AB and CD very close to each other, perpendicular to OP. Now $ABCD$ is a ring.
4) Let $CP = x$, $\angle COP = \theta$ and $\angle AOC = d\theta$
5) Thickness of the ring, $AC = Rd\theta$ [$\because$ from $\angle AOC$]

   Radius of the ring, $CE = R\sin\theta$ [$\because$ from $\angle OEC$]

   Circumference of the ring $= 2\pi \times CE = 2\pi R\sin\theta$
6) Area of the ring $= 2\pi R\sin\theta \times Rd\theta$ [$\because$ A = circumference × thickness]

   $= 2\pi R^2 \sin\theta\, d\theta$
7) Charge on the ring $(dq) = 2\pi R^2 \sin\theta\, d\theta \times \sigma$ [$\because$ Area of ring × Surface density]
8) But $\sigma = \dfrac{q}{4\pi R^2}$; $\therefore$ $dq = 2\pi R^2 \sin\theta\, d\theta \times \dfrac{q}{4\pi R^2} = \dfrac{q\sin\theta\, d\theta}{2}$
9) The potential at p due to the charge on the ring at a distance x is $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{x}$

   (Every point of the narrow ring $ABCD$ is the same distance x from the point p)

   $dV = \dfrac{q\sin\theta\, d\theta}{8\pi\epsilon_0 x} \rightarrow (1)$
10) From figure, $x^2 = R^2 + r^2 - 2Rr\cos\theta$;

    On diffferentiating, $2x\,dx = 2Rr\sin\theta\, d\theta$

    $\therefore$ $\sin\theta\, d\theta = \dfrac{x\,dx}{Rr} \rightarrow (2)$
11) Writing (2) in (1), we have $dV = \dfrac{qx\,dx}{8\pi\epsilon_0 Rrx} = \dfrac{q\,dx}{8\pi\epsilon_0 Rr} \rightarrow (3)$

$\therefore$ Potential due to whole spherical shell, $V = \int_{r-R}^{R+r} dV = \int_{r-R}^{r+R} \frac{q\,dx}{8\pi\epsilon_0 Rr} = \frac{q}{8\pi\epsilon_0 Rr}[x]_{r-R}^{r+R}$

$= \frac{q}{8\pi\epsilon_0 Rr}[r + R - r + R] = \frac{q}{8\pi\epsilon_0 Rr} \times 2R \qquad \therefore\ V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

Case (ii) : When P lies on the surface :

In this case, $r = R$.

$\therefore$ Potential at the surface $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Case (iii) : When P lies inside the shell :

1) In this case, the limits of integration becomes $x = R - r$ and $x = R + r$.

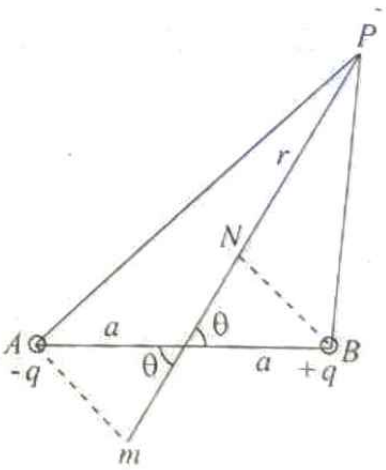
2) Hence $V = \int_{R-r}^{R+r} \frac{q\,dx}{8\pi\epsilon_0 Rr} = \frac{q}{8\pi\epsilon_0 Rr}\int_{R-r}^{R+r} dx = \frac{q}{8\pi\epsilon_0 Rr}[R + r - R + r]$

$\therefore\ V = \frac{q}{8\pi\epsilon_0 Rr} \times 2r = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Thus the potential at an internal point is the same as that on the surface.

Derive an expression for the potential due to a dipole.

When two charges equal in magnitude but opposite in sign are separated by a small distance the system is called an electric dipole.



Let us consider two electric charges $+q$ and $-q$, which are separated by a distance $2a$ as shown in the fig. These two electric charges form a dipole. The dipole moment of this dipole is $2aq$.

To determine the potential at a point P at a distance r from the centre O of the dipole, let us join the points A and B with P. Join P with the centre of the dipole O and extend the line. Draw perpendiculars AM and BU. P makes an angle θ with the dipole.

Now potential at P due to charge $+q$ at B

$$V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{BP}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{NP}\right)$$

Potential at P due to charge $-q$ at A

$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{-q}{AP}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{-q}{MP}\right)$$

$\therefore$ Resultant potential at P is given by

$$V = V_B - V_A = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{NP} - \frac{q}{MP}\right] \rightarrow (1)$$

From right angled triangle AMO,

$OM = AO \cos\theta = a\cos\theta$

$MP = MO + OP = a\cos\theta + r = r + a\cos\theta$

From right angled triangle ONB

$ON = OB\cos\theta = a\cos\theta$

$NP = OP - ON = (r - a\cos\theta)$

substituting these values in equation (1) we get,

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r - a\cos\theta} - \frac{q}{r + \cos\theta}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r + a\cos\theta - r + a\cos\theta}{r^2 - a^2\cos\theta}\right] = \frac{q}{4\pi\epsilon_0}\left(\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

$$= \frac{P\cos\theta}{4\pi\epsilon_0(r^2 - a^2\cos^2\theta)} \qquad (\because 2aq = p = \text{dipole moment})$$

As $r >> a$, $a^2\cos^2\theta$ term may be neglected in comparison with r^2.

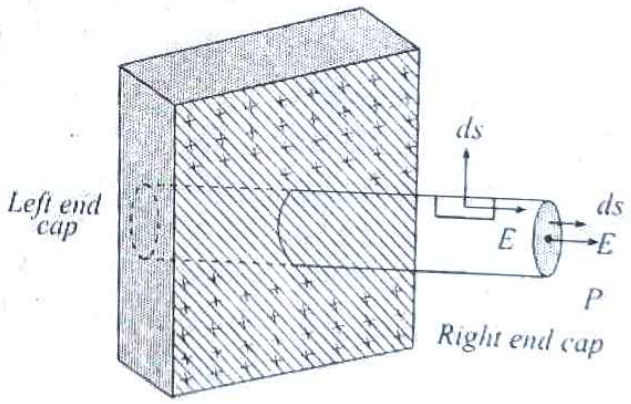
Hence $\boxed{V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}}$

SHORT ANSWER TYPE QUESTIONS

1. Derive an expression for electric field due to infinite sheet of charge.

[ANU 19, 17; AdNU 18; SVU 18; VSU 17; YVU 18]

A. 1) Consider a portion of thin, non conducting, infinite sheet of charge. Let σ be the surface charge density i.e., charge per unit area.



2) Let P_1 and P_2 are two symmetrical points on either side of a thin plate at a distance 'r'.

3) Imagine cylindrical Gaussion Surface with area of crossection A which is passing through P_1 and P_2.

4) By symmetry E is everywhere perpendicular to the plane.

5) The contribution of electric flux due to curved surface of Gaussian cylinder is zero because E and ds are perpendicular to each other.

6) But end caps of Gaussian Surface contribute electric flux, because E and ds are parallel to each other. The total electric flux by the end caps is given by

$$\phi_E = \oint E \cdot ds + \oint E \cdot ds$$

$$= \oint E ds + \oint E ds \quad [\because \theta = 0]$$

$$= EA + EA = 2EA$$

7) According to Gauss's law,

$$\phi_E = 2EA = \frac{Q}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0} \quad [\because \sigma = \frac{Q}{A}]$$

$$E = \frac{\sigma}{2\epsilon_0}$$

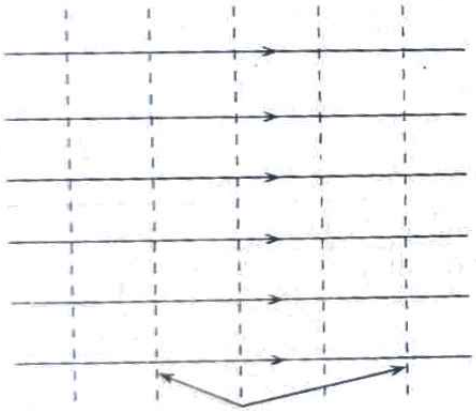
This is the expression for electric field of a thin, non conducting, infinite sheet.

2. Explain briefly equipotential surface. **[KU 18; SKU 18; SVU 17]**

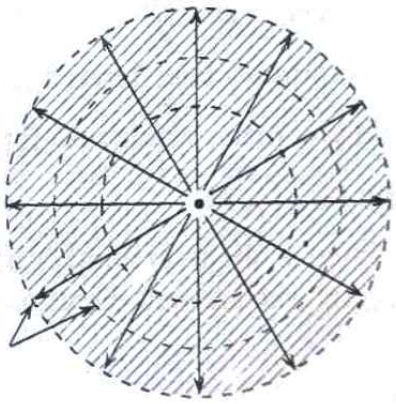
A. **Equipotential surface :** The locus of all points which have the same electric potential is called equipotential surface.

Explanation :

1) In case of uniform field, where the lines of forces are straight and parallel, the equipotential surfaces are planes perpendicular to the lines of force as shown in figure (a). As the potential difference between any two points on the equipotential surface is zero. Hence no work is done intaking a charge from one point to another point.



(a)



(b)

2) The equipotential surfaces are a family of concentric spheres for a sphere of charge or a point charge as shown in figure (b).

3) The important points in case of equipotential surface are

i) The electric field along the equipotential surface is zero.

ii) The electric field is normal to the surface.

iii) The workdone in moving a charge on the equipotential surface is zero.

iv) When the charge is in infinite, the equipotential surface is plane.

v) The equipotential surface act as wave-fronts in optics.

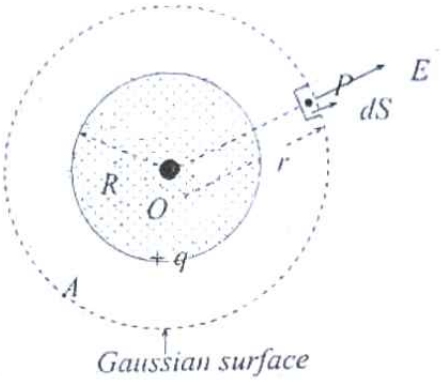
3. What is Gaussion Surface. Obtain coulombs law from Gauss's law. **[ANU J16]**

A. **Gaussian Surface :** The surface with same electric intensity at every point is called Gaussian Surface.

Coulombs law from Gauss law :

1) Consider an isolated point charge q as shown in figure.
2) Construct an imaginary surface. Let E be electric intensity at any point on surface at a distance r from q.
3) According to Gauss's law $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$; $\Rightarrow \oint E ds \cos\theta = \frac{q}{\epsilon_0}$ $[\because \theta = 0]$

$$E \oint ds = \frac{q}{\epsilon_0}$$



4) But $\oint ds = 4\pi r^2 \Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0}$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

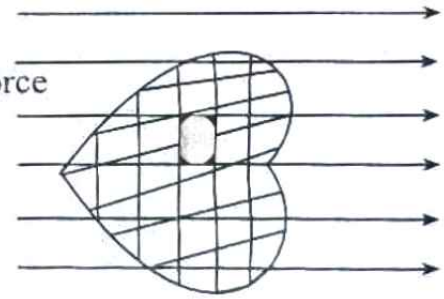
5) If q_0 is the second charge placed on the spherical surface, it experiences a force $F = Eq_0$

$$\therefore F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

This is coulomb's law, derived from Gauss's law.

4. What do you mean by electric flux? Explain **[SKU 17]**

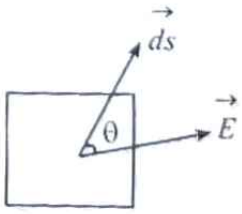
A. **Electric flux (ϕ_E) :**



It is defined as the total number of electric lines of force crossing the surface in a direction normal to the surface.

Explanation :

Now imagine an arbitrary closed surface placed in the field. Let the surface is divided into number of elementary squares. Each square of surface area ds. The scalar product $E \cdot ds$ is defined as the electric flux for the surface. The total flux through the entire surface is given by



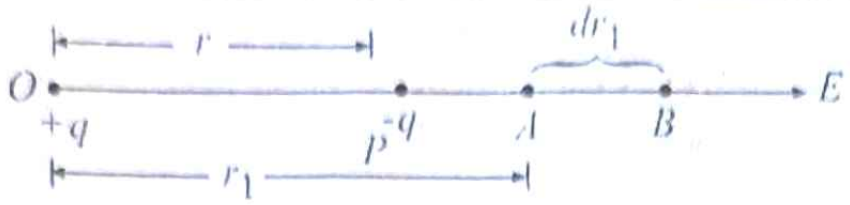
$$\phi_E = \oint \vec{E} \cdot d\vec{s} = \oint E\, ds \cos\theta$$

Where θ is angle between $\vec{E}$ and $\vec{ds}$

For a closed surface ϕ_E is taken as positive, it the lines of force point outward and negative, it they point inward.

5. Define potential and find potential due to a point charge.

A. **Potential Definition :** It is the work done in moving a unit positive charge from infinity to a point in the field.



Consider a positive charge q at '0', due to which an electric field is produced.

Consider a point P at a distance r from 'O'. Let $\vec{OP}$ be $\vec{r}$.

At some other point A, the electrostatic force on unit positive charge is $\frac{q \times 1}{4\pi\epsilon_0 r_1^2}\hat{r}_1$,

where r_1 is the unit charge along $\vec{OP}$.

At work done against the electrosatic force in bringing the unit positive charge force r_1 to $r_1 + dr_1$ is $dw = \frac{-q}{4\pi\epsilon_0 r_1^2} \cdot dr_1$

Total work done by the external force in bring the unit +ve charge from infinity to a point P on OP is obtained by integrating the above equation from α to r.

$$\therefore\ W = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 r_1^2}\, dr_1 = \left[\frac{q}{4\pi\epsilon_0 r_1}\right]_{\infty}^{r} = \frac{q}{4\pi\epsilon_0 r}$$

This gives the potential at P due to point charge q

$$\therefore\ V = \frac{q}{4\pi\epsilon_0 r}.$$

6. Derive an expression for the intensity of electric field to a point charge.

A. **Intensity of electric field (*E*) :** It is defined as the force experienced by a unit positive charge placed at that point in an electric field. $E = \frac{F}{q_0}$, where q_0 is positive test charge.

Expression for the intensity due to a point charge : Let us consider a point charge q placed at O in vaccum. Around this charge electric field is produced.

Consider a point P at a distance 'r' from 'O', Let q_0 be the positive test charge. According to coulomb's law, force between two charges is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \rightarrow (1)$$

$$\frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Intensity of electric field at that point $P = E = \frac{E}{q_0}$

$$\therefore\ E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}.$$

PROBLEMS

*1. If a point charge q is placed at the centre of a cube, what is the flux linked (a) with the cube ? (b) with each face of the cube ?

Sol : a) Charge at the centre of a cube = q

According to Gauss's law, the total flux $= \frac{1}{\epsilon_0} \times$ Charge $= \frac{q}{\epsilon_0}$

b) Number of faces by cube = 6

According to Gauss's law, the total flux due to six faces $= \frac{1}{\epsilon_0} q$

$\therefore$ The flux due to one face is $= \frac{q}{6\epsilon_0}$.

2. a) How much electric flux will come out through a surface $S = 10j$ kept in electro static field $E = 2i + 4j + 7k$?

b) If 1 coulomb charge is placed at the centre of a cube of side 10 cm. Calculate the flux coming out of any face of the cube.

Sol : Given $E = 2i + 4j + 7k$; $S = 10j$

a) $\therefore$ Electric flux $\phi = E.S = (2i + 4j + 7k) \cdot 10j$

$= 40$ units

b) Net flux $\phi = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0}$ $[\because q = 1C]$

The electric flux through each face of the cube

$$= \frac{1}{6} \times \frac{1}{\epsilon_0} = \frac{1}{6\epsilon_0} = 1{\cdot}884 \times 10^{-10}\ \text{N} - \text{m}^2/\text{C}^2.$$

3. The electric field in a region is radially outward and varies with distance r as $E = 250r\ V/m^2$. Calculate the charge contained in a sphere of radius 0·2 m centred at the origin.

Sol : According to Gauss's law, we have $\phi = E_{\text{Surface}} S = \frac{q}{\epsilon_0}$

$\Rightarrow q = E_{\text{Source}} S\epsilon_0 = E_{\text{Surface}}\, 4\pi r^2 \epsilon_0$

But $E_{\text{Surface}} = 250r = 250 \times 0{\cdot}2 = 50\ \text{V/m}^2$

$\therefore q = 50 \times \frac{1}{9 \times 10^9} \times (0{\cdot}2)^2$ $[\because \frac{1}{4\pi\epsilon_0} = 9 \times 10^9]$

$= 2{\cdot}22 \times 10^{-10}$ c.

4. A charge of 4×10^{-8} c is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of a radius 5 cm. Find the electric field at a point 2 cm away from the centre.

Sol : Given $q = 4 \times 10^{-8}$ c; $r = 2$ cm $= 2 \times 10^{-2}$ m

According to Gauss's law, $\phi = \frac{q}{\epsilon_0} = ES \Rightarrow q = ES\,\epsilon_0 = E\,4\pi r^2 \epsilon_0$

$$\therefore\ E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} = 9 \times 10^9 \times \frac{4 \times 10^{-8}}{(2 \times 10^{-2})^2} = 9 \times 10^5 \text{ N/C.}$$

***5.** What is the electric potential at the surface of nucleus of gold ? The radius of nucleus is $6{\cdot}6 \times 10^{-15}$ m. The atomic number of gold is 79.

[KU J16]

Sol : Given $Z = 79$; $q = 79 \times (1{\cdot}6 \times 10^{-19})$ c $(\because q = ne)$

$$r = 6{\cdot}6 \times 10^{-15} \text{ m; } V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r} = \frac{9 \times 10^9 \times (79 \times 1{\cdot}6 \times 10^{-19})}{6{\cdot}6 \times 10^{-15}}$$

$\therefore\ V = 17 \times 10^6$ volts.

6. A point charge is placed at a point A. The charge is $1{\cdot}5 \times 10^{-8}$ c. What is the radius of equipotential surface having a potential of 30 v ?

Sol : Given that $q = 1{\cdot}5 \times 10^{-8}\ C$; $V = 30$ volt

We know that $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$$30 = 9 \times 10^9 \times \frac{1{\cdot}5 \times 10^{-8}}{r} \text{ or } r = \frac{9 \times 10^9 \times 1{\cdot}5 \times 10^{-8}}{30} = 4{\cdot}5 \text{ m.}$$

7. A sperical oil drop, radius 10^{-4} cm has on its at a certain time, a total charge of 40 electrons. Calculate the energy that would be required to place on addition electron on drop. Charge on an electron $= 1{\cdot}6 \times 10^{-19}$ c.

Sol : Charge on oil drop $q = Ze = 40 \times 1{\cdot}6 \times 10^{-19} = 64 \times 10^{-19}$ c

Potential of the oil drop $V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r} = 9 \times 10^9 \times \frac{64 \times 10^{-19}}{(10^{-4} \times 10^{-2})} = 576 \times 10^{-4}$ volt

$\therefore$ Energy required $= V \times$ Charge of an electron

$= 576 \times 10^{-4} \times 1{\cdot}6 \times 10^{-19} = 921{\cdot}6 \times 10^{-23} J$

***8.** At distances of 5 cm and 10 cm from the surface, the potentials are 600 v and 420v. Find the potential on its surface.

Sol : Let r be the radius of the sphere. Then $V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r} \rightarrow (1)$

Given that $V_1 = \frac{1}{4\pi\epsilon_0} \times \frac{q}{(r + 0{\cdot}05)} = 600 \rightarrow (2)$

and $V_2 = \frac{1}{4\pi\epsilon_0} \times \frac{q}{(r + 0{\cdot}1)} = 420 \rightarrow (3)$

Dividing equation (2) by equation (3), we get

$$\frac{r + 0{\cdot}1}{r + 0{\cdot}05} = \frac{600}{420} \text{ or } r = \frac{0{\cdot}2}{3} \text{ m}$$

Dividing equation (1) by equation (2), we get

$$\frac{V}{600} = \frac{r + 0{\cdot}05}{r} = \frac{\frac{0{\cdot}2}{3} + 0{\cdot}05}{\left(\frac{0{\cdot}2}{3}\right)}$$

$V = 1050$ volt.

*9. A spherical drop of water carrying a charge of 3×10^{-10}c has potential of 500 volts at its surface. What is the radius of the drop ? If two such drops of the same charges and radius combine to form a single spherical drop, what is potential at the surface of the new drop ?

Sol : Given that $V = 50$; $q = 3 \times 10^{-10}$ c

The potential of the sphere is given by $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$$500 = 9 \times 10^9 \times \frac{3 \times 10^{-10}}{r} \Rightarrow r = 0{\cdot}0054 \text{ m} = 0{\cdot}54 \text{ cm}$$

Volume of the drop $= \frac{4}{3}\pi r^3$; Volume of 2 drops $= \frac{8}{3}\pi r^3$

Let r^1 be the radius of the new drop, then

$$\frac{4}{3}\pi r^{1^3} = \frac{8}{3}\pi r^3 \text{ or } r^1 = 2^{\frac{1}{3}} r$$

$$\text{Charge on new drop} = \frac{2q}{4\pi\epsilon_0 r^1} = \frac{(6 \times 10^{-10})(9 \times 10^9)}{2^{1/3} \times 0{\cdot}0054} = 794 \text{ volts.}$$

10. Two spheres of radius 6 cm have charges 10^{-8} coulomb and -3×10^{-8} coulomb. Find the potential of the 2 spheres and potential at the mid point of the centres if the distance between centres is 2 metre.

Sol : Given that $r = 6$ cm $= 6 \times 10^{-2}$ m

Potential of the sphere with charge $10^{-8}c = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

$$= (9 \times 10^9)\left(\frac{10^{-8}}{6 \times 10^{-2}}\right) = 1{\cdot}5 \times 10^3 \text{ volts}$$

Potential of the sphere with charge $-3 \times 10^{-8}c$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right) = (9 \times 10^9)\frac{(-3 \times 10^{-8})}{6 \times 10^{-2}} = -4{\cdot}5 \times 10^3 \text{ volts}$$

Potential at the mid point of line of centre

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{10^{-8}}{1} - \frac{3 \times 10^{-8}}{1}\right] = (9 \times 10^9)(-2 \times 10^{-8}) = -180 \text{ volts.}$$

11. A thin spherical shell of metal has a radius of 0·25 m and carries a charge of 0·2 micro-coulomb. Calculate the electric intensity at a point (i) inside the shell, (ii) just outside the shell and (iii) 3·0 metre from the centre of the shell.

Sol : Given that $q = 0{\cdot}2$ micro coulomb $= 0{\cdot}2 \times 10^{-6}$c

$$R = 0{\cdot}25 \text{ m}; r = 3{\cdot}0 \text{ m}$$

i) Electric field intensity inside the thin shell

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{0}{r^2} \quad [\because q = 0 \text{ inside thin shell}]$$

ii) Electric field intensity just outside the shell

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = 9 \times 10^9 \times \frac{0{\cdot}2 \times 10^{-6}}{(0{\cdot}25)^2} = 2{\cdot}88 \times 10^4 \text{ N/C}$$

iii) Electric field intensity at 3·0 m from the centre of the shell

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{0{\cdot}2 \times 10^{-6}}{(3)^2} = 200 \text{ N/C.}$$

12. A gold foil with surface density 160 mg/cm^2 kept on a charged, conducting surface just rises as its weight is balanced by the electric force. Calculate the surface charge density on the conducting surface.

Sol : Given that mass/area = 160 mg/cm^2 $= \dfrac{160 \times 10^{-3} \times 10^{-3}}{(10^{-2})^2 m^2}$ kg $= 160 \times 10^{-2}$ kg/m^2.

$\epsilon_0 = 8{\cdot}854 \times 10^{-12}; g = 9{\cdot}8$ m/s^2

Force on conducting surface F = wt. per unit area balanced by electric force

$$\frac{\sigma^2}{2\epsilon_0} = \frac{mg}{A}; \quad \frac{\sigma^2}{2\epsilon_0} = 160 \times 10^{-2} \times 9{\cdot}8$$

$$\Rightarrow \sigma^2 = 160 \times 10^{-2} \times 9{\cdot}8 \times 2 \times 8{\cdot}854 \times 10^{-12}$$

$$\therefore \sigma = 1{\cdot}666 \times 10^{-5} \text{ c/m}^2.$$

13. A hollow metallic sphere of 0·1 m radius is given a charge of 10 micro-coulomb. Calculate the potential on the surface of the sphere. **[KU J15]**

Sol : Given that $R = 0{\cdot}1$ m $= 10^{-1}$ m ; $q = 10 \times 10^{-6}$c $= 10^{-5}$ c

$\therefore$ Potential on the surface of hollow metal sphere,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = 9 \times 10^9 \times \frac{10^{-5}}{10^{-1}} = 9 \times 10^5 \text{ V.}$$

***14.** Eight charged drops of water, each of radius 1 mm and having a charge of 10^{-10}c, combine to form a bigger drop. Determine the potential of bigger drop.

Sol : Given that radius of water drop $r = 1$ mm $= 10^{-3}$ m

UNIT I

2. DIELECTRICS

LONG ANSWER TYPE QUESTIONS

1. Define D, E and P. Establish the relation between D, E and P. Hence deduce the relation between dielectric constant and susceptibility.

[ANU 19, 17; AdNU 18, 17; AU 18, 17; BRAU 18, 17; KU 18, 17; RU 17; SKU 17; SVU 18, 17; VSU 18; YVU 18, 17]

A. The three electric vectors are 1) Electric intensity 2) Dielectric polarisation 3) Electric displacement.

1) **Electricity intensity (E) :**

The force experienced by a unit positive charge placed in a electric field is called electric intensity at that point.

i.e., $E = \dfrac{q}{K \in_0 A}$

2) **Dielectric polarisation (P) :**

The electric dipole moment per unit volume is called as dielectric polarisation. It is numerically equally to induced surface charge (q') per unit area (A).

i.e., $P = \dfrac{q'}{A}$

3) **Dielectric displacement (D) :**

Charge per unit area is called dielectric displacement D. or It is defined as a vector quantity whose surface integral over any charge surface, the flux of D, is equal to the free charge only within the surface.

i.e., $\oint D\, dS = q \Rightarrow D = \dfrac{q}{A}$

Relation between D, E and P :

1) When a dielectric slab is placed between the plates of a parallel plate capacitor, the medium is polarised.
2) Now induced surface charges appear. The charge is negative on the surface nearer the positive plate of the capacitor and positive charge nearer the negative plate.
3) Let q^1 be the induced surface charge.
4) Now the charge q on the plate of the capacitor and induced surface charge q^1 are related as

$$\frac{q}{K \in_0 A} = \frac{q}{\in_0 A} - \frac{q^1}{\in_0 A} \rightarrow (1)$$

$$\frac{q}{\in_0 A} = \frac{q}{K \in_0 A} + \frac{q^1}{\in_0 A}$$

$$\frac{q}{A} = \epsilon_0\left(\frac{q}{K\,\epsilon_0 A}\right) + \left(\frac{q^1}{A}\right) \rightarrow (2)$$

5) But $\frac{q}{K\,\epsilon_0 A} = E$; $\frac{q^1}{A} = P$ and $D = \frac{q}{A}$

$$\therefore \frac{q}{A} = \epsilon_0 E + P$$

Hence $D = \epsilon_0 E + P \rightarrow (3)$

This is the relation between D, E and P.

Relation between dielectric constant and susceptibility :

1) We know that $E = \frac{q}{K\,\epsilon_0 A}$ where K is dielectric constant

$$\frac{q}{A} = K\,\epsilon_0 E \Rightarrow D = K\epsilon_0 E \rightarrow (4) \quad [\because D = \frac{q}{A}]$$

$$\therefore D = \epsilon E \quad [\because \epsilon = K\,\epsilon_0]$$

2) Putting the value of (4) in equation (3), we get

$$K\,\epsilon_0 E = \epsilon_0 E + P$$

$$P = K\,\epsilon_0 E - \epsilon_0 E = \epsilon_0 (K-1)E$$

$P = \chi E$ where $\chi = \epsilon_0 (K-1) =$ susceptibility

$$\chi = \epsilon_0 (K-1) = K\,\epsilon_0 - \epsilon_0 = \epsilon - \epsilon_0 \Rightarrow \epsilon = \chi + \epsilon_0$$

3) Now $K = \frac{\epsilon}{\epsilon_0} = \frac{\epsilon_0 + \chi}{\epsilon_0} \quad \therefore \quad K = 1 + \frac{\chi}{\epsilon_0}$

This is the relation between dielectric constant and susceptibility.

2. State and prove boundary conditions at the dielectric surface. **[ANU M16; AU 18]**

A. The rules governing the behaviour of ***E*** and ***D*** at the boundary between two dielectrics are known as boundary conditions. There are 2 boundary conditions.

1) **First boundary condition :**

The normal component of electric displacement ***D*** is the same or continuous across the charge free boundary between two dielectrics.

Explanation :

1) Let $A'\,B'$ be the surface (boundary) separating two media of absolute permitivities ϵ_1 and ϵ_2.
2) Imagine a small pill box (small cylinder) $A'\,B'$ having their surfaces A' and B' parallel to AB. i.e., A', B' are very close to the boundary.
3) If $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are the dielectric displacements in the two media, their normal components to the surface are $D_1 \cos\theta_1$ and $D_2 \cos\theta_2$. where θ_1, θ_2 are the angles which the displacement vectors make with the normal to the surface in the two media.

$$V = \frac{q}{\epsilon_0 A}(d-t) + \frac{q}{K\epsilon_0 A}\cdot t$$

$$= \frac{q}{\epsilon_0 A}\left(d - t + \frac{t}{K}\right) = \frac{q}{\epsilon_0 A}\left(d - t\left(1 - \frac{1}{K}\right)\right) \rightarrow (4)$$

The capacitance of the capacitor is given by

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\epsilon_0 A}\left(d - t\left(1 - \frac{1}{K}\right)\right)} = \frac{\epsilon_0 A}{\left[d - t\left(1 - \frac{1}{K}\right)\right]} \quad \therefore C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)}$$

When the dielectric is completely filled, then $t = d$. In this case, the capacitance is given by

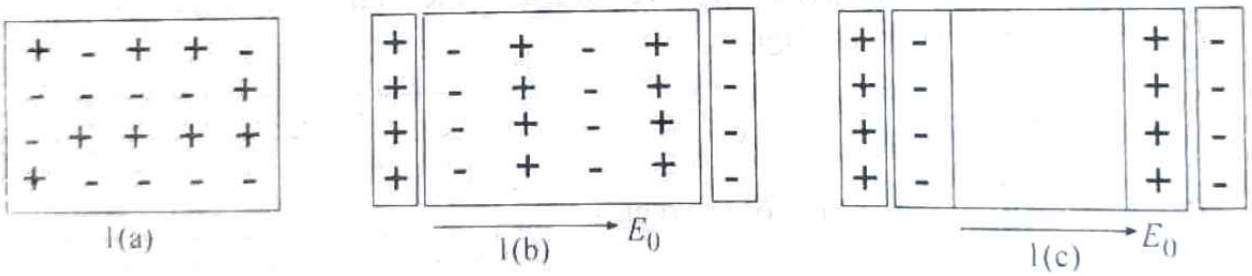
$$C = \frac{\epsilon_0 AK}{Kd - Kd + d} = \frac{\epsilon_0 AK}{d}.$$

SHORT ANSWER TYPE QUESTIONS

1. Explain the effect of electric field on non-polar and polar dielectrics.

A. **Non-Polar dielectric in Electric field :** The dielectrics which are polarised only when they are placed in an electric field are called non-polar dielectrics.

When a non-polar dielecric slab is placed in electric field, the centre of a positive charge is pulled towards the negative plate of the condenser and vice versa. Thus, the net effect of the applied field is to separate the positive charges from the negative charges. This effect is known as polarisation of dielectric.



When a non-polar dielectric is out of an external electric field the positive and negative charges are arranged randomnly as shown in the fig (a).

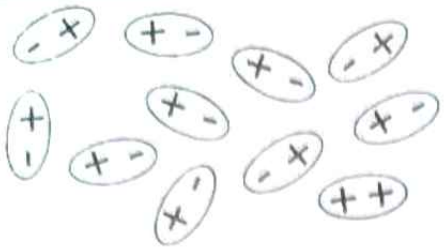
When this non polar dielectric is placed in the electric field of intensity E_0 surface charges appear as shonw in fig (b).

The induced surface charges appear in such a way, as shown in fig (c), that the electric field set up by them (E^1) opposes the external electric field E_0. The resultant field E in dielectric is the vector sum of E^1 and E_0. $E = E_0 - E^1$.

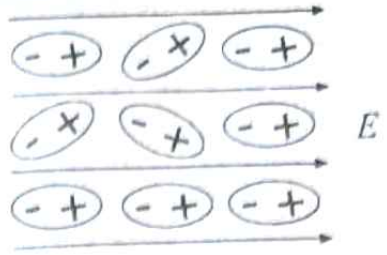
Thus, if the dielectric is placed in an electric field, induced surface charges appear which tend to weaker the original field within the dielectric.

Polar dielectric in Electric field :

The dielectrics which have permanent dipole moments with their random orientations as shown in the fig (a).



2(a)



2(b)

In the presence of an electric field, the partial alignment of dipoles takes place as shown in the fig 2(a). As the molecules are always in thermal agitation, the alignment will never be perfect. The alignment increases with the increase of electric field. The dipole moment of a polar molecule in an electric field will be $P_p + P_i$, where P_p is the permanent dipole moment, and P_i is the induced dipole moment.

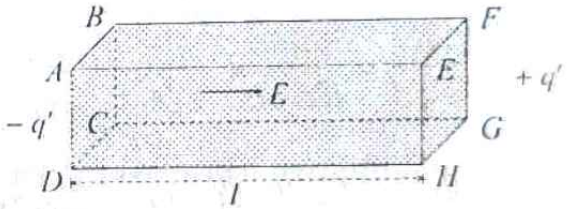
2. Define polarisation and show that it is equal to the charge density. **[BRAU 18]**

A. **Dielectric polarisation :**

The electric dipole moment per unit volume is called as dielectric polarisation P.

Charge density : Induced charge (q') per unit area is called charge density. i.e., charge density $= \frac{q'}{A}$

Explanation :



1) Suppose a dielectric slab of area of cross-section A and length l is placed in an electric field as shown in fig.
2) Let the induced charges on farces $ABCD$ and $EFGH$ be $-q'$ and $+q'$ respectively. The dipole moment p will be $q'l$. As volume of the slab is Al, hence the electric dipole moment per unit volume i.e., dielectric polarisation P is given by

$$P = \frac{q'l}{Al} = \frac{q'}{A}$$

3) Hence dielectric polarisation is numerically equal to the surface charge density.

3. Define dielectric constant and electric susceptibility. **[AdNU 18, 17; SKU 18; YVU 18]**

A. **Dielectric constant (K) :**

The ratio of capacity of a capacitor with dielectric to capacity of a capacitor without dielectric is called dielectric constant,

i.e $K = \frac{C}{C_0}$ or in the other words $K = \frac{V_0}{V} = \frac{F_0}{F} = \frac{\epsilon}{\epsilon_0}$

Electric susceptibility (χ) :

The ratio of polarisation vector to the electric intensity in the dielectric is called Electric susceptibility. i.e., $\chi = \frac{P}{E}$

Relation between K and χ :

$$\chi = \epsilon_0 (K - 1)$$

$$\chi = K\epsilon_0 - \epsilon_0$$

$$\chi = \epsilon - \epsilon_0$$

$$\epsilon = \chi + \epsilon_0$$

$$K = \frac{\epsilon}{\epsilon_0} = \frac{\chi + \epsilon_0}{\epsilon_0} = 1 + \frac{\chi}{\epsilon_0}$$

4. Derive the relation between D, E and P.

A. **Relation between D, E and P :**

1) When a dielectric slab is placed between the plates of a parallel plate capacitor, the medium is polarised. Now induced surface charges appear.
2) The charge is negative on the surface nearer the positive plate of the capacitor and positive charge nearer the negative plate.
3) Let q^1 be the induced surface charge.
4) Now the charge q on the plate of the capacitor and induced surface charge q^1 are related as

$$\frac{q}{K\epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q^1}{\epsilon_0 A} \rightarrow (1)$$

$$\frac{q}{\epsilon_0 A} = \frac{q}{K\epsilon_0 A} + \frac{q^1}{\epsilon_0 A}$$

$$\frac{q}{A} = \epsilon_0 \left(\frac{q}{K\,\epsilon_0 A}\right) + \left\{\frac{q^1}{A}\right\} \rightarrow (2)$$

5) But $\frac{q}{K\,\epsilon_0 A} = E$; $\frac{q^1}{A} = P$ and $D = \frac{q}{A}$

$$\therefore \frac{q}{A} = \epsilon_0 E + P$$

Hence $D = \epsilon_0 E + P \rightarrow (3)$

This is the relation between D, E and P.

PROBLEMS

1. The area of the plate of a parallel plate condenser is 100 sq. cm. The distance between plates is 1 cm. A potential difference of 100 volts is applied. A slab of thickness 0·5 cm and dielectric constant 7 is placed between plates. Calculate the values of E, D, P in air and dielectric.

Sol : Given that $A = 100$ sq. cm $= 10^2 \times 10^{-4}$ sq. m $= 10^{-2}$ sq. m ; $d = 1$ cm $= 10^{-2}$ m

$V = 100$ volts; $t = 0{\cdot}5$ cm $= 0{\cdot}5 \times 10^{-2}$ m; $K = 7$

In air : $E = \frac{V}{d} = \frac{100}{10^{-2}} = 10^4$ V/m

$D = \epsilon_0 E = 8{\cdot}9 \times 10^{-12} \times 10^4 = 8{\cdot}4 \times 10^{-8}$ c/m^2

Polarisation $P = 0$

In dielectric :

Permittivity $\epsilon = K\epsilon_0 = 7 \times (8{\cdot}9\ 10^{-12}) = 62{\cdot}3 \times 10^{-12}$

$$D = 8{\cdot}9 \times 10^{-8} \text{ c/m}^2$$

$$E = \frac{D}{\epsilon} = \frac{8{\cdot}9 \times 10^{-8}}{62{\cdot}3 \times 10^{-12}} = 1429 \text{ V/m}$$

Polarisation $P = D - \epsilon_0 E = (8{\cdot}9 \times 10^{-8}) - (8{\cdot}9 \times 10^{-12})(1429)$

$$= 7{\cdot}628 \times 10^{-8} \text{ c/m}^2$$

2. The electric susceptibility of a medium is 948×10^{-11}. Calculate the permeability (or absolute permeability) and relative permeability.

Sol : Given that $\chi = 948 \times 10^{-11}$

The relative permeability $\mu_r = 1 + \chi = 1 + (948 \times 10^{-11})$

Permeability of the medium $\mu = \mu_r \mu_0$

$= [1 + 948 \times 10^{-11}] \times (4\pi \times 10^{-7}) \approx 4\pi \times 10^{-7}$

3. The dielectric constant of water is 78. Calculate its electrical permittivity.

Sol : Given that $K = 78$ **[ANU 2019]**

$$\epsilon = K\epsilon_0 = 78 \times 8{\cdot}85 \times 10^{-12} = 690{\cdot}3 \times 10^{-12} \text{ F/m}$$

*4. The dielectric constant of medium is 4. Electric field in the dielectric is 10^6 V/m. Calculate electric displacement and polarisation. Take $\epsilon_0 = 9 \times 10^{-12}$ F/m **[ANU 18; VSU 18]**

Sol : Given that $K = 4$; $E = 10^6$ V/m; $\epsilon_0 = 9 \times 10^{-12}$ F/m

$$D = K\epsilon_0 E = 4 \times (9 \times 10^{-12}) \times 10^6 = 36 \times 10^{-6} \text{ c/m}^2$$

$$P = (K - 1)\ \epsilon_0\ E = (4 - 1) \times 9 \times 10^{-12} \times 10^6 = 27 \times 10^{-6} \text{ c/m}^2$$

5. The electric susceptibility of a material is $36 \times 10^{-12}\ c^2/\text{N} - \text{m}^2$. Calculate the value of dielectric constant and absolute permittivity of the material. Take $\epsilon_0 = 9 \times 10^{-12}$ F/m

Sol : Given that $\chi = 36 \times 10^{-12}\ c^2\ \text{N} - \text{m}^2$

$\epsilon_0 = 9 \times 10^{-12}$ F/m

Dielectric constant $K = 1 + \frac{\chi}{\epsilon_0} = 1 + \frac{36 \times 10^{-12}}{9 \times 10^{-12}}$

$$= 1 + 4 = 5$$

And permittivity of material $\epsilon = K\epsilon_0 = 5 \times (9 \times 10^{-12})$

$$= 45 \times 10^{-12} \text{ F/m.}$$

*6. The electric susceptibility of a material is $35{\cdot}4 \times 10^{-12}$ $c^2/N - m^2$. What are the values of dielectric constant and the permittivity of the material ?

Sol : Given that $\chi = 35{\cdot}4 \times 10^{-12}$; $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ F/m

i) Dielectric constant $K = 1 + \frac{\chi}{\epsilon_0} = 1 + \frac{35{\cdot}4 \times 10^{-12}}{8{\cdot}85 \times 10^{-12}}$

$$= 1 + 4 = 5$$

ii) The permittivity is $\epsilon = K\epsilon_0 = (8{\cdot}85 \times 10^{-12}) \times 5$

$$= 44{\cdot}3 \times 10^{-12} \text{ c}^2/\text{N} - \text{m}^2.$$

7. The dielectric constant of helium at 0°C and 1 atmospheric pressure is 1·000074. Calculate the dipole moment induced in each helium atom when the gas is in electric field of 1 volt per meter. Given $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ $c^2/N - m$ and molecular density of helium $\simeq 2{\cdot}69 \times 10^{25}$ molecules/meter at N.T.P.

Sol : We know that $P = \epsilon_0 (K - 1)E$

If P be the dipole moment induced in each helium atom and n be the number of helium atoms per metre^3, then $P = np$

$\therefore \; np = \epsilon_0 (K - 1)E$

$$p = \frac{\epsilon_0 (K - 1)E}{n} = \frac{(8{\cdot}85 \times 10^{-12})(1{\cdot}000074 - 1) \times 1}{2{\cdot}69 \times 10^{25}} = 2{\cdot}43 \times 10^{-41} \text{ c} - \text{m..}$$

8. The dielectric constant of helium at 0°C is 1·000074. Calculate its electrical susceptibility at this temperature.

Sol : Given that $K = 1{\cdot}000074$; $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ $c^2/N - m^2$

$$\chi = (K - 1)\epsilon_0 = (1{\cdot}000074 - 1) \times 8{\cdot}85 \times 10^{-12} = 6{\cdot}548 \times 10^{-16} \text{ c}^2/\text{N} - \text{m}^2.$$

9. The thickness of dielectric between parallel plates of a condenser is 5 mm. Dielectric constant is 3. Electric field in the dielectric is 10^6 V/m. Calculate the surface charge density on the condenser plate, surface charge density on the dielectric, polarisation, electric displacement and energy density. **[ANU 201**

Sol : Given that $t = 5$ mm $= 5 \times 10^{-3}$ m; $K = 3$; $E = 10^6$ V/m

Electric displacement $D = \epsilon E = K\epsilon_0 E = 3 \times (8{\cdot}9 \times 10^{-12}) \times 10^6 = 26{\cdot}7$ C/m^2

$\therefore$ Surface charge density on the plate of condenser $\sigma = 26{\cdot}7 \times 10^{-6}$ c/m^2

Polarisation $P = \epsilon_0 (K - 1)E = (8{\cdot}9 \times 10^{-12})(3 - 1)(10^6)$

$$= 17{\cdot}8 \times 10^{-6} \text{ c/m}^2 = 17{\cdot}8 \; \mu \text{ c/m}^2$$

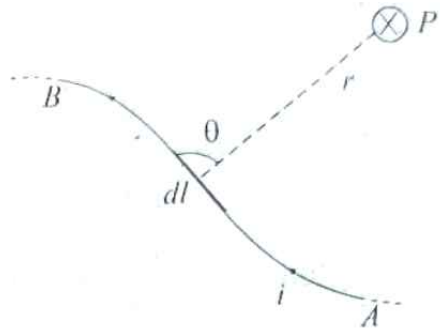
UNIT II 3. MAGNETOSTATICS

LONG ANSWER TYPE QUESTIONS

1. State and explain Biot Savart's law. Derive an expression for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
[AdNU 18, 17; AU 17; BRAU 18; KU 18; SKU 18, 17; SVU 18; VSU 17; YVU 18, 17]

Biot-Savart's law :

According to Biot-Savarts law, the magnetic induction dB at P due to length of element dl depends on the following factors.



1) It is directly proportional to the current flowing through the conductor i.e., $dB \propto i$.

2) It is directly proportional to the length of the element. i.e., $dB \propto dl$

3) It is directly proportional to the sine of the angle θ. i.e., $dB \propto \sin\theta$

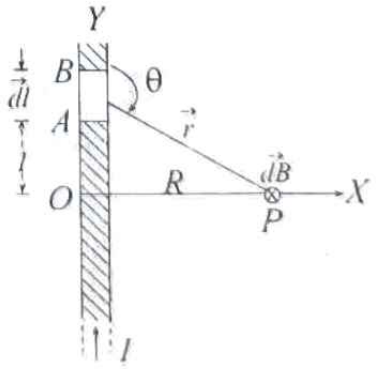
4) It is inversely proportional to the square of the distance of point from the element.

i.e., $dB \propto \frac{1}{r^2}$

5) Combining all these factors, $dB \propto \frac{idl\sin\theta}{r^2} \Rightarrow dB = \frac{\mu_0}{4\pi} \times \frac{idl\sin\theta}{r^2}$

Magnetic field due to long straight conductor carrying current :

1) Consider an infinitely long conductor placed in vaccum carrying a current i ampere as shown in fig.



2) Let P be a point on X-axis at a distance R from conductor.

3) Let O is the foot of the perpendicular from P to the conductor.

4) Consider a small element AB of length dl of infinite conductor at a distance l from 'O'.

5) Let $\vec{r}$ be the distance of the element from the point P.

6) Suppose θ be the angle in clock wise direction which is the direction of current makes with the line joining the element to point P.

7) The magnitude of the $\vec{dB}$ due to element AB at point P is given by

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \sin\theta}{r^2} \rightarrow (1)$$

8) The magnetic induction due to the whole conductor is given by

$$B = \int dB = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{\sin\theta \, dl}{r^2} \rightarrow (2)$$

9) From fig. $r = (l^2 + R^2)^{1/2}$ and $\sin\theta = \sin(\pi - \theta) = \frac{R}{r}$ $\therefore$ $\sin\theta = \frac{R}{(l^2 + R^2)^{1/2}}$

Substituting the value of r and $\sin\theta$ in equation (2), we get

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{R dl}{(l^2 + R^2)^{3/2}} \rightarrow (3)$$

10) Let $l = R\tan\alpha$ $\quad \therefore dl = R\sec^2\alpha \, d\alpha$

The limits of integration under this substitution become $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

11) Hence $B = \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{R \cdot R\sec^2\alpha \, d\alpha}{(R^2\tan^2\alpha + R^2)^{3/2}}$

$$= \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{R^2\sec^2\alpha \, d\alpha}{R^3(1 + \tan^2\alpha)^{3/2}} = \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{R^2\sec^2\alpha \, d\alpha}{R^3\sec^3\alpha}$$

$$= \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{d\alpha}{R\sec\alpha} = \frac{\mu_0 i}{4\pi R} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \cos\alpha \, d\alpha = \frac{\mu_0 i}{4\pi R} [\sin\alpha]_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} = \frac{\mu_0 i}{4\pi R}[1 + 1]$$

$$\therefore B = \frac{\mu_0 i}{2\pi R} \text{ Wb/m}^2 \text{ or tesla}$$

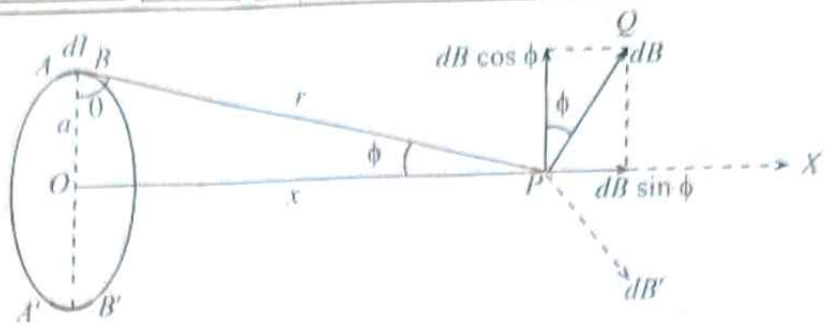
12) This is the expression for the magnetic field induction near a long straight conductor.

2. Derive the expression of magnetic field on the axis of a current loop.

[ANU 17; AdNU 18; AU 18; BRAU 17; RU 17; SVU 17]

A. 1) Consider a circular coil of radius 'a' and carrying a current 'i'.
2) Let P be a point at a distance 'x' from its centre O.
3) Let a small element AB of length dl.

4) Let r be the distance of element from point P and θ be the angle which the direction of current makes with the line joining the element to the point 'O'.



5) The magnetic field dB at point P due to current element AB of length dl is given by

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \frac{idl}{r^2} \qquad [\because \theta = 90^\circ]$$

6) The vector dB at the point P can be resolved into
   i) $dB \cos \phi$ perpendicular to axis of the coil
   ii) $dB \sin \phi$ along the axis of the coil.

7) If we take another element $A'B'$ diametrically opposite to AB of same length, it will also produce dB at point P.

8) The direction of dB now will be opposite to the previous one. This can also be resolved into components $dB \cos \phi$ and $dB \sin \phi$.

9) The components along the axis will add up while the components perpendicular to the axis will cancel.

10) Similarly, if we divide the whole circular coil into a number of elements, the vertical components will cancel while the components along the axis will add up.

11) $\therefore$ Magnetic field along the axis $= B = \int dB \sin \phi$

$$\therefore B = \frac{\mu_0 i}{4\pi r^2} \int dl \sin \phi = \frac{\mu_0 i}{4\pi r^2} \int dl \cdot \left(\frac{a}{r}\right) \qquad [\because \sin \phi = \frac{a}{r}]$$

$$B = \frac{\mu_0 ia}{4\pi r^3} \int dl$$

12) But $\int dl =$ Circumference of the coil $= 2\pi a$ from fig, $r = (a^2 + x^2)^{1/2}$

$$\therefore B = \frac{\mu_0 ia}{4\pi (a^2 + x^2)^{3/2}} \times 2\pi a = \frac{\mu_0 ia^2}{2(a^2 + x^2)^{3/2}}$$

13) If there are N turns in the coil, then $B = \dfrac{\mu_0 Ni a^2}{2(a^2 + x^2)^{3/2}}$ Wb/m^2 or Tesla.

14) The direction of B is along the axis of the coil.

i) At the centre of the coil $x = 0$, $B = \dfrac{\mu_0 Nia^2}{2a^3} = \dfrac{\mu_0 Ni}{2a}$

ii) At very far off from the loop $x >> a$ and $(a^2 + x^2)^{3/2} \simeq x^3$

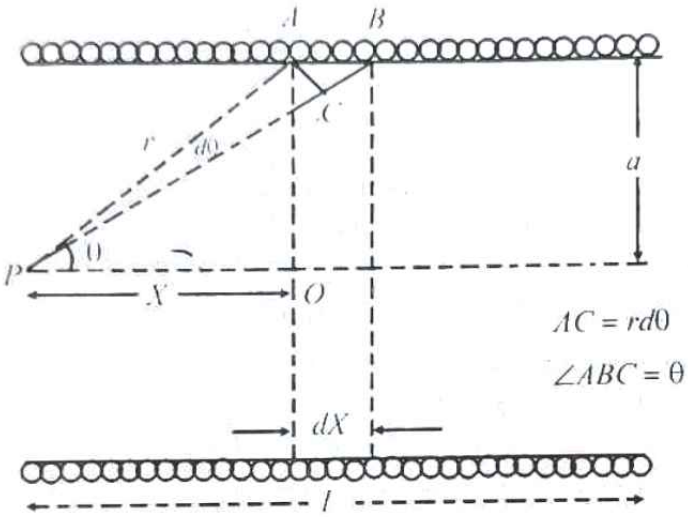
$$\therefore \; B = \frac{\mu_0 N i a^2}{2x^3}$$

3. Derive an expression for the magnetic field inside a long solenoid carrying a current i and show that the filed at the ends of such a solenoid is half of that in the middle. **[YVU 2018]**

A. 1) Consider a long solenoid of length l meter and radius a meter.

2) Let N be the total number of turns in the solenoid.

3) Then the number of turns n per meter will be $\left(\frac{N}{l}\right)$.



4) Suppose the solenoid carries a current i ampere. We shall calculate 3 cases as given below.

Case (i) : Field at an inside point :

1) Let P is a point inside the solenoid at a distance 'r' from O on its axis.

2) Consider the solenoid is divide into a number of narrow equidistant coils.

3) Consider one such coil of width dx. There will be ndx turns.

4) The field at P due to elementary coil of the width dx carrying a current i is given by $dB = \dfrac{\mu_0 (ndx)\, ia^2}{2(a^2 + x^2)^{3/2}}$ Weber / m^2

5) From triangle ABC, we have $\sin\theta = \dfrac{rd\theta}{dx}$ or $dx = \dfrac{rd\theta}{\sin\theta}$

6) From $\Delta\, APO$, $a^2 + x^2 = r^2 \;\therefore\; (a^2 + x^2)^{3/2} = r^3$

$$\Rightarrow dB = \frac{\mu_0 n \left(\frac{rd\theta}{\sin\theta}\right) ia^2}{2r^3} = \frac{\mu_0 nia^2 d\theta}{2r^2 \sin\theta} = \frac{\mu_0 ni\, d\theta}{2\sin\theta}\left(\frac{a}{r}\right)^2$$

$$= \frac{\mu_0 nid\theta}{2\sin\theta}(\sin^2\theta)\left[\because \left(\frac{a}{r}\right)^2 = \sin^2\theta\right]$$

$$= \frac{\mu_0 nid\theta \sin\theta}{2}$$

Applications : 1) It is used to determine the sign of the charge carriers.

2) The density of charge carriers can be found from this effect.

3) It can be used to measure the drift velocity of charge carriers.

4) Hall effect is useful in understanding the electrical conduction in metals and semi conductors.

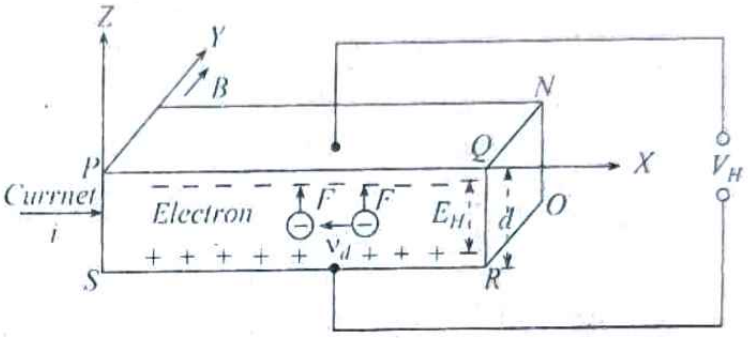
SHORT ANSWER TYPE QUESTIONS

*1. What is Hall effect? Explain it.

[ANU 17; BRAU 18, 17; RU 17; SKU 17; SVU 17]

1) When a magnetic field is applied perpendicular to a current carrying conductor, a potential difference is developed between the points on opposite side of the conductor. This effect is called Hall effect.

2) **Hall Coefficient :** Consider a uniform, thick metal strip placed with its length parallel to X - axis.



3) Let a current i is passed in the conductor along X - axis and a magnetic field B is established along Y- axis.

4) Due to B. the charge carries electrons experience a force in + ve direction of Z.

5) Hence the upper side will be charged negatively while the lower side will be positively. So potential difference or Hall e.m.f is produced.

6) If the charge carries are holes or protons, the sign of e.m.f is reversed.

7) Thus we can find the nature of charge carries by determining the sign of Hall e.m.f which can be measure with potentiometer.

8) Due to displacement of charge carries (electrons) transverse field is generated. This is called Hall electric field E_H as shown in fig.

9) This field acts inside the conductor to oppose the sideway drift of the charge carrier.

10) When the equilibrium is reached, the magnetic deflecting forces on the charge carriers are balanced by the electric forces due to electric field.

11) Magnetic deflecting force $= q(V_d \times B)$

Hall electric deflecting force $= qE_H$

12) As the net force on the charge carriers become zero.

$q(V_d \times B) + q\,E_H = 0$

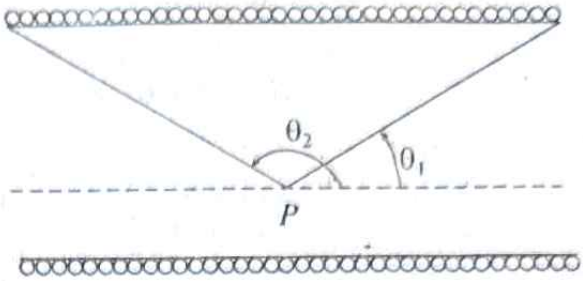
$E_H = (-V_d \times B) = +V_d B$

[$\because$ Writing in terms of magnitude only]

13) But $Vd = J/nq$ Here J = current density

7) $B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \frac{\mu_0 ni \sin\theta \, d\theta}{2} = \frac{\mu_0 ni}{2} [-\cos\theta]_{\theta_1}^{\theta_2} = \frac{\mu_0 ni}{2} [\cos\theta_1 - \cos\theta_2] \rightarrow (1)$

Here θ_1 and θ_2 are the semivertical angles subtended at P by first and last turn respectively of the solenoid as shown in fig.



8) At any axial point P when it is well inside a very long solenoid, $\theta_1 = 0$ and $\theta_2 = \pi$.

9) Hence $B = \frac{\mu_0 ni}{2} [\cos 0 - \cos\pi] = \frac{\mu_0 ni}{2} [1 - (-1)]$

$\therefore \; B = \mu_0 ni$

This is the field at the centre of a long solenoid.

Case (ii) : Field at an axial end point :

In this case, $\theta_1 = 0$ and $\theta_2 = 90°$

$$B = \frac{\mu_0 ni}{2} [\cos 0 - \cos 90°] = \frac{\mu_0 ni}{2}$$

This shows that the field at either end is one half of its magnitude at the centre.

Case (iii) : Field at the centre of a solenoid of finite length :

1) Consider that point P is at the centre i.e., it is a distance $\frac{l}{2}$ from either end.

2) In this case, $\cos\theta_1 = \frac{\frac{l}{2}}{\left\{a^2 + \left(\frac{d}{2}\right)^2\right\}^{1/2}} = \frac{l}{(4a^2 + l^2)^{1/2}}$

3) And $\cos(\pi - \theta_2) = \frac{(l/2)}{\left\{a^2 + \left(\frac{d}{2}\right)^2\right\}^{1/2}} = \frac{l}{(4a^2 + l^2)^{1/2}}$

and $\cos\theta_2 = \frac{l}{(4a^2 + l^2)^{1/2}}$

4) Putting these values in equation (3), we get

$$B = \frac{\mu_0 ni}{2} \left[\frac{l}{(4a^2 + l^2)^{1/2}} + \frac{l}{(4a^2 + l^2)^{1/2}} \right] = \frac{\mu_0 ni}{2} \times \frac{2l}{(4a^2 + l^2)^{1/2}} = \frac{\mu_0 i nl}{(4a^2 + l^2)^{1/2}}$$

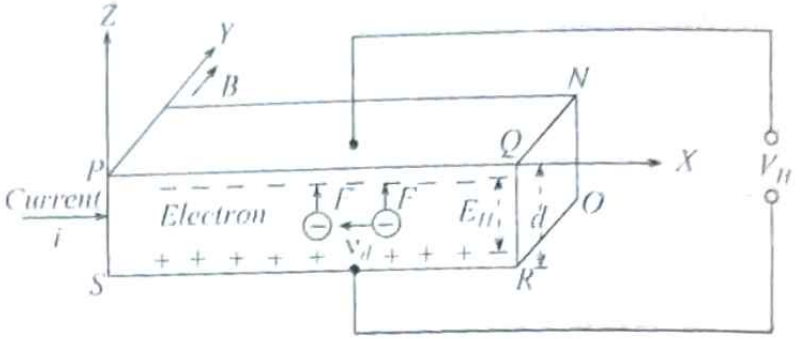
$$\therefore \; B = \frac{\mu_0 iN}{(4a^2 + l^2)^{1/2}}$$

This expression gives the field at the centre of the solenoid of finite length.

4. Explain Hall effect. Derive an expression for Hall Coefficient. Write the applications of Hall effect. **[ANU 19, 18; AdNU 18, 17; AU 18, 17; BRAU 18; RU 18, 17; SKU 18, 17; SVU 17; VSU 17]**

A. **Hall effect :** When a magnetic field is applied perpendicular to a current carrying conductor, a potential difference is developed between the points on opposite side of the conductor. This effect is called Hall effect.

Expression of Hall coefficient :



1) Consider a uniform, thick metal strip placed with its length parallel to X-axis.
2) Let a current i is passed in the conductor along X- axis and a magnetic field B is established along Y- axis.
3) Due to B, the charge carries electrons experience a force in + ve direction of Z.
4) Hence the upper side will be charged negatively while the lower side will be positively. So potential difference or Hall e.m.f is produced.
5) If the charge carries are holes or protons, the sign of e.m.f is reversed.
6) Thus we can find the nature of charge carries by determining the sign of Hall e.m.f which can be measure with potentiometer.
7) Due to displacement of charge carries (electrons) transverse field is generated. This is called Hall electric field E_H as shown in fig.
8) This field acts inside the conductor to oppose the sideway drift of the charge carrier.
9) When the equilibrium is reached, the magnetic deflecting forces on the charge carriers are balanced by the electric forces due to electric field.
10) Magnetic deflecting force $= q(V_d \times B)$
    Hall electric deflecting force $= qE_H$
11) As the net force on the charge carriers become zero.

$$q(V_d \times B) + q\,E_H = 0$$

$$E_H = (-V_d \times B) = +V_d B$$

[$\because$ Writing in terms of magnitude only]

12) But $Vd = J/nq$ Here J = current density
    n = number of charge carriers per unit volume.

$$\therefore\ E_H = \left(\frac{1}{nq}\right) JB \text{ where } R_H = \frac{1}{nq} = \text{Hall coefficient.} \quad \therefore\ R_H = \frac{E_H}{JB}$$

13) If the R_H negative the charge carries are electrons end positive the charge carriers are holes.

Applications : 1) It is used to determine the sign of the charge carriers.

2) The density of charge carriers can be found from this effect.

3) It can be used to measure the drift velocity of charge carriers.

4) Hall effect is useful in understanding the electrical conduction in metals and semi conductors.

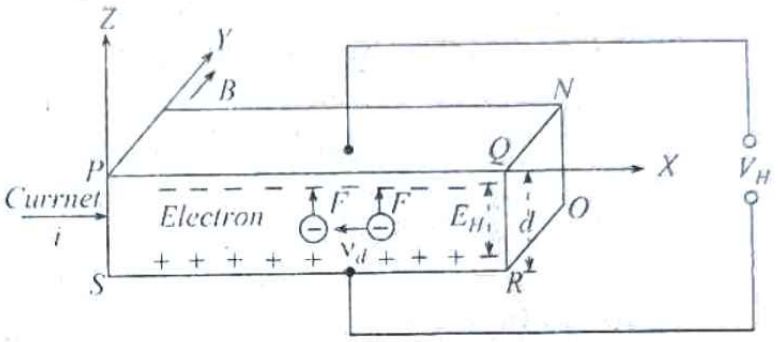
SHORT ANSWER TYPE QUESTIONS

What is Hall effect? Explain it.

[ANU 17; BRAU 18, 17; RU 17; SKU 17; SVU 17]

1) When a magnetic field is applied perpendicular to a current carrying conductor, a potential difference is developed between the points on opposite side of the conductor. This effect is called Hall effect.

2) **Hall Coefficient :** Consider a uniform, thick metal strip placed with its length parallel to X - axis.



3) Let a current i is passed in the conductor along X - axis and a magnetic field B is established along Y- axis.

4) Due to B. the charge carries electrons experience a force in + ve direction of Z.

5) Hence the upper side will be charged negatively while the lower side will be positively. So potential difference or Hall e.m.f is produced.

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n = number of charge carriers per unit volume.

$$\therefore\ E_H = \left(\frac{1}{nq}\right) JB \text{ where } R_H = \frac{1}{nq} = \text{Hall coefficient.}$$

$$\therefore\ R_H = \frac{E_H}{JB}$$

If the R_H negative the charge carries are electrons end positive the charge carriers are holes.

2. Derive an expression for force on a current carrying conductor.

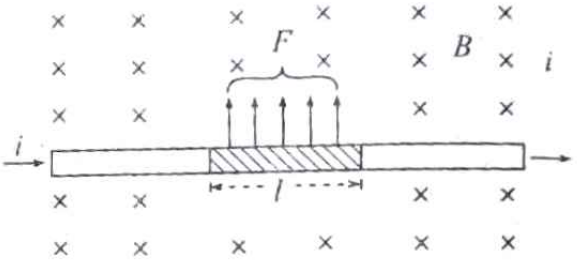
A. 1) The electric current in a conductor is due to the motion of conduction electrons. When such a conductor is placed in a magnetic field B, it exert side way force on the conductor carrying a current.

2) If the charge is infinitely small, then $dF = dq\,(V \times B)$
where V is the displacement of the charge per unit time.

$$\therefore\ dF = dq\left(\frac{dl}{dt} \times B\right) = \frac{dq}{dt}(dl \times B)$$

$$\left[\because V = \frac{dl}{dt}\right]$$

$$dF = i\,(dl \times B)$$



3) On integrating the above equation, we get $F = i(l \times B) = Bil \sin\theta$

Case (i) If $\theta = 90^\circ$, $F = Bil$

Case (ii) If $\theta = 0^\circ$, $F = 0$

3. Define Hall effect. Mention the applications of this effect. **[AU 17; SKU 18]**

A. **Hall effect :** When a magnetic field is applied perpendicular to a current carrying conductor, a potential difference is developed between the opposite sides of the conductor.

Applications : 1) It is used to determine the sign of the charge carriers.

2) The density of charge carriers can be found from this effect.

3) It can be used to measure the drift velocity of charge carriers.

4) Hall effect is useful in understanding the electrical conduction in metals and semi conductors.

4. Explain Biot - Savarts law. **[ANU 19; AdNU 18, 17; VSU 18, 17; YVU 18, 17]**

A. **Biot-Savart's law :**

According to Biot-Savarts law, the magnetic induction dB at P due to length of element dl depends on the following factors.

1) It is directly proportional to the current flowing through the conductor.
i.e., $dB \propto i$

2) It is directly proportional to the length of the element.

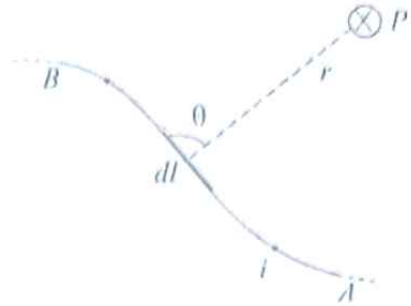
i.e., $dB \propto dl$

3) It is directly proportional to the sine of the angle θ.

i.e., $dB \propto \sin\theta$

4) It is inversely proportional to the square of the distance of point from the element.

i.e., $dB \propto \frac{1}{r^2}$



5) Combining all these factors, $dB \propto \frac{idl\sin\theta}{r^2} \Rightarrow dB = \frac{\mu_0}{4\pi} \times \frac{idl\sin\theta}{r^2}$

6) In vector form $dB = \frac{\mu_0}{4\pi}\frac{idl \times \bar{r}}{r^3}$.

5. Explain the divergence and curl of magnetic field.

A. **div. B :** Divergence of magnetic field B explains the non existance of isolated magnetic poles. It is also known as Gauss law of magnetism.
According to Biot-Savart law,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3}$$

By integrating the above equation, we get the magnetic field due to a whole current loop.

$$\oint d\vec{B} = \int \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times \vec{r}}{r^3} \rightarrow (1)$$

taking divergence both sides, we get $\nabla \cdot B = \frac{\mu_0}{4\pi} I \oint \vec{\nabla} \cdot \left(\frac{d\vec{l} \times \vec{r}}{r^3}\right) \rightarrow (2)$

using $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$

$$\nabla \cdot \vec{B} = \frac{\mu_0 I}{4\pi}\left[\oint \frac{\vec{r}}{r^3} \cdot (\vec{\nabla} \times d\vec{l}) - d\vec{l} \cdot \left(\vec{\nabla} \times \frac{\vec{r}}{r^3}\right)\right] \rightarrow (3)$$

since $d\vec{l}$ is not the function of x, y, z, so $\vec{\nabla} \times d\vec{l} = 0 \rightarrow (4)$

$$\nabla \times \frac{\vec{r}}{r^3} = -\vec{\nabla} \times \vec{\nabla}\left(\frac{1}{r}\right)$$

we know curl of gradiant is zero

$$\vec{\nabla} \times \frac{\vec{r}}{r^3} = -\vec{\nabla} \times \vec{\nabla}\left(\frac{1}{r}\right) = 0 \rightarrow (5)$$

using equations (4) and (5), equation (3) becomes $\vec{\nabla} \cdot \vec{B} = 0$.

Thus, divergence of $\vec{B}$ vector is zero.

Hence it proves that the magnetic field due to a closed current loop is zero that explains the non existance of single magnetic pole.

Curl $\vec{B}$: It is the differential form of ampere's law.

According to Ampere's circuital law.

$$\oint B \cdot dl = \mu_0 I \rightarrow (1)$$

But we know that, $I = \iint \vec{J} \cdot \vec{dS} \rightarrow (2)$

From eqns. (1) and (2)

$$\oint B \cdot dl = \mu_0 \iint \vec{J} \cdot \vec{dS} \rightarrow (3)$$

But according to strokes theorem, the closed line integral of the vector B is equal to the surface integral of its curl.

$$\oint B \cdot dl = \iint (\vec{\nabla} \times \vec{B}) \cdot \vec{dS} \rightarrow (4)$$

From (3) and (4) $\iint (\vec{\nabla} \times \vec{B}) \cdot \vec{dS} = \mu_0 \iint \vec{J} \cdot \vec{dS}$

$$\vec{\nabla} \times \vec{B} = \vec{J}$$

which is the differential form of Ampere's law. Thus the curl of a magnetic field at any point is equal to the current density at that point.

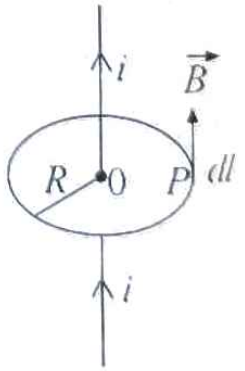
6. State and prove Ampere's circuital law.

A. **Ampere's law :** The line integral of the intensity of magnetic induction field B around any closed path in air or vaccum is equal to μ_0 times the net current I through the area bounded by the closed path.

$$\oint B \cdot dl = \mu_0 I$$

where μ_0 is the permeability of free space.

Proof : Consider a long straight conductor carrying a current i as shown in the figure.



Taking the point O as centre, if we consider a circular path of Radius R, then the magnetic induction B at point of the circular path will be

$$B = \frac{\mu_0 i}{2\pi R} \text{ (Biot Savart law)}$$

The direction will be along the tangent of circular path. Regarding B, two points should be remembered.

i) The magnitude of B is constant at all points on the circle.

ii) This is parallel to the circuit element dl.

The line integral of B along the circular path is given by

$$\oint B \cdot dl = \oint B dl \cos 0^\circ = B \int dl = B\,(2\pi R)$$

where $\oint dl = 2\pi R$ = circumference of the circle.

Substituting the value of B from equation (1) in equation (2) we get

$$\oint B \cdot dl = \frac{\mu_0 i}{2\pi R} \cdot 2\pi R = \mu_0 i$$

Thus, the line integral of $\oint B \cdot dl$ is μ_0 times the current through the area bounded by the circle.

This is Ampere's law.

PROBLEMS

*1. A copper strip 2 cm wide and 1 mm thick is placed in a magnetic field with $B = 1{\cdot}5$ w/m^2 with its thickness parallel to B. If a current of 200 amp is set up the strip, what hall potential is developed across the strip ? The number of conduction electrons in the copper strip is $8{\cdot}4 \times 10^{28}$ / m^3

Sol : Given that,

$B = 1{\cdot}5$ Wb/m^2, $i = 200$ amp; $n = 8{\cdot}4 \times 10^{28}$/m^2; $q = 1{\cdot}6 \times 10^{-19}$ c; $t = 1$ mm $= 10^{-3}$m

$$\text{Hall potential } V_H = \frac{iB}{Nqt} = \frac{200 \times 1{\cdot}5}{(8{\cdot}4 \times 10^{28}) \times (1{\cdot}6 \times 10^{-19}) \times 10^{-3}}$$

$$= 22{\cdot}32 \times 10^{-6} \text{ volt} = 22{\cdot}32\ \mu\text{ v.}$$

*2. Two straight parallel wire each of length 10 cm are one above the other separated by 5 cm distance. The same current flows through the wires in opposite directions such that the upper wire is supported without falling. If the mass of the wire is 5 gm, find the current in wire.

Sol : Given that $l = 10$ cm $= \frac{10}{100}$ m; $d = 5$ cm $= \frac{5}{100}$ m; $m = 5$ gm $= 5 \times 10^{-3}$ kg;

Let $i_1 = i_2 = i$.

We know that $F = \frac{\mu_0 i_1 i_2 l}{2\pi d}$

$$\frac{4\pi \times 10^{-7} \times i^2}{2\pi \times \left(\frac{5}{100}\right)} \times \frac{10}{100} = 5 \times 10^{-3} \times 9{\cdot}8$$

$$i^2 = \frac{5 \times 9{\cdot}8 \times 10^7}{4 \times 1000}$$

$\therefore\ i = 350$ amp.

3. A current of 20 amp flows through each of the parallel long wires which are 4 cm apart. Compute the force exerted per unit length of each wire.

Sol : Given that $i_1 = i_2 = 20$ amp; $d = 4$ cm $= 4 \times 10^{-2}$ m; $\mu_0 = 4\pi \times 10^{-7}$

Force exerted per unit length is, $\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{4\pi \times 10^{-7} \times 20 \times 20}{2\pi \times 4 \times 10^{-2}} = 2 \times 10^{-3}$ Newton.

*4. An infinitely long conductor carries a current of 10 mA. Find the magnetic field at a point 10 cm away from it.

Sol : Given that, $i = 10$ mA $= 10 \times 10^{-3}$ A $= 10^{-2}$ A; $R = 10$ cm $= 0{\cdot}1$ m

$\therefore$ Magnetic field $H = \frac{B}{\mu_0} = \frac{1}{\mu_0} \times \frac{\mu_0 i}{2\pi R} = \frac{i}{2\pi R} = \frac{10^{-2}}{2\pi \times 0{\cdot}1} = \frac{10^{-1}}{2\pi} = 0{\cdot}01591$ A/m.

5. A long straight wire carries a current of 10*A*. An electron travels with a velocity of 5×10^6 m/s parallel to the wire 0·1 m from it, and in a direction opposite to the current. What force does the magnetic field of current exert on the electron ?

Sol : Given that $i = 10A$; $V = 5 \times 10^6$ m/s; $R = 0{\cdot}1$ m

$\therefore$ The magnetic induction B about straight conductor is given by

$$B = \frac{\mu_0 i}{2\pi R} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0{\cdot}1} = 2 \times 10^{-5} \text{ wb/m}^2$$

When the electron is moving parallel to the wire, then V is perpendicular to B.

Hence, $F = qVB \sin 90^\circ = qVB$

$$= (1{\cdot}6 \times 10^{-19}) \times (5 \times 10^6) \times (2 \times 10^{-5}) = 1{\cdot}6 \times 10^{-17} \text{ Newton.}$$

6. An infinitely long conductor carries a current of 100 mA. Find the magnetic field at a point 10 cm away from it.

Sol : Given that, $i = 100$ mA $= 100 \times 10^{-3} A = 10^{-1}$ A; $R = 10$ cm $= 10^{-1}$ m

$$\therefore B = \frac{\mu_0 i}{2\pi R} = \frac{4\pi \times 10^{-7} \times 10^{-1}}{2\pi \times 10^{-1}} = 2 \times 10^{-7} \text{ Tesla.}$$

*7. In Bohr model of hydrogen atom the electron circulates round the nucleus in a path of radius $5{\cdot}1 \times 10^{-11}$ m at a frequency 'f' of $6{\cdot}8 \times 10^{15}$ rev/s.

a) What is the value of B at the centre of the orbit ?

b) What is the equivalent dipole moment ?

Sol : Given that $r = 5{\cdot}1 \times 10^{-11}$ m; $f = 6{\cdot}8 \times 10^{15}$ rev/s

a) We know that $i = \frac{\text{charge (q)}}{\text{time (t)}} = \text{charge } (q) \times \text{frequency } (f)$

$$= (1{\cdot}6 \times 10^{-19})(6{\cdot}8 \times 10^{15}) = 1{\cdot}1 \times 10^{-3} \text{ A}$$

The magnetic field at the centre of the orbit,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi iN}{r} = 10^{-7} \times \frac{2\pi \times (1{\cdot}1 \times 10^{-3}) \times 1}{5{\cdot}1 \times 10^{-11}} = 13{\cdot}6 \text{ weber/m}^2 \simeq 14 \text{ Tesla}$$

b) Equivalent dipole moment $= iA = i \times \pi r^2 = (1{\cdot}1 \times 10^{-3}) \times \pi \times (5{\cdot}1 \times 10^{-11})^2$

$$= 90 \times 10^{-24} \text{ A} - \text{m}^2.$$

A long straight copper tube having inside radius of 1 cm and outside radius of 2 cm carries a current of 200 amp. Compute the magnetic field at a distance of 0·5 cm and 4 cm from the axis.

: Given that $r_1 = 1$ cm and $r_2 = 2$ cm.

The distance $r = 0{\cdot}5$ cm lies inside the tube. Hence $B_1 = 0$

Now, consider the distance $r = 4$ cm. This distance is outside the tube. Hence

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times (200)}{2\pi \times (4 \times 10^{-2})} = 10^{-3} \text{ weber/m}^2.$$

A current of 1 amp is flowing in a circular coil of radius 10 cm and 20 turns. Calculate the intensity of magnetic field at a distance 10 cm on the axis of the coil and at the centre. **[SVU 18; VSU 18]**

: Given that $r = 10$ cm $= 10^{-1}$ m; $n = 20$; $a = 10$ cm $= 10^{-1}$ m

$$B_{\text{centre}} = \frac{\mu_0 in}{2a} = \frac{4\pi \times 10^{-7} \times 10 \times 20}{2 \times 10} = 4\pi \times 10^{-5} \text{ weber/m}^2$$

Magnetic induction at a distance 10 cm from centre is given by

$$B^1 = \frac{\mu_0}{2} \frac{nia^2}{(x^2 + a^2)^{\frac{3}{2}}} = \frac{4\pi \times 10^{-7}}{2} \times \frac{20 \times 1 \times (0{\cdot}1)^2}{[(0{\cdot}1)^2 + (0{\cdot}1)^2]^{\frac{3}{2}}}$$

$$= \frac{2\pi \times 10^{-7} \times (0{\cdot}1)^2}{(0{\cdot}1)^3 \times 2^{3/2}} = \frac{\pi \times 10^{-7}}{(0{\cdot}1)\,2} = \frac{3{\cdot}14 \times 10^{-7} \times 10}{2}$$

$$= 1{\cdot}57 \times 10^{-6} \text{ Tesla.}$$

). A circular coil of wire of diameter 10 cm and having 100 turns carries a current of 1 amp. If it is placed in a uniform magnetic field of induction $0{\cdot}5T$. Calculate the maximum torque on the coil.

: Given that $d = 10$ cm; $a = 5$ cm $= 5 \times 10^{-2}$ m; $n = 100$; $i = 1$ amp; $B = 0{\cdot}5$ T

$$\therefore\ \tau_{\max} = BinA = Bin(\pi a^2) = 0{\cdot}5 \times 1 \times 100 \times 3{\cdot}14 \times (5 \times 10^{-2})^2 = 0{\cdot}3928 \text{ N} - \text{m}.$$

. Two similar coils of wire having a radius of 7 cm and 60 turns have a common axis and are 18 cm apart. Find the strength of magnetic field at a point midway between them on their common axis when a current of 0·1 amp is passed through them.

: Given that, $a = 7$ cm $= 0{\cdot}07$ m; N = 60; $i = 0{\cdot}1$ A; $x = \frac{18}{2} = 9$ cm $= 0{\cdot}09$ m

$\therefore$ The strength of the magnetic field $H = \frac{B}{\mu_0} = \frac{Nia^2}{2(a^2 + x^2)^{\frac{3}{2}}}$

$$= \frac{60 \times 0{\cdot}1 \times (0{\cdot}07)^2}{2[(0{\cdot}07)^2 + (0{\cdot}09)^2]^{\frac{3}{2}}} = \frac{6 \times 0{\cdot}0049}{2(0{\cdot}0049 + 0{\cdot}0081)^{\frac{3}{2}}}$$

$= 9{\cdot}917$ amp – turn/metre

$\therefore$ Strength of magnetic field due to two coils $= 2H = 19{\cdot}834$ amp – turn/metre.

12. A solenoid of length 100 cm has 1000 turns would on it. Calculate the magnetic field at the middle point of its axis when a current of 2 amp is passed through it. **[KU 2018]**

Sol : Given that, $l = 100$ cm $= 1$ m; $N = 1000$; $i = 2$ amp

$\therefore$ Number of turns/meter, $n = \frac{N}{l} = \frac{1000}{1} = 1000$

Now, $B = \mu_0 ni = (4\pi \times 10^{-7}) \times (1000) \times 2 = 0{\cdot}002573$ wb/m^2.

13. A long solenoid has 20 turns per cm. Calculate the magnetic induction at the interior point on the axis for a current of 20 mA.

Sol : Given that, $n = 20$ turns/cm $= 20 \times 100 = 2000$ turns/m;

$i = 200$ mA $= 200 \times 10^{-3}$ A $= 2 \times 10^{-1}$ A

$\therefore$ Magnetic induction at the interior point

$B = \mu_0 ni = (4\pi \times 10^{-7}) \times 2000 \times 2 \times 10^{-1} = 16\pi \times 10^{-5} = 50{\cdot}24 \times 10^{-5}$ weber/m^2.

***14.** A solenoid of 1000 turns is would uniform on a glass tube of 50 cm long and 10 cm in diameter. Find the strength of magnetic field at the centre of solenoid when 0·1 amp current is flowing through it. **[ANU 2017]**

Sol : Given that, $N = 1000$; $l = 50$ cm $= \frac{1}{2}$ m

Number of turns per meter $n = \frac{N}{l} = \frac{1000}{1/2} = 2000$

$i = 0{\cdot}1$ amp

$\therefore$ Magnetic field at the centre of solenoid is $H = \frac{B}{\mu_0}$

$= \frac{\mu_0 ni}{\mu_0} = ni = 2000 \times 0{\cdot}1 = 200$ A – $\frac{\text{turns}}{\text{metre}}$.

15. 2 cm is closely wound with 200 turns. Calculate the magnetic field intensity at either end of solenoid when the current in the windings is 5 amp.

Sol : Given that, $N = 200$ turns; $l = 20$ cm $= 2 \times 10^{-1}$ m; $i = 5$ amp

$n = \frac{N}{l} = \frac{200}{2 \times 10^{-1}} = 1000$ turns/metre

$\therefore$ The field at the end of the solenoid $H = \frac{B}{\mu_0} = \frac{\mu_0 ni}{\mu_0 2} = \frac{1000 \times 5}{2} = 2500$ amp/m.

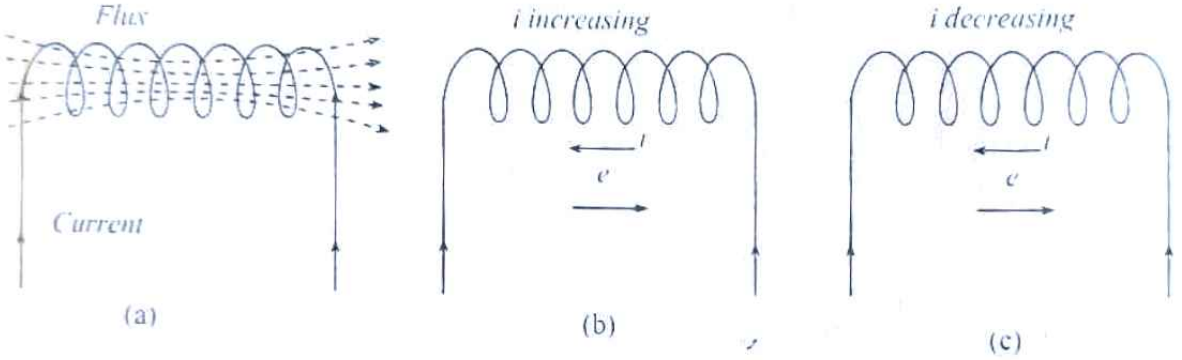
16. A wire of 60 cm length and mass 10 gm is suspended by a pair of flexible leads in a magnetic field of induction 0·60 weber/m^2. What are the magnitude and direction of the current required to remove the tension in the supporting leads ($\theta = 90^0$)

UNIT II 4. ELECTROMAGNETIC INDUCTION

LONG ANSWER TYPE QUESTIONS

What is self inductance ? Define coefficient of self induction and obtain an expression for self inductance of a solenoid. **[ANU 17; AdNU 17; BRAU 17; RU 17; SKU 18; SVU 18, 17; YVU 17]**

Self inductance : The phenomenon in which an e.m.f is induced in a coil due to the change of current or magnetic flux through it is, called self inductance. It is discovered by J. Henry.



Explanation :

1) When a current flows in a coil, magnetic field is set up in it as shown in fig(a).
2) If the current passing through the coil is changed, an induced e.m.f is set up in the coil.
3) By Lenz's law, the direction of induced e.m.f is such as to oppose the change in current.
4) When the current is increasing, the induced e.m.f is against the current shown in fig (b).
5) When the current is decreasing, the induced e.m.f is in the direction of the current shown in fig(c).
6) So the induced e.m.f. opposes any change of the original current. This phenomenon is called self induction.

Def. of coefficient of self induction :

1) The total magnetic flux ϕ_B linked with a coil is proportional to the current i, flowing in it
   i.e., $\phi_B \propto i \Rightarrow \phi_B = Li$ where L is a constant called the coefficient of self induction.
2) It is defined as the ratio of magnetic flux linked with coil to unit current flows through it.
   And e.m.f, $e = -\dfrac{d\phi_B}{dt} = \dfrac{-d(Li)}{dt} = \dfrac{-Ldi}{dt}$
3) It is also defined as the ratio of induced e.m.f in the coil to rate of change of current in the coil. S.I unit of self inductance is Henry.

Self inductance of a long solenoid :

Definition of solenoid :

1) A long wire is bent into the form a spiral is called solenoid.
2) Suppose there is an air core solenoid of length l, number of turns N, area of cross-section A and current flowing i.

   Then $B = \frac{\mu_0 N_i}{l}$

3) $\therefore$ Flux through each turn $= BA = \frac{\mu_0 NiA}{l}$

4) Total flux through N turns $\phi = N \times \frac{\mu_0 NiA}{l}$

   $\therefore \phi = \frac{\mu_0 N^2 iA}{l} \rightarrow (1)$

5) But $\phi = Li \rightarrow (2)$
6) From equation (1) and (2), we have

   $Li = \frac{\mu_0 N^2 iA}{l}$

   $\therefore L = \frac{\mu_0 N^2 A}{l}$ Henry.

7) If $\frac{N}{l} = n =$ number of turns per unit length.

   Then $L = \mu_0 n^2 Al$. Henry.

2. Define Faraday's laws of electromagnetic induction. Express the Faraday's law in integral and differential forms.

[ANU 18; BRAU 17; KU 18; RU 18; SKU 18; VSU 18; YVU 17]

A. **Faraday's laws :**

1) When the magnetic flux linked with a circuit is changed, an induced e.m.f is set up in the circuit.

2) The magnitude of the induced e.m.f is directly proportional to the negative rate of change of magnetic flux linked with the circuit.

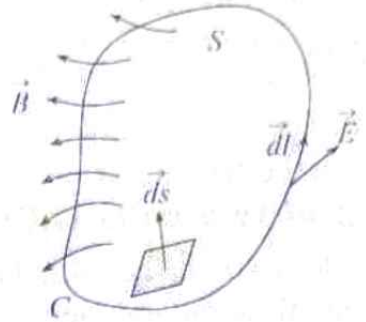
If ϕ_B be the magnetic flux linked with circuit at any instant and e be the induced e.m.f, then $e = -\left(\frac{d\phi_B}{dt}\right)$. This law is called Neumann's law. If there are N turns in the coil, then $e = N\left(\frac{d\phi_B}{dt}\right) \rightarrow (1)$

Vector form of Faraday's law (integral and differential forms) :

1) Suppose a closed circuit C of any shape which encloses a surface S is kept in a magnetic flux density B as shown in fig.
2) The magnetic flux through a small area dS is $\vec{B} \cdot \vec{ds}$.

3) Now the flux through the entire circuit is

$$\oint_B = \int_S \vec{B} \cdot \vec{ds} \rightarrow (2)$$



4) When magnetic flux is changed, an electric field is induced around the circuit.

5) The line integral of the electric field gives the induced e.m.f in the closed circuit.

Thus $e = \int \vec{E} \cdot \vec{dl}$ – (3)

where $\overline{E}$ is the electric field at an element dl of the circuit.

6) Writing equation (2) and (3) in (1), we get

$$\oint \vec{E} \cdot \vec{dl} = \frac{-d}{dt} \int_S \vec{B} \cdot \vec{ds} \rightarrow (4)$$

7) This is integral form of Faraday's law.

8) According to Stokes theorem, we have $\oint \vec{E} \cdot \vec{dl} = \int_S (\nabla \times E) \cdot dS \rightarrow (5)$

9) From equation (4) and (5), we have $\int_S (\nabla \times E) \cdot ds = \frac{-d}{dt} \int_S \vec{B} \cdot \vec{ds}$

$$\int_S (\nabla \times E) \cdot ds = -\int_S \frac{d\vec{B}}{dt} \cdot ds$$

10) It follows that $\nabla \times E = \frac{-\partial B}{\partial t}$ (or) $curl\, E = -\left(\frac{\partial B}{\partial t}\right)$

This is the differential form of Faraday's law.

3. Explain the terms self inductance and mutual inductance. Prove that $M = \sqrt{L_1 L_2}$ where the symbols have their usual meanings. Define the coefficient of coupling.

[ANU 17; AdNU 18; AU 17; KU 18; VSU 18]

A. **Self inductance :**

1) The total magnetic flux ϕ_B linked with a coil is proportional to the current i, flowing in it. i.e.,$\phi B \propto i \Rightarrow \phi_B = Li$ where $L =$ coefficient of self induction or self inductance.

2) It is defined as the ratio magnetic flux linked with coil of unit current flows through it. It's S.I unit is Henry.

Mutual inductance :

1) Let a current i in primary coil P produces a magnetic flux ϕ_B in the secondary coil S.

2) If P and S coils are in relative fixed positions, the flux linked with 'S' coil is proportional to the current in 'P' coil.
3) Thus $\phi_B \propto i \Rightarrow \phi_B = Mi$ where M = coefficient of mutual induction or mutual inductance.
4) It is defined as the ratio of flux linked with 'S' circuit to a unit current flowing through the 'P' circuit.

Coupling of two coils with flux linkage or coefficient of coupling :

1) Consider two closed coils having number of turns N_1 and N_2. Let i_1, i_2 be the currents flowing in them.
2) By definition of self inductance $L_1 = \frac{N_1 \phi_1}{i_1} \rightarrow (1)$ and $L_2 = \frac{N_2 \phi_2}{i_2} \rightarrow (2)$

   where ϕ_1 and ϕ_2 are magnetic fluxes.
3) When current i_1 is passed in the 1st coil, magnetic flux ϕ_{21}, linked with 2nd coil. Suppose M_{21}, is the mutual inductance due to i_1 in the 1st coil, then $N_2 \phi_{21} = M_{21} i_1$
4) When current i_2 is passed in the 2nd coil, magnetic flux ϕ_{12} linked with 1st coil. Suppose M_{12} is the mutual inductance due to i_2 in the 2nd coil, then $N_1 \phi_{12} = M_{12} i_2$.
5) If two coils are coaxially wound on the same core, then the coupling is perfect.
6) Now $\phi_{12} = \phi_2$ and $\phi_{21} = \phi_1$

   The situation is known as maximum flux linkage situation.
7) In this situation, applying the reciprocity theorem, we have $M_{12} = M_{21} = M_{\text{Maximum}}$
8) Then $N_1 \phi_1 = M_{\text{Max.}} i_1$ and $N_1 \phi_2 = M_{\text{Max.}} i_2 \rightarrow (3)$
9) Multiplying equation (1) and (2), we get

$$M_{\max}^2 i_1 i_2 = N_1 N_2 \phi_1 \phi_2 \Rightarrow M_{\max}^2 = \frac{N_1 \phi_1}{i_1} \times \frac{N_2 \phi_2}{i_2} \Rightarrow M_{\max}^2 = L_1 L_2$$

$$\therefore M_{\max} = \sqrt{L_1 L_2}$$

11) Now we express the mutual inductance between two coils as $M = K\sqrt{L_1 L_2}$ where K is called coefficient of coupling.
12) It is defined as the ratio of maximum flux from one coil links with the other to the square root of product of self inductance of 1st and 2nd coil.

4. Describe the principle, construction, working and theory of a transformer with the help of a neat diagram. **[ANU 19; AU 18; BRAU 18; SVU 18; YVU 18]**

A. **Transformer :** A device, decrease or increase of voltage is called transformer.

Principle : The transformer is based on the principle of mutual induction.

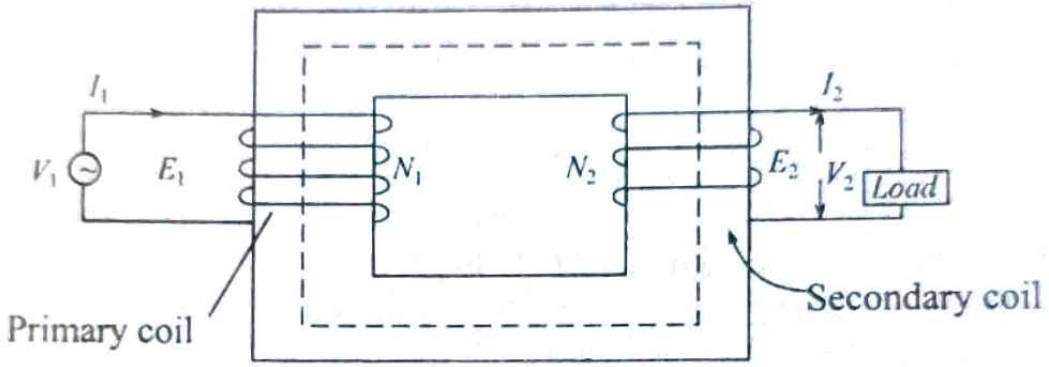
Construction :

1) It consists of two separate coils wound around a closed soft iron core.
2) To prevent the loss of energy due to eddy currents the core is a laminated one.

3) One of the coils is of low resistance and is connected to A.C.Source of supply. It is called the primary coil.
4) The other is called the secondary coil. It is connected to the load which may be some device which draws current and uses electric power.
5) In a step up transformer the secondary coil has more turns than the primary, and in step-down transformer, the secondary coil has less turns than the primary.

Working :

1) The alternating emf in the primary coil induced an alternating emf in the secondary coil.
2) The presence of iron core in the primary and secondary makes the flux linkage between the two coils very large.
3) The alternating emf in the primary coil makes the magnetic flux in the core also to vary periodically.
4) This varying magnetic flux in iron core induces an alternating emf in the secondary.



Theory :

1) Let the flux ϕ through the iron (core of the transformer) be changing at the rate of $\frac{d\phi}{dt}$.
2) If n_1 be the number of turns in primary, the flux linked with the primary is $\phi_1 = n\phi$ and the induced emf in the primary is $E_1 = \frac{-d\phi_1}{dt} = -n_1 \frac{d\phi}{dt}$.
3) Similarly the flux linked with the secondary is $\phi_2 = n_2\phi$ and the emf in the secondary is $E_2 = \frac{-d\phi_2}{dt} = -n_2 \frac{d\phi_1}{dt}$

$$\therefore \frac{E_2}{E_1} = \frac{n_2}{n_1} \rightarrow (1)$$

4) Now since the resistance of the primary is low, we can take the induced emf E_1 to be very nearly equal to the applied voltage to be open (its resistance is large) so that terminal voltage V_2 is equal to the induced emf E_2.

$$\therefore \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{n_2}{n_1} \rightarrow (2)$$

5) In a step-up transformer, $V_2 > V_1$ and there fore $n_2 > n_1$. In a step-down transformer, $V_2 < V_1$ and therefore $n_2 < n_1$.

6) Now $\frac{n_2}{n_1}$ is called the transformation ratio.

7) In the case of an ideal transformer there are no losses i.e., the power input is equal to the power output.

8) Suppose the current in the primary and the secondary are I_1 and I_2 respectively.

9) Power input $= E_1 \times I_1$

Power output $= E_2 \times I_2$

$$\therefore \quad E_1 I_1 = E_2 I_2 \text{ (or) } \frac{E_2}{E_1} = \frac{I_1}{I_2} \rightarrow (3)$$

10) In a step-up transformer $E_2 > E_1$ but $I_2 < I_1$.

In a step-down transformer $E_2 < E_1$ but $I_2 > I_1$.

11) Thus, whatever is the gain in voltage is the corresponding loss in current and vice versa.

i.e., $\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{n_2}{n_1} \rightarrow (4)$

12) Efficiency : It is defined as the ratio of power out put to the power input.

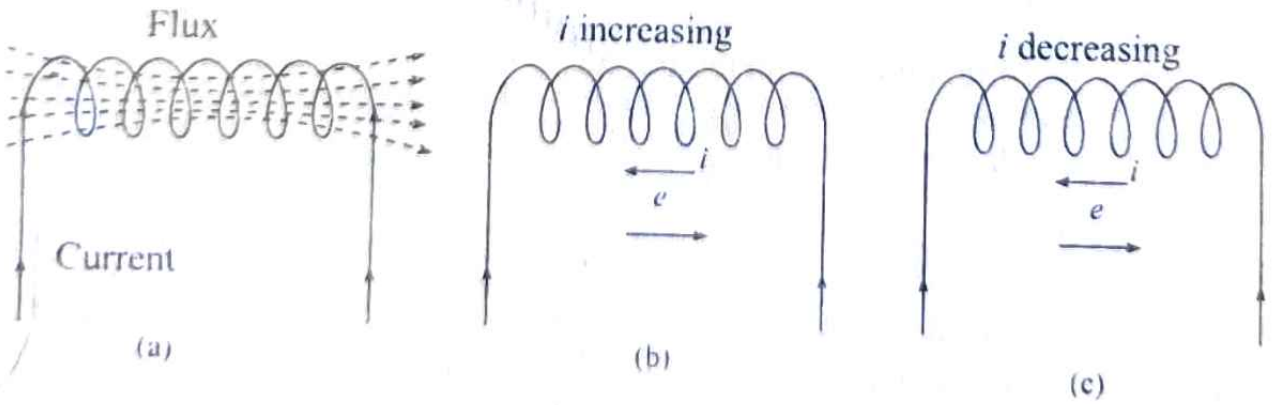
i.e., $\eta = \frac{E_2 I_2}{E_1 I_1} = \frac{V_2 I_2}{V_1 I_1}$.

SHORT ANSWER TYPE QUESTIONS

1. What is self inductance ? Explain. Give its S.I unit.

A. **Self inductance :** The phenomenon in which an e.m.f is induced in a coil due to the change of current or magnetic flux through it is called self inductance. It is discovered by J. Henry.

Explanation :



1) When a current flows in a coil, magnetic field is set up in it as shown in fig(a).
2) If the current passing through the coil is changed, an induced e.m.f is set up in the coil.

3) By Lenz's law, the direction of induced e.m.f is such as to oppose the change in current.
4) When the current is increasing, the induced e.m.f is against the current fig (b)
5) when the current is decreasing, the induced e.m.f is in the direction of the current fig(c)
6) So the induced e.m.f. opposes any change of the original current. This phenomenon is called self induction

Def. of coefficient of self induction :

1) The total magnetic flux ϕ_B linked with a coil is proportional to the current i flowing in it
i.e.. $\phi_B \propto i \Rightarrow \phi_B = Li$ where L is a constant called the coefficient of self induction.
2) It is defined as the ratio of magnetic flux linked with coil to unit current flows through it.

And e.m.f, $e = -\dfrac{d\phi_B}{dt} = \dfrac{-d(Li)}{dt} = \dfrac{-Ldi}{dt}$

3) It is also defined as the ratio of induced e.m.f in the coil to rate of change of current in the coil.
S.I unit of self inductance is Henry.

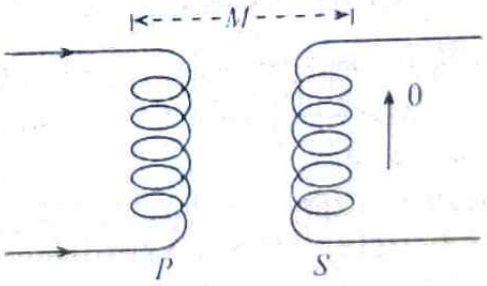
2. What is mutual inductance? Explain it.

A. **Definition of mutual inductance :**

When the current in the circuit changes, there is change in the magnetic flux linked with neighbouring circuit is called mutual induction and the pair of circuits which show it are said to have mutual inductance.

Explanation :

1) Let a current i in primary coil P produces a magnetic flux ϕ_B in the secondary coil S.
2) If 'P' and 'S' coils are in relative fixed positions, the flux linked with 'S' coil is proportional to the in 'P' coil.



3) Thus $\phi_B \propto i \Rightarrow \phi_B = Mi$ where $M =$ coefficient of mutual induction or mutual inductance.
4) It is defined as the ratio of flux linked with 'S' circuit to a unit current flowing through the 'P' circuit.
5) The induced e.m.f in the secondary S is given by

$$e = \frac{-d\phi_B}{dt} = \frac{-d}{dt}(Mi) = -M\frac{di}{dt}$$

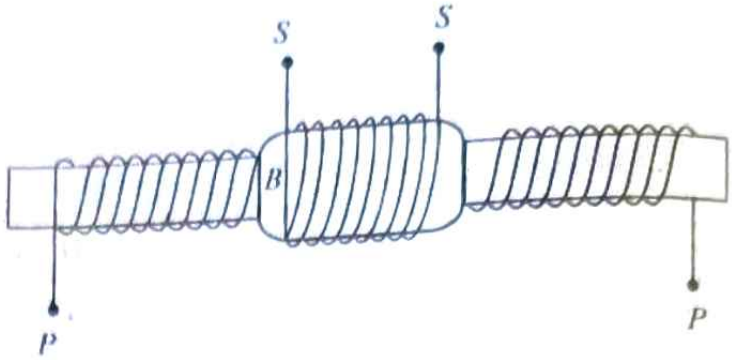
6) If it also defined as the ratio of induced e.m.f in the S circuit to unit rate of decay of current in the P circuit. S.I unit of mutual inductance is Henry.

3. Derive the expression for mutual inductance of two solenoids or two given coils ? **[RU 2017]**

A. 1) Consider two co-axial solenoids A and B.

2) A is the primary solenoid and B is the secondary solenoid.

3) It is assumed that there is no leakage of magnetic flux.



4) Number of turns of primary solenoid $= N_1$

Area of cross section $= A$

Length of primary coil $= l$

5) Number of turns of the secondary coil $= N_2$

Current flowing through the primary $= i$

6) Magnetic field at any point inside $= \dfrac{\mu_0 N_1 i}{l}$

7) $\therefore$ Magnetic flux through each turn of the secondary $= \dfrac{\mu_0 N_1 i A}{l}$

8) Total magnetic flux through N_2 turns of the secondary, $\phi = \dfrac{\mu_0 N_1 i A N_2}{l}$

9) But $\phi = Mi$

$$\therefore \; Mi = \frac{\mu_0 N_1 N_2 i A}{l}$$

$$\therefore \; M = \frac{\mu_0 N_1 N_2 A}{l}$$

4. Derive an expression for the energy stored in a magnetic field.

[AdNU 18; AU 18, 17; BRAU 18; RU 18; SKU 18; VSU 18, 17; YVU 18]

A. 1) When the current in a coil is switched on, self-induction opposes the growth of the current .

2) The current flows against back e.m.f and does work against it.

$$dw = eidt = -L\frac{di}{dt}idt \qquad \left[\because e = -L\frac{di}{dt}\right]$$

3) Hence total workdone in bringing the current from zero to a steady maximum value i_0 is,

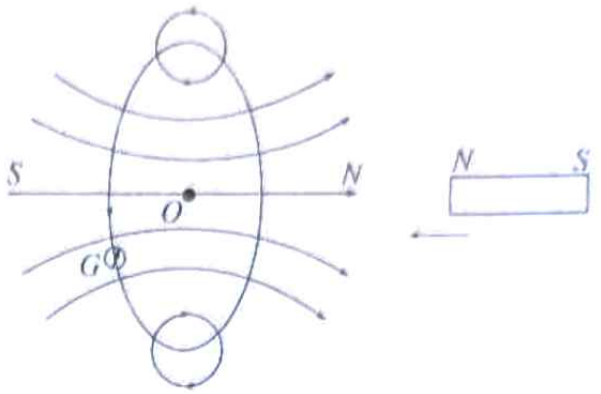
$$W = L\int_0^{i_0} i\frac{di}{dt}\cdot dt = L\int_0^{i_0} i\,di = \frac{1}{2}L\,i_0^2$$

4) This is the stored energy in a magnetic field.

5. State and explain Lenz's law. **[AdNU 17; BRAU 17]**

A. **Statement :** The direction of induced e.m.f or current in a closed circuit is such that it opposes the original cause that produces it.

Explanation of Lenz's law :



1) Suppose the north pole of a magnet is moved towards a coil connected to a galvanometer as shown in in fig.
2) As the magnet is pushed towards the circuit, an induced current is set up in the coil.
3) Due to this current the coil behave as a magnet.
4) The face of the coil towards *N*-pole of the magnet becomes a north pole.
5) So there will be a force of repulsion between them.
6) Due to this force of repulsion, the motion of the magnet is opposed.This causes a change of magnetic flux in the coil.
7) Thus the direction of induced current is such as to oppose the motion of the magnet.

6. Define Coefficient of self induction and coefficient of mutual induction. **[VSU 2017]**

A. i) **Defination of coefficient of self induction:**

1) It is defined as the ratio of magnetic flux linked with coil to unit current flows through it.

And e.m.f, $e = -\frac{d\phi_B}{dt} = \frac{-d(Li)}{dt} = \frac{-Ldi}{dt}$

2) It is also defined as the ratio of induced e.m.f in the coil to rate of change of current in the coil.
3) S.I unit of self inductance is Henry.

ii) **Defination of coefficient of mutual induction:**

1) It is defined as the ratio of flux linked with '*S*' circuit to a unit current flowing through the '*P*' circuit.
2) The induced e.m.f in the secondary *S* is given by

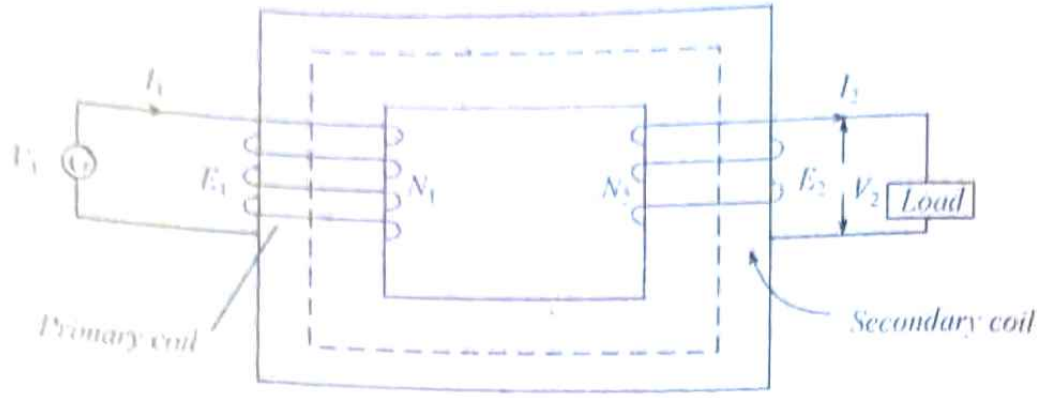
$$e = \frac{-d\phi_B}{dt} = \frac{-d}{dt}(Mi) = -M\frac{di}{dt}$$

3) If it also defined as the ratio of induced e.m.f in the *S* circuit to unit rate of decay of current in the *P* circuit. S.I unit of mutual inductance is Henry.

7. Write a note on transformer. **[AdNU 2017]**

A. **Transformer :**

1) A transformer is an A.C static device which transfers electric power from one circuit to another.
2) It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current.



3) It works on the principle of mutual induction.
4) Fig shows a basic transformer
5) The transformer consists of primary coil and secondary coil wound on a common core.
6) They are electrically insulated but magnetically linked.
7) The energy from one coil is transferred to other coil by means of magnetic coupling.
8) There are 2 types of transformers. 1. Step-up transformer ($N_2 > N_1$) 2. Step-down transformer ($N_1 > N_2$)
9) The principle of transformer is given by

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

10) The efficiency of a transformer is defined as the ratio of output power to the input power.

i.e., $\eta = \frac{\text{Output power}}{\text{Inputpower}} = \frac{V_2 I_2}{V_1 I_1}$

8. List the various losses in a transformer and explain how they can be minimised.

A. There are four main energy losses in transformers

i) **Copper losses :** Primary and secondary coils are made with copper. Heat is produced in the primary and secondary coils due to Joule's heating effect. The power loss $= I^2R$. Where I is the current flowing and R is the resistance. This loss is called copper loss. This is minimised by using wires of large diameters so that they have low resistance.

ii) **Eddy current losses or Iron losses :** When the flux linked with the core of the transformer changes, closed circuits of currents will be induced in the core. These closed circuits of currents are known as Eddy currents. The formation of eddy currents in the core results in the loss of energy in the form of heat energy developed in the core. This loss can be minimised by having the core made up of several thin sheets of an alloy of steel known as stalloy. The different sheets are bound together to form a laminated core.

iii) **Hysteresis losses :** There is a loss of power in magnetising an iron core and taking it through a complete cycle of magnetisation. This loss in power is called hysteresis loss. This is minimised by using Iron-Silicon core (soft-iron core).

iv) **Magnetic flux leakage :** In an actual transformer the magnetic flux is not confined entirely to the iron core but some the flux lines return through air. This leakage takes place both in the primary and the secondary coils. This is minimised by using a shell shaped core.

9. Write about eddy currents.

Eddy currents are loops of electrical current induced within conductors by a changing magnetic field in conductor according to Faraday's law of electromagnetic induction.

Eddy currents flow in closed loops within conductors. in planes perpendicular to the magnetic field. They can be induced within nearby stationary conductors by a time varying magnetic field created by an AC.

The magnitude of the current in a given loop is proportional to the strength of the magnetic field, the area of the loop and the rate of change of flux and inversely proportional to the resistivity of the material.

Eddy currents are used to heat objects in induction heating furnaces and equipment and to detect cracks and flows in metals in metal parts using eddy current testing instruments.

10. Explain electromagnetic damping.

When a charge is passed through the galvanometer, the coil sets swinging. Because the coil moves in the field of permanent magnet, an induced current is set up in it. This current gives rise to a couple on the coil. Now the coil soon comes to rest and its motion is said to be damped. The damping which arises due to the induced current in the coil during its motion in permanent magnetic field is called electromagnetic damping.

PROBLEMS

1. Calculate the self inductance of a solenoid of length 1 metre and area of cross-section 0·01 sq – m^2 with 2000 turns.

Sol : Given that, $l = 1$ m; $A = 0{\cdot}01$ sq – m; $N = 2000$ turns

$$\therefore\ L = \mu_0 n^2 Al = (4\pi \times 10^{-7})\left(\frac{2000}{1}\right)^2 (0{\cdot}01) \times 1$$

$$= 4\pi \times 10^{-7} \times 4 \times 10^6 \times 10^{-2} = 16 \times 3{\cdot}14 \times 10^{-3} = 0{\cdot}05027 \text{ henry.}$$

2. A solenoid of length of 0·50 m wound with 5000 turns/m of wire has a radius 4 m. Calculate the self inductance of solenoid.

Sol : Given that $n = 5000$ turns/m, $l = 0{\cdot}50$ m and $r = 0{\cdot}04$ m

$$\therefore\ L = (4\pi \times 10^{-7})(5000)^2 \times \pi(0{\cdot}04)^2 \times 0{\cdot}50 = 79{\cdot}02 \times 10^{-3} \text{ henry}$$

$$= 79{\cdot}02 \text{ mH.}$$

*3. What is self inductance of a 50cm long solenoid with 2cm diameter and having 200 turns ? where $\mu_0 = 4\pi \times 10^{-7}$ H/m **[KU M16]**

Sol : Given that, $l = 50$ cm $= \frac{1}{2}$ m; $d = 2$ cm; $r = 1$ cm $= 1 \times 10^{-2}$ m; $N = 200$ turns

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$$L = \frac{\mu_0 N^2 A}{l} = (4\pi \times 10^{-7})\left(\frac{200}{0.5}\right)^2 \times (\pi \times (0.01)^2) \times 0.5$$

$= 3.158 \times 10^{-5}$ henry.

4. A coil has 600 turns. Its self inductance is 100 mH. Find the self inductance of another same type of coil having 500 turns.

Sol : Given that $N_1 = 600$ turns ; $L_1 = 100$ mH; $N_2 = 500$ turns ; $L_2 = ?$

We know that $\frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2$

$$\frac{L_2}{100} = \left(\frac{2500}{600}\right)^2 \Rightarrow L_2 = \frac{25}{36} \times 100$$

$\therefore L_2 = 69.44$ mH.

5. An air core solenoid has 1000 turns and is one meter long. Its cross-sectional area is 6×10^{-4} Sq-m. Calculate its self inductance ($\mu_0 = 4\pi \times 10^{-7}$ h/m).

Sol : Given that, $l = 1$ m; $A = 6 \times 10^{-4}$ m^2, $N = 1000$; $n = \frac{N}{l} = \frac{1000}{1} = 1000$

and $\mu_0 = 4\pi \times 10^{-7}$ h/m

$\therefore$ Self inductance, $L = \mu_0 n^2 l A = (4\pi \times 10^{-7}) \times (1000)^2 \times 1 \times 6 \times 10^{-4}$

$= 75.36 \times 10^{-8}$ henry.

6. Calculate the self inductance of an air cored toroid of mean radius 20 cm and a circular cross-section of area 5 cm^2. The total number of turns of the toroid is 3000. **[ANU J16]**

Sol : Given that $N = 3000$; $A = 5$ cm$^2 = 5 \times 10^{-4}$ m^2

$R = 20$ cm $= 0.2$ m; $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Wb}}{\text{amp} - \text{m}}$

$\therefore$ The self inductance of an air cored toroid $L = \mu_0 \left[\frac{N^2 A}{2\pi R}\right]$

$$= (4\pi \times 10^{-7})\left[\frac{(3000)^2 \times (5 \times 10^{-4})}{2 \times 3.14 \times 0.2}\right] = 4.5 \times 10^{-3} \text{ henry} = 4.5 \text{ mH}.$$

7. A 20 henry inductor carries a steady current of 2 amp. How can a 200 volt self induced e.m.f can be made to appear in the inductor?

Sol : Given that $L = 20$ henry ; $e = 200$ volt

$e = -L\frac{di}{dt} = L\frac{di}{dt}$ (numerically)

$\frac{di}{dt} = \frac{e}{L} = \frac{200}{20} = 10$ amp/sec.

UNIT III

5. ALTERNATING CURRENTS

LONG ANSWER TYPE QUESTIONS

1. An A.C. voltage is applied to a series L-R circuit. Derive the equation for the current, impedence and phase angle with the help of phase diagram.

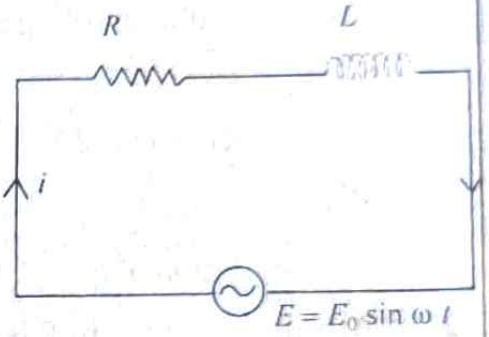
[ANU 19; AdNU 17; BRAU 18; VSU 17]

Circuit consists of resistor of resistance R and inductor of inductance L connected in series. An A.C. voltage $E = E_0 \sin \omega t$ is applied to it. Let i be the current at that instant of time t.

Back e.m.f. developd across inductor is $-L\dfrac{di}{dt}$

Total e.m.f = $= E_0 \sin \omega t - L\dfrac{di}{dt}$

According to ohm's law, this must be equal to iR



$$E_0 \sin \omega t - L\frac{di}{dt} = iR$$

$$L\frac{di}{dt} + iR = E_0 \sin \omega t \rightarrow (1)$$

Let the trail solution of equation (1) is

$$i = i_0 \sin(\omega t - \phi) \rightarrow (2)$$

Differentiating, we get $\dfrac{di}{dt} = i_0 \omega \cos(\omega t - \phi) \rightarrow (3)$

Substituting equations (2) and (3) in equation (1)

$$\omega L i_0 \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin \omega t$$

$$= E_0 \sin\{(\omega t - \phi) + \phi\}$$

$$= E_0 \sin(\omega t - \omega)\cos\phi + E_0 \cos(\omega t - \phi)\sin\phi \rightarrow (4)$$

Comparing the coefficients of $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ both sides

$$\omega L i_0 = E_0 \sin\phi \rightarrow (5)$$

$$R i_0 = E_0 \cos\phi \rightarrow (6)$$

Squaring equations (5) & (6), then adding

$$i_0^2[R^2 + (\omega L)^2] = E_0^2(\sin^2\phi + \cos^2\phi)$$

$$i_0^2[R^2 + (\omega L)^2] = E_0^2$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + (\omega L)^2}} \rightarrow (7)$$

Impedance $(Z) = \frac{E_0}{i_0}$

$Z = \sqrt{R^2 + (\omega L)^2} \rightarrow (8)$

Dividing equation (5) by equation (6)

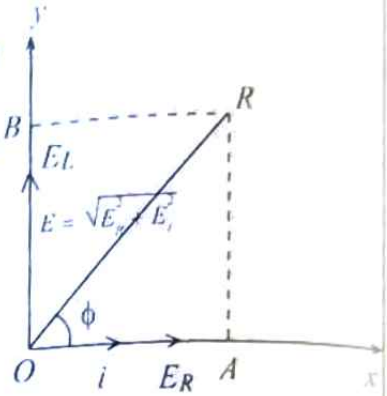
$$\frac{\omega L i_0}{R i_0} = \frac{E_0 \sin\phi}{E_0 \cos\phi}$$

$$Tan\ \phi = \frac{\omega L}{R}\ ;\ \phi = Tan^{-1}\left(\frac{\omega L}{R}\right) \rightarrow (9)$$

Vector diagram :

The voltage across resistance always remains in phase with current but voltage across inductance lead over current 90°.

In figure E_R and E_L are mutually at right angles. The vector OA represents E_R while the vector OB represents E_L. By the parallelogram law of vectors, the diagonal OR represents resultant voltage E.



$\therefore\ E^2 = E_R^2 + E_L^2$

$(iZ)^2 = (iR)^2 + (i\,\omega L)^2$

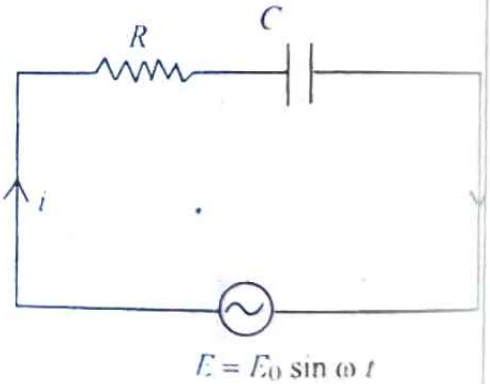
$Z^2 = R^2 + (\omega L)^2$

$Z = \sqrt{R^2 + (\omega L)^2}$.

2. An alternating e.m.f is applied to a circuit having capacitance and resistance in series. Derive an expression for the current and impedance. Draw a vector diagram. **[BRAU 18, 17]**

A. Circuit containing resistance R and a capacitance C in series, connected to an A.C. voltage $E = E_0 \sin\omega t$.

Let q be the charge and i be the current at that instant of time t. Back e.m.f across capacitor is $-\frac{q}{C}$.



Total e.m.f $= E_0 \sin\omega - \frac{q}{C}$

According to ohms law, this must be equal to iR

$$E_0 \sin\omega t - \frac{q}{C} = iR$$

$$iR + \frac{q}{C} = E_0 \sin\omega t$$

Differentiating, $R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = E_0\,\omega\cos\omega t$

$$R\frac{di}{dt} + \frac{i}{C} = E_0\,\omega\cos\omega t \rightarrow (1) \qquad \left(\because i = \frac{dq}{dt}\right)$$

Let the solution of eq (1) is $i = i_0 \sin(\omega t - \phi) \rightarrow (2)$

Differentiating, $\frac{di}{dt} = i_0\,\omega \cos(\omega t - \phi) \rightarrow (3)$

Substituting equation (2) and (3) in equation (1)

$$i_0\,\omega R \cos(\omega t - \phi) + \frac{i_0}{C}\sin(\omega t - \phi) = E_0\,\omega \cos \omega t$$

$$= E_0\,\omega \cos\{(\omega t - \phi) + \phi\}$$

$$= E_0\,\omega \cos(\omega t - \phi)\cos\phi - E_0\,\omega \sin(\omega t - \phi)\sin\phi) \rightarrow (4)$$

Comparing the coefficients of $\cos(\omega t - \phi)$ and $\sin(\omega t - \phi)$ both sides

$$i_0\,\omega R = E_0\,\omega \cos\phi$$

$$i_0 R = E_0 \cos\phi \rightarrow (5)$$

$$\frac{i_0}{C} = -E_0\,\omega \sin\phi$$

$$-\frac{i_0}{\omega C} = E_0 \sin\phi \rightarrow (6)$$

Squaring equations (5) and (6), then adding

$$i_0^2\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right] = E_0^2\,[\cos^2\phi + \sin^2\phi]$$

$$i_0^2\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right] = E_0^2$$

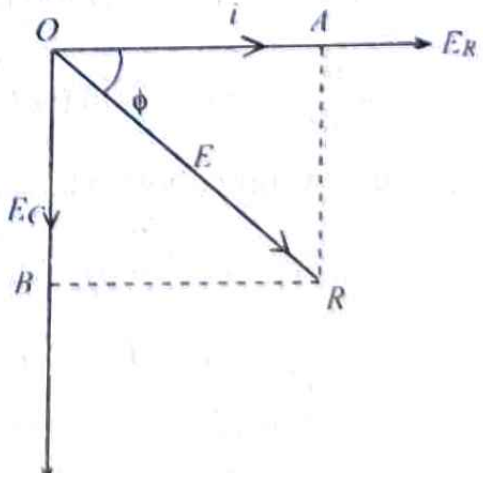
$$i_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \rightarrow (7)$$

Impedance $(Z) = \frac{E_0}{i_0} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \rightarrow (8)$

Dividing equation (6) by equation (5), $Tan\,\phi = \frac{1/\omega C}{R}$

$$\phi = Tan^{-1}\left(\frac{1}{\omega C R}\right) \rightarrow (9)$$

Phase diagram :



The voltage across resistance always remains same in phase with current but voltage across condenser lags behind by 90°. The vector OA represents E_R and the vector OB represents E_C. The resultant vector OR represents E.

$\therefore E^2 = E_R^2 + E_C^2$

$$(iZ)^2 = (iR)^2 + \left(\frac{1}{\omega C}\right)^2$$

$$Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \rightarrow (10)$$

3. Derive an expression for the current in the A.C circuit containing resistance, inductance and capacitance all in series with the applied alternating e.m.f. Explain what is meant by the resonance of the LCR series A.C circuit ?

[ANU 19, 18, 17; AdNU 18; AU 18, 17; BRAU 17; RU 18, 17; SKU 18; SVU 18, 17; VSU 17; YVU 18]

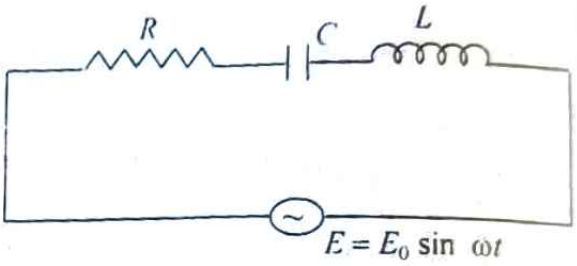
A. A conductor of resistance R, an inductance L, and a capacitance C are connected in series and the combination is connected across a source of alternating e.m.f $E = E_0 \sin \omega t$. Let i be the current in the circuit at any instant t and q be the charge.

The potential difference across the condenser $= -\frac{q}{C}$

Back E.m.f due to inductance $= -L\frac{di}{dt}$

Effective e.m.f

$$= E_0 \sin \omega t - \frac{q}{C} - L\frac{di}{dt}$$



According to Ohm's law this must be equal to Ri.

$$E_0 \sin \omega t - \frac{q}{C} - L\frac{di}{dt} = Ri$$

$$L\frac{di}{dt} + Ri + \frac{q}{C} = E_0 \sin \omega t$$

Differentiating, we get $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = E_0\omega \cos \omega t$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = E_0\, \omega \cos \omega t \rightarrow (1)$$

Let the trial solution of equation (1) is $i = i_0 \sin(\omega t - \phi) \rightarrow (2)$

where i_0 and ϕ are constants.

Differentiating equation (2), $\frac{di}{dt} = i_0\, \omega \cos(\omega t - \phi)$

and $\frac{d^2i}{dt^2} = -i_0\, \omega^2 \sin(\omega t - \phi)$

Substituting above values in equation (1), We get

$$-Li_0\, \omega^2 \sin(\omega t - \phi) + Ri_0\, \omega \cos(\omega t - \phi) + \left(\frac{i_0}{C}\right) \sin(\omega t - \phi)$$

$$= E_0\, \omega \cos \omega t = E_0\, \omega \cos\{(\omega t - \phi) + \phi\}$$

$$= E_0\, \omega \cos(\omega t - \phi) \cos \phi - E_0\, \omega \sin(\omega t - \phi) \sin \phi \rightarrow (3)$$

Comparing the coefficients of sin ($\omega t - \phi$) and cos ($\omega t - \phi$) on both sides of equation (3)

$$-Li_0\omega^2 + \frac{i_0}{C} = -E_0\omega\sin\phi$$

$$\left(-L\omega^2 + \frac{1}{C}\right)i_0 = -E_0\omega\sin\phi$$

$$\left(\omega L - \frac{1}{\omega C}\right)i_0 = E_0\sin\phi \rightarrow (4)$$

and $R\omega i_0 = E_0\omega\cos\phi$

$$R i_0 = E_0\cos\phi \rightarrow (5)$$

Squaring and adding equations (4) and (5)

$$i_0^2\left[R^2 + \left(\omega L - \frac{1}{\omega}C\right)^2\right] = E_0^2\left[\sin^2\phi + \cos^2\phi\right]$$

$$i_0^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] = E_0^2$$

$$i_0^2 = \frac{E_0^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \rightarrow (6)$$

Thus $i = \dfrac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\sin(\omega t - \phi) \rightarrow (7)$

Dividing equation (4) by equation (5), we get

$$\phi = Tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \frac{\omega L - \frac{1}{\omega C}}{R} \rightarrow (8)$$

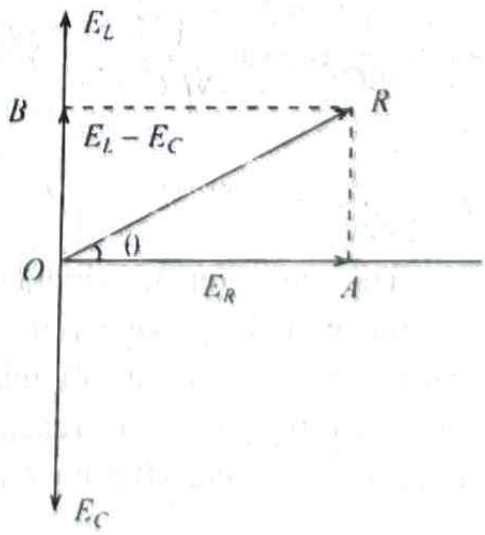
Impedance

$$(Z) = \frac{E_0}{i_0} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \rightarrow (9)$$

or $Z = \sqrt{R^2 + (X_L - X_C)^2}$

where $X_L = \omega L$ = Inductive reactance

$X_C = \dfrac{1}{\omega C}$ = Capacitive reactance



Vector diagram :

In series LCR circuit the P.D across the resistance E_R is in phase with the current. i_0 is

the peak value of the current. This is representedn by the vector $OA = Ri_0$. The voltage across the inductance $E_l = \omega L i_0$, which is 90° advance of the current. The voltage across the capacitor $E_c = \frac{i_0}{\omega C}$, which is 90° lags the current. Thus E_l and E_c are antiphase. The resultant of the two represented by

$E = E_l - E_c = i_0\left(\omega L - \frac{1}{\omega C}\right)$. The resultant of E_R and E can be obtained by vector addition method. This is represented by OR.

$$E_0^2 = (OR)^2 = (Ri_0)^2 + i_0^2\left(\omega L - \frac{1}{\omega C}\right)^2$$

$$= i_0^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

$$OR = i_0\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} = E_0$$

$$\text{Thus } Z = \frac{E_0}{i_0} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}}$$

$$\text{From figure } \tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

Series resonant circuit :

So at a particular frequency, the total reactance in the circuit is zero $\left(\omega L = \frac{1}{\omega C}\right)$. The current becomes maximum. Such a condition is called resonance and the circuit under this condition is termed as series resonant circuit.

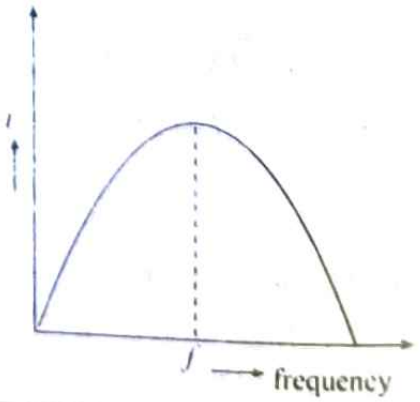
$$\text{At resonance } \left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\text{The corresponding frequency is given by } \omega L = \frac{1}{\omega C}$$

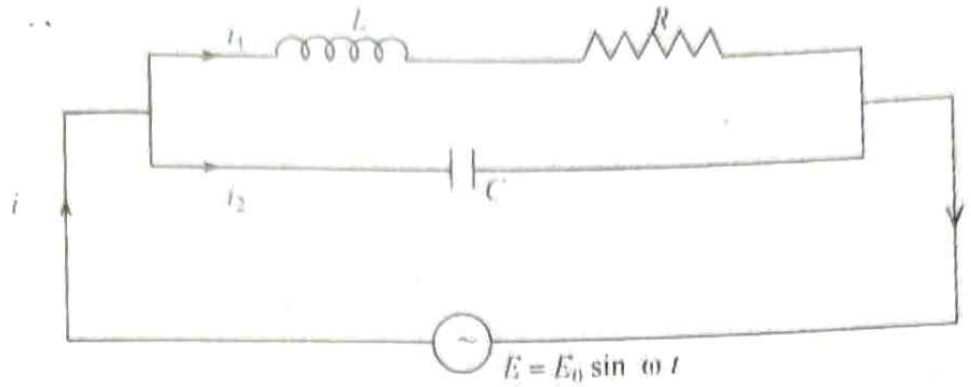
$$\omega^2 = \frac{1}{LC} \text{ or } \omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The variation of current with frequency is shown in the above figure. The current increases as the frequency increases until the resonant frequency is reached where it is maximum. There after the current falls.



Give the theory of parallel resonant circuit ? Define quality factor. **[KU 2018]**



Consider a parallel resonant circuit consisting of an inductance L and resistance R in series in one branch and a condenser of capacitance C in another branch as shown in the figure. A source of an alternating e.m.f $E = E_0 \sin \omega t$ is connected to the circuit. If the total current at any instant is i, the current through the inductance branch and capacitance branch are i_1 and i_2.

From Kirchoff's law.

$i = i_1 + i_2 \rightarrow (1)$

Let Z be the impedance of the circuit.

Z_1 = Impedance of inductance and resistance $= R + J\omega L$

Z_2 = Impedance of condenser $= \dfrac{1}{J\omega C}$

Using equation (1), $\dfrac{E_0}{Z} = \dfrac{E_0}{Z_1} + \dfrac{E_0}{Z_2}$

$$\frac{E_0}{Z} = \frac{E_0}{R + J\omega L} + \frac{E_0}{1/J\omega C}$$

$$\frac{1}{Z} = \left(\frac{1}{R + J\omega L} + J\omega C\right)$$

Now admittance $(Y) = \dfrac{1}{Z} = \dfrac{1}{R + J\omega L} + J\omega C$

$$Y = \frac{(R - J\omega L)}{(R - J\omega L)(R + J\omega L)} + J\omega C$$

$$= \frac{R - J\omega L)}{(R^2 + \omega^2 L^2)} + J\omega C$$

$$Y = \frac{R + J(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

The magnitude of admitance is given by

$$|Y| = \sqrt{\frac{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}{R^2 + \omega^2 L^2}}$$

The admitance is minimum if $\omega C R^2 + \omega^3 L^2 C - \omega L = 0$

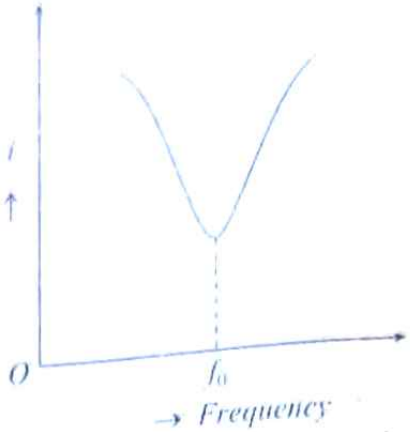
i.e., $CR^2 + \omega^2 L^2 C - L = 0$

or $\omega^2 L^2 C = L - CR^2$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$



At this frequency, the admittance is minimum and hence the current is minimum. Such a frequency is called as resonant frequency. This circuit is known as parallel resonant circuit.

Quality factor :

This is defined as 2π times the ratio of energy stored to the average energy loss per period

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy loss per period}}$$

$$Q = \frac{\omega L}{R} \text{ (or) } \frac{1}{\omega c R}.$$

SHORT ANSWER TYPE QUESTIONS

1. Write a short note on Q- factor. **[ANU 17, J16, M16; AdNU 18; RU 18, 17; SKU 17; YVU 18, 17]**

A. The quality factor is defined as 2π times the ratio of the energy stored to the average energy loss per period.

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy loss per period}}$$

$$= 2\pi f \cdot \frac{\text{Energy stored}}{\text{Power loss per period}}$$

$$Q = 2\pi f \cdot \frac{\frac{1}{2} L i_0^2}{\frac{1}{2} i_0^2 R} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

Similarly it can be shown that $Q = \frac{1}{\omega CR}$. We have seen that the power factor $(\cos\phi) = \frac{\omega L}{R}$ in an $L - R$ circuit, $\cos\phi = \frac{1}{\omega CR}$ in a $C - R$ circuit.

$\therefore$ Quality factor is equal to the power factor of the circuit.

2. Derive an expression for the power in an alternating current circuit. Explain the power factor. **[RU 2018]**

A. The power in an electric circuit is the rate at which electrical energy is consumed in the circuit. Let an e.m.f $E = E_0 \sin \omega t$ be applied to the circuit. The current in the circuit $i = i_0 \sin(\omega t - \phi)$ where ϕ is the phase difference between A.C voltage and current. The instantaneous power is given by $Ei = E_0 i_0 \sin \omega t \sin(\omega t - \phi)$

$$\text{Average power } (P) = \frac{\int_0^T Eidt}{\int_0^T dt} = \frac{1}{T}\int_0^T E_0 i_0 \sin \omega t \sin(\omega t - \phi)dt$$

$$P = \frac{1}{T}\int_0^T \frac{E_0 i_0}{2}[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi]dt$$

$$= \frac{1}{T}\frac{E_0 i_0}{2}[\int_0^T \cos\phi\, dt - \int_0^T \cos(2\omega t - \phi)dt]$$

$$= \frac{1}{T}\cdot\frac{E_0 i_0}{2}\left[t\cos\phi\Big|_0^T - \left\{\frac{\sin(2\omega t - \phi)}{2\omega}\right\}_0^T\right]$$

$$= \frac{1}{T}\cdot\frac{E_0 i_0}{2}[T\cos\phi - \frac{1}{2\omega}\{\sin(2\omega T - \phi) - \sin\phi\}]$$

$$= \frac{1}{T}\cdot\frac{E_0 i_0}{2}\left[T\cos\phi - \frac{1}{2\omega}\left\{\sin\left(2\cdot\frac{2\pi}{T}\cdot T - \phi\right) - \sin\phi\right\}\right]$$

$$P = \frac{1}{T}\frac{E_0 i_0}{2}\cdot T\cos\phi = \frac{E_0 i_0}{2}\cos\phi$$

$$P = \frac{E_0}{\sqrt{2}}\cdot\frac{i_0}{\sqrt{2}}\cos\phi = E_{rms}\times i_{rms}\times\cos\phi$$

$$\Rightarrow P = E_{rms}\times i_{rms}\cos\phi$$

Here $\cos\phi$ is known as power factor.

3. Write a short note on power factor.

A. "Power factor is defined as the ratio of true power to the apparent power".

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{E_{rms}\cdot i_{rms}\cos\phi}{E_{rms}\cdot i_{rms}}$$

Power factor $= \cos\phi$

In a pure resistive circuit, $\phi = 0$, power factor $= 1$

In a pure inductance circuit, $\phi = 90^0$, power factor $= 0$

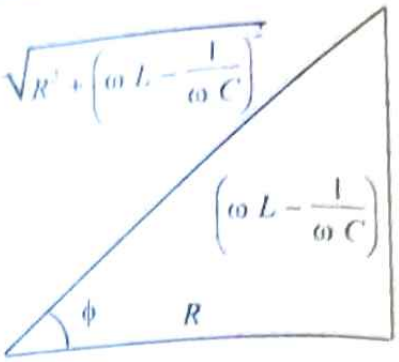
In a pure capacitance circuit $\phi = 90^0$, power factor $= 0$

In an $L - R$ circuit, $\cos\phi = \frac{\omega L}{R}$

In an $C - R$ circuit, $\cos\phi = \dfrac{1}{\omega CR}$

In an $L - C - R$ circuit, $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$

$\therefore$ Power factor $(\cos\phi) = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$



4. Write the advantages of A.C over D.C ?

A. 1) Using a transformer with A.C we can either step up or step down the voltage at our choice. We can get a wide range of A.C voltages. This type of transformation is not possible with D.C.

2) A.C induction motors and other appliances are mechanically more robust and electrically simple.

3) The line losses in A.C power transmission are very low even over long distances.

4) A.C can be readily converted into D.C. When required, by using rectifiers.

5) In A.C. circuits the current can be reduced without any appreciable loses of power, with the use of a choke or a capacitor.

5. Derive the relation between RMS, average and peak values of voltage in an A.C. circuit.

A. i) **R.M.S. value of voltage (or) current:**

The R.M.S value is the square root of the average of the sum of the square of the instantaneous values taken for one cycle.

$$E_{r\text{-}m\text{-}s} = \frac{E_0}{\sqrt{2}} \text{ and } i_{r\text{-}m\text{-}s} = \frac{i_0}{\sqrt{2}}$$

ii) Average value (or) Mean value : (V_{ave})

The average value is obtained by integrating the instantaneous value over half - cyle

Average value $= \dfrac{2}{\pi} \times E_0$

$$E_{av} = \frac{2}{\pi} \times E_0$$

Where $E_0 =$ Maximum voltage.

iii) Peak (or) Maximum value : The value of alternating voltage (or) current which has higher magnitude in a cycle is called Maximum value (or) Peak value.

E_0 and i_0 represents the maximum values of voltage and currents.

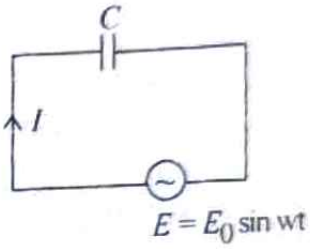
6. Distinguish between $L - C - R$ series resonance and parallel resonance circuits.

[BRAU 18; PU 18]

Series resonant circuit	Parallel resonant circuit
1) Series resonant frequency is independent of resistance $f_r = \frac{1}{2\pi\sqrt{LC}}$	1) Parallel resonant circuit depends on resistance $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
2) At resonance current is maximum.	2) At resonance current is minimum.
3) At resonance impendance is minimum.	3) At resonance impedance is maximum.
4) A series resonant circuit called the acceptor circuit.	4) A parallel resonant circuit called the rejector circuit.
5) At resonance the circuit exhibits a voltage magnification equal to Q - factor.	5) At resonance the circuit exhibits current magnification equal to Q - factor.

7. Obtain relation between voltage and current in an a.c circuit containing pure capacitance.

Let us consider a capacitor of capacity C to which an A.C. source is connected. The alternating e.m.f. applied is given by



$E = E_0 \sin wt \rightarrow (1)$

Let the instantaneous current through the capacitor at any time 't' is I. q is the charge on the capacitor at that time 't'.

The potential across the capacity $V = \frac{q}{C}$, which is equal to e.m.f 'E'.

$\therefore\ E = E_0 \sin wt = \frac{q}{C}$

$q = E_0\, c \sin wt.$

differeniating w.r.t. time, $\frac{dq}{dt} = \frac{d}{dt}[E_0 C \sin wt] = E_0\, Cw \cos wt.$

$\therefore\ I = \frac{dq}{dt} = E_0\, cw \cos wt$

$I = E_0\, cw \sin\left(wt + \frac{\pi}{2}\right);\ I = I_0 \sin wt + \frac{\pi}{2} \rightarrow (2)$

Here $I_0 = E_0 cw$, which represents maximum current.

From equations (1) & (2) the current leads the e.m.f. by $\frac{\pi}{2}$.

PROBLEMS

1. If A.C main supply is given to be 220 volts, what would be the average e.m.f during positive half cycle ?

Sol : $E_{rms} = 220$ volt

$E_{rms} = \frac{E_0}{\sqrt{2}}$ or $E_0 = \sqrt{2} \times E_{rms}$

$E_0 = \sqrt{2} \times 220 = 311{\cdot}1$ volt

The average e.m.f during a half cycle is given by

$$E_{average} = \frac{2E_0}{\pi} = \frac{2 \times 311{\cdot}2}{3{\cdot}14} = 198{\cdot}2 \text{ volt}$$

2. Calculate the current in an A.C. series circuit of inductance $0{\cdot}01H$, resistance 5Ω, with an e.m.f of 200 v rms with 50 Hz frequency.

Sol : Given that $L = 0{\cdot}01H$, $R = 5\Omega$, $E_{rms} = 200$ V

$f = 50$Hz

$\omega = 2\pi f = 2\pi \times 50 = 100\pi$

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{5^2 + (100\pi)^2 \times (0{\cdot}01)^2}$

$Z = \sqrt{25 + 9{\cdot}868} = \sqrt{34{\cdot}868}$ $Z = 5{\cdot}906\ \Omega$

Current $(I_{rms}) = \frac{E_{rms}}{Z} = \frac{200}{5{\cdot}906} = 33{\cdot}86$ amp.

3. An alternating voltage of $110V - 50$ cycles is applied to a circuit containing an inductance of $0{\cdot}02H$, and a resistance of 10 ohms. Determine the current and the phase lag.

Sol : $V_{rms} = 110$ V, $f = 50$ cycles/sec

$\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314$ rad/sec

$L = 0{\cdot}02H$, $R = 10\Omega$

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{10^2 + (314 \times 0{\cdot}02)^2}$

$= \sqrt{100 + (6{\cdot}28)^2} = \sqrt{139{\cdot}4} = 11{\cdot}8\ \Omega$

$\therefore$ The current $(I_{rms}) = \frac{V_{rms}}{Z} = \frac{110}{11{\cdot}8} = 9{\cdot}32$ ampere

The phase angle $\phi = Tan^{-1}\left(\frac{\omega L}{R}\right) = Tan^{-1}\left(\frac{6{\cdot}28}{10}\right)$

$\phi = 32^\circ 8'$

We know that the current in an inductor will be lagging behind voltage and hence the phase lag is $32^\circ 8'$.

4. A potential difference with a frequency of 50 Hz is applied to a coil of 1000Ω and inductance $2H$, calculate the power factor of the circuit. **[July 2016]**

Sol : $f = 50$ Hz, $R = 1000\ \Omega$, $L = 2H$

$\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314$ rad/sec

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{(10^3)^2 + (314)^2(2)^2} = \sqrt{1394384}$

$Z = 1180{\cdot}8\ \Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{1000}{1180.8} = 0.8467$.

5. How large an inductance should be connected in series with a 120 V and 60 W bulb, if it is to operate normally when the combination is connected to a 220V, 50Hz line mains ?

Sol : From the specifications of the lamp

$$I = \frac{60\ W}{120V} = 0.5 \text{ amp}$$

$$R = \frac{120V}{0.5\ A} = 240\ \Omega$$

Even when the bulb is connected to 220V mains.

$$Z = \frac{220V}{0.5A} = 440\ \Omega$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$R = 240\ \Omega, f = 50Hz, \omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/sec

$$Z^2 = R^2 + \omega^2 L^2$$

$$(440)^2 = (240)^2 + (314)^2 L^2$$

$$(314)^2 L^2 = (440)^2 - (240)^2 = 680 \times 200$$

$$L = \sqrt{\frac{680 \times 200}{314 \times 314}} = 1.174 \text{ henry.}$$

6. A.C voltage of 200V with frequency 50 Hz is connected in series with a resistance 60 Ω, capacitance of $4\mu F$ and an inductance of 3H. Calculate i) Impedance of the circuit and ii) Potential difference across the inductance.

Sol : $E = 200V, f = 50Hz, R = 60\ \Omega, C = 4\mu F, L = 3H, \omega = 2\pi \times 50 = 100\pi$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(60)^2 + [3 \times 100\pi - 1/(4 \times 10^{-6} \times 100\pi)^2}$$

$$= \sqrt{3600 + (942.3 - 795.6)^2}$$

$$= \sqrt{3600 + (146.7)^2} = \sqrt{3600 + 21520}$$

$$Z = \sqrt{25120} = 158.5\ \Omega$$

$$\text{Current } (i) = \frac{\text{e.m.f}}{\text{Impedance}} = \frac{200}{158.5} = 1.262 \text{ amp}$$

P.D across inductance $= \omega Li = 100\pi \times 3 \times 1.262 = 1189$ volt.

7. Calculate the resonant frequency of an LCR parallel resonant circuit with $L = 10mH$, $C = 1\mu F$ and $R = 1K\Omega$

Sol : $L = 10mH = 10 \times 10^{-3} H$

$C = 1\mu F = 1 \times 10^{-6} F, R = 1K\ \Omega = 1 \times 10^3\ \Omega$

UNIT III

6. ELECTROMAGNETIC WAVES - MAXWELL'S EQUATIONS

LONG ANSWER TYPE QUESTIONS

1. What are the basic laws of electricity and magnetism on which Maxwell's electromagnetic theory is based. **[AdNU 18; KU 18]**

A. The basic laws of electricity and magnetism on which Maxwell's electromagnetic theory is based are

i) **Gauss's law in electrostatics :**

The total electric flux through a closed surface is equal to the total charge enclosed by the surface.

$$\oint \vec{E} \cdot \vec{ds} = q/\epsilon_0$$

But if ρ be the charge density and dV is the small volume considered.

$$q = \int_V \rho \, dV$$

$$\oint E \cdot ds = \frac{1}{\epsilon_0} \int_V \rho \, dV \text{ or } \oint \epsilon_0 E \cdot ds = \int_V \rho \, dV$$

$$\oint D \cdot ds = \int_V \rho \, dV \quad (\because D = \epsilon_0 E)$$

According divergence theorem

$$\oint D \cdot ds = \int_V (\vec{\nabla} \cdot D) \, dV$$

$$\text{So } \int_V \vec{\nabla} \cdot D \, dV = \int_V \rho \, dV$$

$$\nabla \cdot D = \rho \text{ or } \nabla \cdot E = \rho/\epsilon_0 \text{ or div } E = \rho/\epsilon_0$$

ii) **Gauss's law in magnetism :**

The total flux of magnetic induction over a closed surface is always zero.

$$\oint \vec{B} \cdot \vec{ds} = 0$$

Transforming the surface integral into volume integral

$$\oint B \cdot ds = \int_V \vec{\nabla} \cdot \vec{B} \, dV = 0$$

As the volume is arbitrary, the integral must be zero.

$$\nabla \cdot B = 0$$

iii) **Fraday's law of electromagnetic induction :**

This law states that an electric field is produced by changing magnetic field

$$\oint E \cdot dl = \frac{-d\phi_B}{dt}$$

$$= \frac{-\partial}{\partial t}\int_s B \cdot ds = \int_s \frac{\partial B}{\partial t} ds$$

Applying Stoke's theorem $\oint E\, dl = \int_s (\nabla \times E) \cdot ds$

$$\therefore \int_s (\nabla \times E) \cdot ds = -\int_s \frac{\partial B}{\partial t} ds$$

This equation is true for all surfaces

$$\therefore \nabla \times \vec{E} = \frac{-\partial B}{\partial t} \text{ (or) curl } E = \frac{-\partial B}{\partial t}.$$

iv) **Ampere's law :**

This law states that the amount of workdone in carrying a unit magnetic pole once around a closed arbitrary path linked with the current is μ_0 times the current i.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i$$

Using Stoke's theorem

$$\oint B \cdot dl = \int_s (\nabla \times B) \cdot ds$$

$$\int_s (\nabla \times B) \cdot ds = \mu_0 \int_s J \cdot ds$$

$$\nabla \times B = \mu_0 J$$

Replacing J by $J + \epsilon_0 \frac{\partial E}{\partial t}$

$$\nabla \times B = \mu_0 \left[J + \epsilon_0 \frac{\partial E}{\partial t} \right]$$

2. Deduce the equation of the electromagnetic wave and hence evaluate the velocity of light in free space. **[AdNU 17; AU 18, 17; SKU 18; SVU 17; VSU 17; YVU 18, 17]**

A. Let us apply Maxwell's electromagnetic equations to a homogeneous, isotropic, dielectric medium containing neither electric charges nor magnetic poles. A dielectric is one which offers infinite resistance to a current and hence its conductivity is zero.

Hence $J = 0$, $\rho = 0$, $D = K \epsilon_0 E = \epsilon E$ and $B = \mu_0 \mu_r H = \mu H$

Hence Maxwell's equations for a dielectric become

$\nabla \cdot E = 0 \rightarrow (1)$

$\nabla \cdot B = 0 \rightarrow (2)$

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$$\oint B \cdot dl = \int_s (\nabla \times B) \cdot ds$$

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Hence Maxwell's equations for a dielectric become

$$\nabla \cdot E = 0 \rightarrow (1)$$

$$\nabla \cdot B = 0 \rightarrow (2)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} \rightarrow (3)$$

$$\nabla \times B = \mu \in \frac{\partial E}{\partial t} \rightarrow (4)$$

By obtaining wave equation in dielectric medium by eliminating E in equation and (4)

Taking curl of equation (4), we get

$$\nabla \times \nabla \times B = \nabla \times \mu \in \frac{\partial E}{\partial t} = \mu \in \left(\nabla \times \frac{\partial E}{\partial t}\right)$$

$$= \mu \in \frac{\partial}{\partial t}(\nabla \times E)$$

$$= \mu \in \frac{\partial}{\partial t}\left(\frac{-\partial B}{\partial t}\right) \quad \left(\because \nabla \times E = \frac{-\partial B}{\partial t}\right) \text{ from equation (3)}$$

$$\nabla \times \nabla \times B = -\mu \in \frac{\partial^2 B}{\partial t^2} \rightarrow (5)$$

We know that $\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B$

$$\nabla \times \nabla \times B = \nabla(0) - \nabla^2 B \rightarrow (6) \quad \text{from eq.(2)}$$

Substituting equation (6) in equation (5)

$$-\nabla^2 B = -\mu \in \frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 B = \mu \in \frac{\partial^2 B}{\partial t^2} \rightarrow (7)$$

Similarly from equation (3), $\nabla^2 E = \mu \in \frac{\partial^2 E}{\partial t^2} \rightarrow (8)$

Equations (7) and (8) represents the wave equations.

The general wave equation is $\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \rightarrow (9)$

where v = velocity of wave and y = displacement

Comparing equations (8) and (9)

$$\frac{1}{v^2} = \mu \in$$

$$v^2 = \frac{1}{\mu \in}$$

$$v = \frac{1}{\sqrt{\mu \in}} \rightarrow (10)$$

where μ = permeability of the medium and $\in$ = permittivity of the medium

For free space $v = \frac{1}{\sqrt{\mu_0 \in_0}} \rightarrow (11)$

$\mu_0 = 4\pi \times 10^{-7}$ and $\in_0 = 8{\cdot}85 \times 10^{-12}$

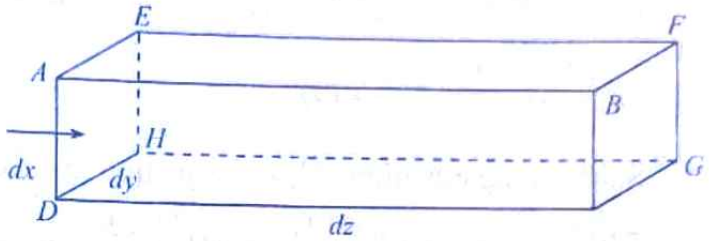
$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8{\cdot}85 \times 10^{-12}}} = 3 \times 10^{8} \text{ m/sec}$$

Thus the velocity of the propagation of variation of E and H is the same as the velocity of light.

3. What is poynting vector ? Derive expression for poynting vector from Maxwell equations. Give its characteristics? **[ANU 19, 18, 17; AdNU 18; AU 18; BRAU 17; RU 17; SKU 18; SVU 18]**

A. **Poynting vector :** The rate at which energy is transmitted through unit area perpendicular to the direction of propagation of energy is called poynting vector and is represented by $\vec{P}$.

Let us consider on elementary volume in the form of a rectangular parallellopiped of sides dx, dy, dz. Its volume is $dx\, dy\, dz$. If the energy is supposed to be propagated in the x-direction, the surface of area $dy\, dz$ is perpendicular to the direction of propagation of energy. If the electromagnetic energy in this volume is U.



Then the rate of change of energy $\left(\frac{\partial u}{\partial t}\right) = -\oint_s \vec{P} \cdot \vec{ds} \rightarrow (1)$

The negative sign is used because the integral on the right hand side of the equation is negative when energy is entering the volume.

$$\oint_s P \cdot ds = \frac{-\partial U}{\partial t} \rightarrow (2)$$

The energy per unit volume due to electric field is $\frac{1}{2} \in_0 E^2$ and due to magnetic field $\frac{1}{2} \mu_0 H^2$.

$$\therefore\ U = \frac{1}{2} \in_0 E^2 + \frac{1}{2} \mu_0 H^2$$

The rate of decrease of energy in volume dV is given by

$$\frac{-\partial}{\partial t}\left(\frac{1}{2} \in_0 E^2 + \frac{1}{2} \mu_0 H^2\right) dV$$

$\therefore$ Rate of decrease of energy for volume V

$$\frac{-\partial U}{\partial t} = \frac{-\partial}{\partial t} \int_V \left(\frac{1}{2} \in_0 E^2 + \frac{1}{2} \mu_0 H^2\right) dV$$

$$= \int_V -\left[\in_0 E\left(\frac{\partial E}{\partial t}\right) + \mu_0 H\left(\frac{\partial H}{\partial t}\right)\right] dV \rightarrow (3)$$

Similarly taking curl B we can show that

$$\frac{\partial E_y}{\partial t} = 0 \text{ or } E_y = \text{constant}$$

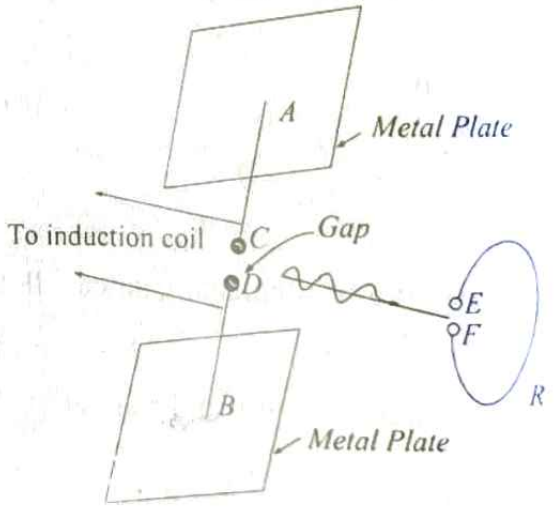
The above mathematical analysis shows that E_x and H_x are constants as regard to time and space. i.e., these represent static components and hence no part of wave motion. Thus $\vec{E} = \hat{j}E_y + \hat{k}E_z$ and $\vec{H} = \hat{j}H_y + \hat{k}E_z$

Since the electric vector $\vec{E}$ and magnetic vector $\vec{H}$ do not contain any x-component, x-direction being the direction of propagation of the wave, both these vectors are perpendicular to the direction of propagation. Hence Maxwell's electromagnetic waves are purely transverse in nature.

5. Describe hertz experiment for the production and detection of electromagnetic waves. **[ANU 19; BRAU 18; RU 18; SVU 18; YVU 18]**

A. The schematic arrangement of Hertz experiment is shown in figure. A and B are two brass square plates. These plates are placed vertically at a distance of 60 cm seperation. These plates serve as the plates of a capacitor. Using large area capacitor plates and seperating them by a large distance makes the region of influence of the electromagnetic waves considerably extended.

The opposite faces of A and B are connected to thick wires carrying highly polished brass spheres C and D. The brass spheres (Buttons or knobs) are seperated by a gap of about 2 cm to 3 cm. An induction coil is connected across the wires attached to the plates. So that the capacitor plates A and B can be charged to a suitably high potential.

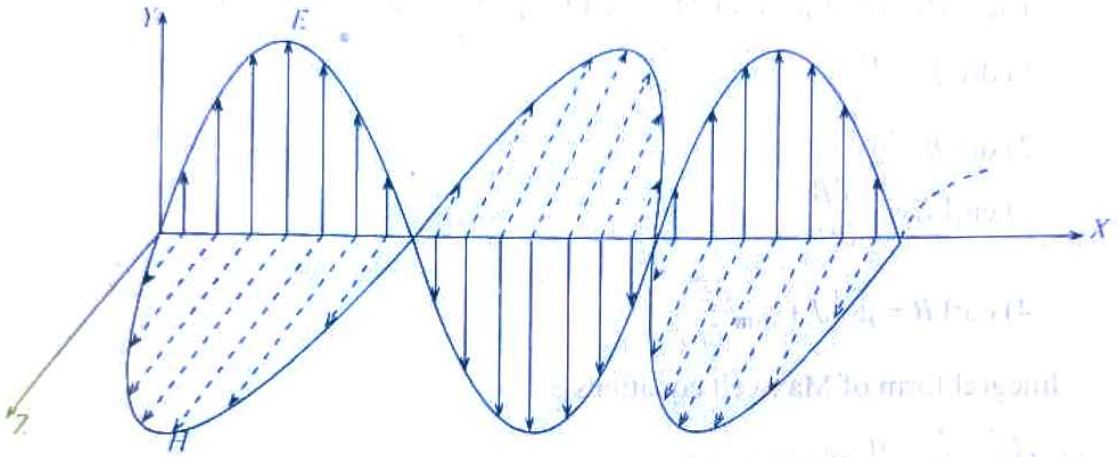


When the D.C voltage supplied by the induction coil becomes sufficiently high, the air in the gap between the brass buttons C and D gets ionized. The air in the gap becomes conducting and provides a path for the electrical discharge between the two plates A and B. The oscillatory discharge through the conducting air produces an electromagnetic waves.

Detection : To detect E.M. waves produced in the spark gap. Hertz employed a slightly open ring R of an almost closed circular thick wire. The two ends of the wire, each cary a small polished brass sphere. These are called buttons E and F. The brass spheres E and F serve as the plates of a capacitor and connecting wire as an inductor. Thus in the detector also we have got a series LC circuit.

If the circular ring R is placed in such a way that the lines of the magnetic field of the EM waves produced in the spark gap passes through it, then an alternating e.m.f will be induced round the circular wire. The oscillatory e.m.f across the buttons E and F causes a small spark to pass between them when the induced e.m.f is of sufficiently high value.

The diameter of the circular wire is altered until the natural frequency of the detector becomes equal with the frequency of EM waves produced in the spark gap between C and D.



In the resonance condition, the amplitude of the oscillations in the detector circuit increases. Consequently a spark will be produced between E and F of the detector. Thus the EM waves can be detected.

SHORT ANSWER TYPE QUESTIONS

1. Explain what is meant by displacement current ?

[BRAU 18; KU 18; RU 18, 17; SKU 17; VSU 17; YVU 17]

A. When the dipoles in a medium are displaced when an electric field is applied, due to displacement of charges a small current is produced which is called displacement current.

Ampere's law in vector form can be expressed as $\nabla \times B = \mu_0 i$

Taking divergence of this equation, we get

$\nabla \cdot (\nabla \times B) = \text{div curl } B = \text{div } \mu_0 J = \mu_0 \text{ div } J$

We know that divergence of curl of a vector is always zero, hence div $J = 0$

Above equation is in contradiction with the equation of continuity which states that

$$\text{div } J + \frac{\partial \rho}{\partial t} = 0$$

So Maxwell realised that the definition of the total current density is incomplete and modified the equation as

curl $B = \mu_0 J$ + something

In order to know this something, maxwell postulated that similar to the electric field due to changing magnetic field. There would be a magnetic field due to the chang-

ing electric field. Thus a changing electric field is equivalent to a current which flows as long as the electric field is changing and produced the same magnetic effect as an ordinary conduction current. This is known as displacement current.

2. Write down the Maxwell's equations in differential form and integral form.

[AdNU 17; RU 17; SVU 17; YVU 18, 17]

A. In 1862 Maxwell formulated the basic laws of electricity and magnetism in the form of four fundamental equations. These equations are known as Maxwell's equations.

The differential form of Maxwell's equations are given below.

1) $\text{div } E = \frac{\rho}{\epsilon_0}$

2) $\text{div } B = 0$

3) $\text{curl } E = \frac{-\partial B}{\partial t}$

4) $\text{curl } B = \mu_0 \left(J + \epsilon_0 \frac{\partial E}{\partial t} \right)$

Integral form of Maxwell equations are

1) $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$ (Gauss law)

2) $\oint \vec{B} \cdot \vec{ds} = 0$ (Gauss law is magnatism)

3) $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$ (Faraday's law)

4) $\oint \vec{B} \cdot \vec{dl} = \mu_0 i$ (Ampers' law).

PROBLEMS

1. Relative permittivity of paraffin $K = 2{\cdot}1$. Find the refraction index and the propagation velocity of e.m.waves through it.

Sol : Given $K = 2{\cdot}1$

$\mu = \sqrt{K} = \sqrt{2{\cdot}1} = 1{\cdot}45$

$\mu = \frac{v_0}{v}$

$v = \frac{v_0}{\mu} = \frac{3 \times 10^8}{1{\cdot}45}$

$v = 2{\cdot}07 \times 10^8$ m/s.

2. Relation permittivity of distilled water is 81. Calculate its refractive index and propagation velocity of e.m.waves through it.

Sol : Given $K = 81$

$\mu = \sqrt{K} = \sqrt{81} = 9$

$$\mu = \frac{v_0}{v}$$

$$v = \frac{v_0}{\mu} = \frac{3 \times 10^8}{9} = 3{\cdot}33 \times 10^7 \text{ m/s.}$$

3. The sun radiates with a power of $3{\cdot}8 \times 10^{26}$ watts. Calculate the magnitude of poynting vector in case of the surface of a sphere having a radius of 7×10^8 m..

[ANU 17]

Sol : Power = $3{\cdot}8 \times 10^{26}$ watts

Radius $(r) = 7 \times 10^8$ m

$$\text{Poynting vector} = \frac{\text{Power}}{\text{Surface area}} = \frac{P}{4\pi r^2}$$

$$= \frac{3{\cdot}8 \times 10^{26}}{4 \times 3{\cdot}14 \times (7 \times 10^8)^2}$$

$$= \frac{3{\cdot}8 \times 10^{10}}{615{\cdot}4}$$

$$= 6{\cdot}175 \times 10^7 \text{ watt/m}^2.$$

4. Calculate the velocity of electromagnetic waves in free space.
[Given $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$, $\mu_0 = 4\pi \times 10^{-7}$]

Sol : $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$

$\mu_0 = 4\pi \times 10^{-7}$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8{\cdot}85 \times 10^{-12}}} = 3 \times 10^8 \text{m/s.}$$

UNIT IV

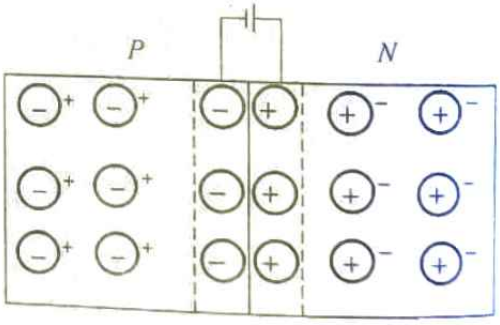
7. BASIC ELECTRONIC DEVICES

LONG ANSWER TYPE QUESTIONS

1. What is a $P-N$ junction diode? Discuss the working of $P-N$ junction diode? Explain forward bias and reverse bias? **[ANU 19, 17; BRAU 18; RU 17; SKU 17; SVU 17; VSU 18, 17; YVU 18]**

A. The function of diode is to generate electrons and to collect them. The region which generates electrons is called a cathode and the one which collects them is called anode. These two function can be performed by a $P-N$ junction. The addition of impurities to germanium (or) silicon during crystal growth can be so adjusted that part of the crystal is n-type and the rest P- type. Then $P-N$ junction is formed. An important property of the $P-N$ junction is that it conducts electric current much more readily in one direction than in the other.

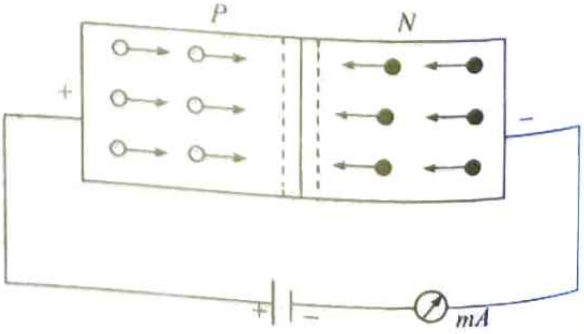
Working : In the absence of external field some electrons in the n-region neutralise holes near the junction and some holes from the P- region move to the n- region. Thus neutralising the charges by recombination. Thus near the junction, over a small region, the n- side is more positive and P- side is negative. This region where the charge carriers are neutralised is called depletion region. The potential difference across the junction is called junction barrier. The potential barrier can be increased (or) decreased by applying an external voltage.



Forward bias :

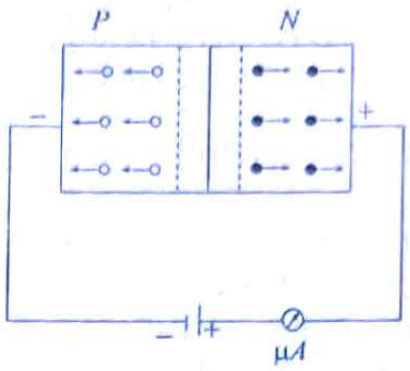
When a positive terminal of a battery is connected to P- region and negative terminal is connected to n- region. The $P-N$ diode is said to be in forward bias.

In the case of forward bias, the holes from P- type semiconductor are repelled by the positive terminal of a battery and move towards the junction simultaneously, the electrons in N- type are repelled by the negative terminal of a battery and move towards the junction. This decreases the depletion layer and the potential barrier. The charge carries cross the junction and electric current flows in the external circuit. This is a low resistance for the $P-N$ junction.



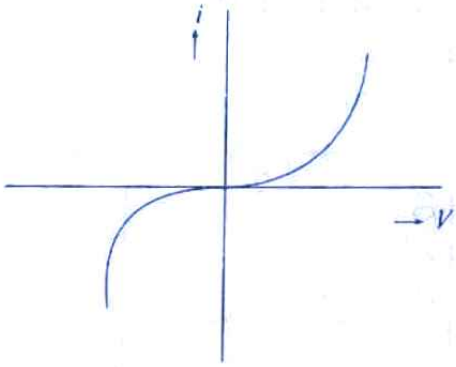
Reverse bias :

When a battery negative terminal is connected to P- region and positive terminal is connected to n-region. The $P-N$ junction diode is said to be in reverse bias.



The holes in P- region are attracted towards the negative polarity and the electrons in the n- region are attracted towards the positive polarity. Thus they are move away from the junction. This increases the potential barrier. Due to thermal agitation a small current still flows across the junction which can be read on only a micro ammeter.

$V-I$ characteristics :



A graph shows the relation between the voltage V and the current i. As the forward bias voltage increases, the current i increases steadily at rearly exponential rate. As the reverse bias voltage increases the reverse current increases first slowly and then rapidly.

2. How is the $P-N$ junction used as a rectifier? **[ANU 18; VSU 18]**

A. **Rectifier :** The process of converting alternating current into direct current is called rectification. The semiconductor diode can be used as a rectifier.

Half wave rectifier :

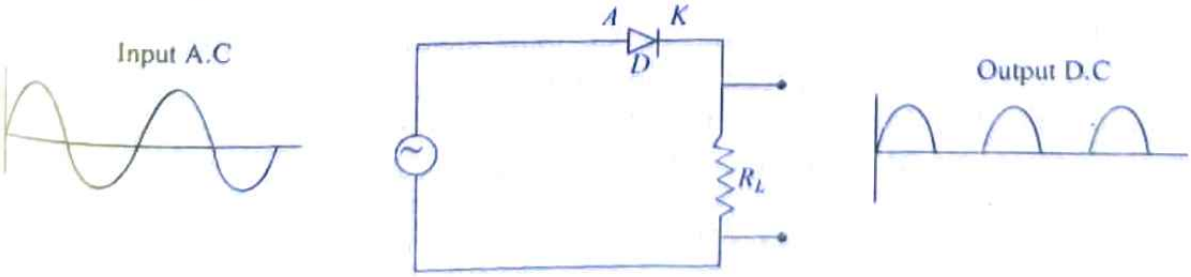


Figure shows the half wave rectifier circuit. The A.C voltage to be rectified is applied to a single diode connected in series with a load resistance R_L. For the positive half cycle of input A.C voltage the diode D is forward biased and hence it conducts. Now a current flows in the circuit and there is a voltage drop across R_L.

This constitutes the output voltage as shown in figure. For the negative half cycle, the diode D is reverse biased and hence it does not conduct. Now no current flows in the circuit. The output is not a steady d.c but only a pulsating d.c wave.

Efficiency of the half wave rectifier $(\eta) = \dfrac{0{\cdot}406\, R_L}{r_f + R_L}$

The maximum value of rectifier efficiency of a half-wave rectifier is 40·6% .

Ripple factor :

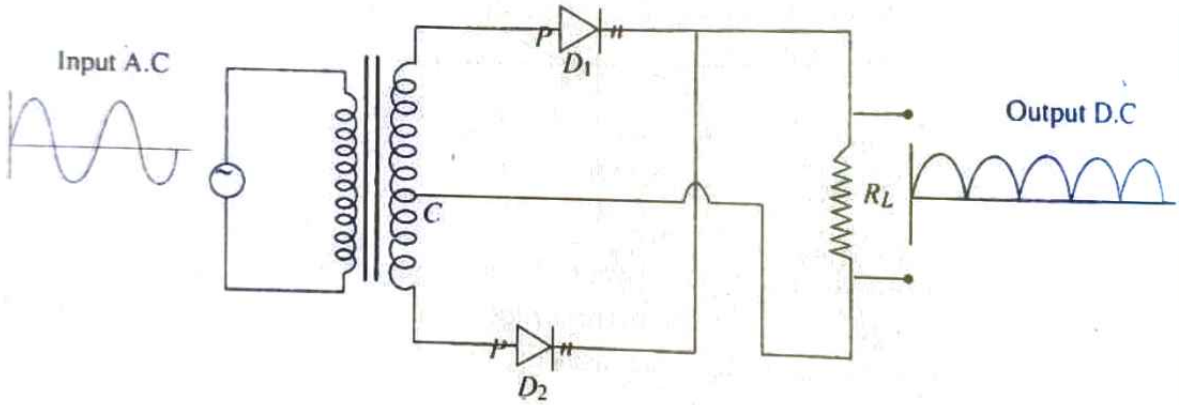
Ripple factor is defined as the ratio of the a.c components to d.c components.

$$\gamma = \frac{\text{rms value of all a.c components}}{\text{d.c value of components}}$$

$$\gamma = \frac{(I_r)_{rms}}{I_{dc}} = \frac{V_{rms}}{V_{dc}} \rightarrow (1)$$

Ripple factor of half wave rectifier is 1·21

Full wave rectifier :



A full wave rectifier can be constructed with two $P-N$ junction diodes as shown in the figure. The secondary of transformer is centre tapped. Its two ends are connected to the P- region of the two diodes D_1 and D_2. The two n- regions are joined and between this terminal and the centre tap, the load resistance R_L is connected.

During the positive half cycle of A.C input the diode D_1 is forward biased i.e., it conducts the diode D_2 remains non-conducting, being reverse biased. During negative half cycle of A.C input the diode D_2 is forward biased ie it conducts. The diode D_1 remains non-conducting being reverse biased.

The efficiency of full wave rectifier is $\eta = \dfrac{0{\cdot}812\, R_L}{r_f + R_L}$

The maximum value of rectifier efficiency of a full wave rectifier is 81·2%

Ripple factor : We know that the form factor of the output voltage of a full wave rectifier is $= \dfrac{I_{rms}}{I_{dc}} = \dfrac{I_m/\sqrt{2}}{2I_m/\pi} = 1{\cdot}11$

The ripple factor γ is $\gamma = \left[\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1\right]^{1/2}$

Ripple factor of full wave rectifier is 0·48.

3. What is the transistor? Describe the operation of *PNP* and *NPN* transistor?

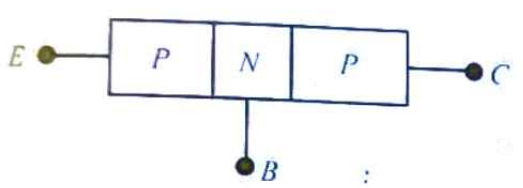
[AdNU 17; AU 18, 17; SVU 18, 17; VSU 17; YVU 18, 17]

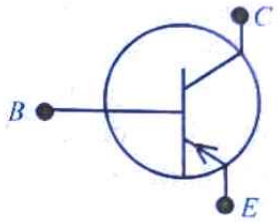
A. "Transistor means transfer of resistance. A transistor is sandwich of one type of semiconductor between two layers of the other type".

There are two types of transistors.

1) $P-N-P$ transistor 2) $N-P-N$ transistor

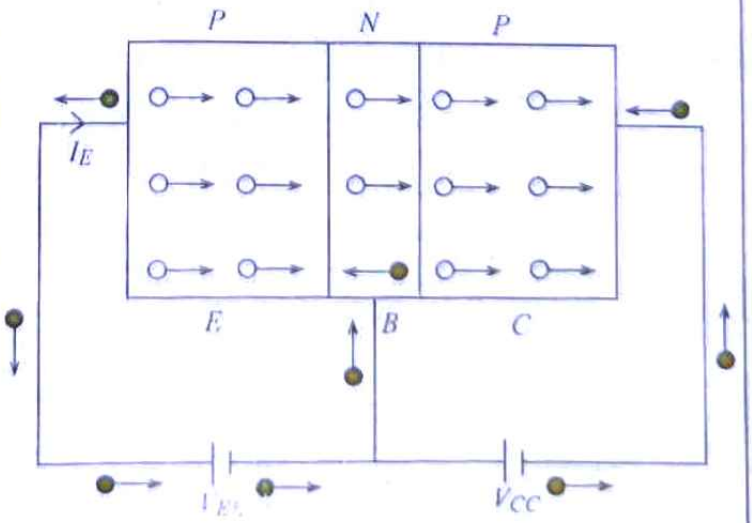
1) P – N – P transistor :





When a layer of *N*- type material sandwiched between two layers of *P*- type material, the transistor is known as $P-N-P$ transistor.

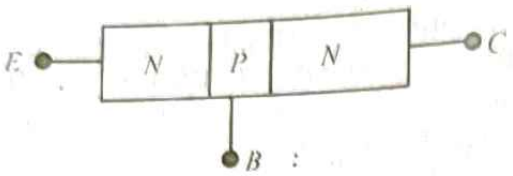
Figure shows a *PNP* transistor with emitter base junction as forward bias and collector-base junction as reverse biased. The holes of *P* region (emitter) are repelled by the positive terminal of battery V_{EE} and move towards the base. This constitute the emitter current I_E. The base region is lightly doped only 2% to 5% of the holes recombine with the free electrons of *N*-region. This constitute the base current I_B. The remaining holes (95% to 98%) enters the collector region. They are swept up by the negative terminal of battery V_{CC}. They constitute the collector current I_C. When one hole reaches the collector, an electron is emitted from the negative terminal of battery and neutralizes the hole. Now a covalent bond near the emitter breaks down. The evolved electron reaches the positive terminal of battery V_{EE} and the hole immediately move towards the emitter junction. This process is repeated again and again. So the current conduction in *PNP* transistor is holes.



$$I_E = I_B + I_C$$

N – P – N transistor :

When a layer of P type material is sandwiched between two layers of N- type material, the transistor is known as NPN transistor.



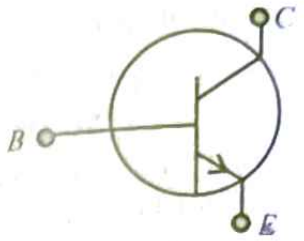
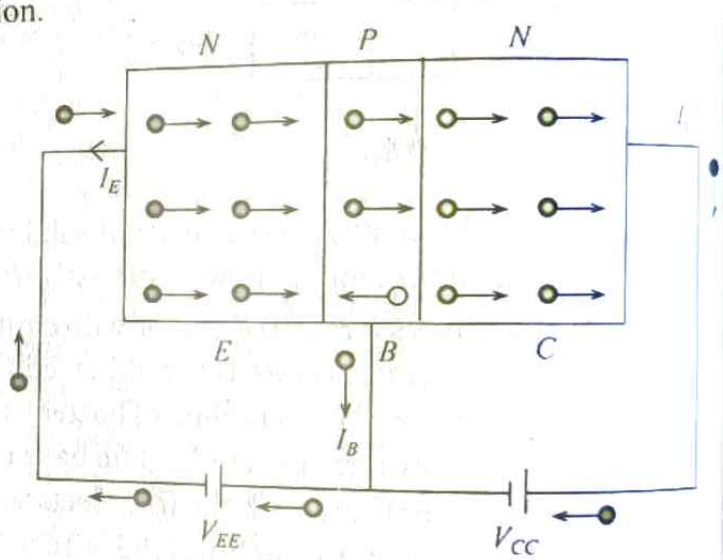


Figure shows a NPN transistor with emitter-base junction as forward biased and collector-base junction as reverse biased. The electrons in n- region (emitter) are repelled from the negative terminal of battery V_{EE} and move towards the junction and enters the base region.

The base region is very thin and lightly doped, a few electrons combine with the holes in P region and are lost as charge carriers. Now the electrons in N region (collector) readily swept up by the positive terminal of V_{CC}. Then an electron from the negative of V_{EE} enters the emitter region. So the current conduction in NPN transistor is carried out by electrons.



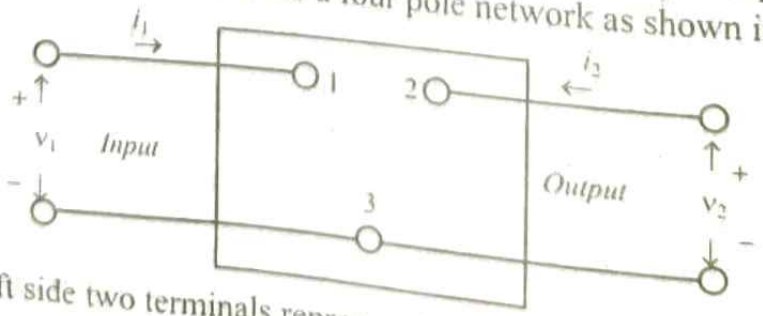
$$I_E = I_B + I_C$$

4. What do you understand by hybrid parameters? Define the h- parameters of CE transistor?

[KU 18; RU 18, 17; YVU 18]

A. "Hybrid means mixed". These parameters have mixed dimensions and for this reason are called hybrid parameters".

h - parameters are very useful at high frequencies. Due to this fact at high frequencies a transistor has low input impedance and high output impedance. A transistor can be considered to be a four pole network as shown in figure.



The left side two terminals represent input terminals. The right side two terminals represent out put terminals.

For each pair of terminals, the variables are the voltage (v) and current (i). In this case i_1 and v_2 taken as independent variables while v_1 and i_2 are taken as dependent variables.

Hence $\left.\begin{array}{l} v_1 = f(i_1, v_2) \\ i_2 = f(i_1, v_2) \end{array}\right\} \rightarrow (1)$

The same can be expressed as

$v_1 = h_{11} i_1 + h_{12} v_2 \rightarrow (2)$

$i_2 = h_{21} i_1 + h_{22} v_2 \rightarrow (3)$

here h_{11}, h_{12}, h_{21} and h_{22} are nothing but four hybrid parameters.

By appropriate choice of open circuit ($i_1 = 0$) and short circuit ($v_2 = 0$) conditions applied to equation (2) and (3)

Hybrid parameters are now defined as

$$h_{11} = h_i = \left|\frac{\partial v_1}{\partial i_1}\right|_{v_2=0} = \left|\frac{v_1}{i_1}\right|_{v_2=0} = \text{Input impedance with output short}$$

$$h_{12} = h_r = \left|\frac{\partial v_1}{\partial v_2}\right|_{i_1=0} = \left|\frac{v_1}{v_2}\right|_{i_1=0} = \text{Reverse voltage ratio with input open}$$

$$h_{21} = h_f = \left|\frac{\partial i_2}{\partial i_1}\right|_{v_2=0} = \left|\frac{i_2}{i_1}\right|_{v_2=0} = \text{Forward current gain with output short}$$

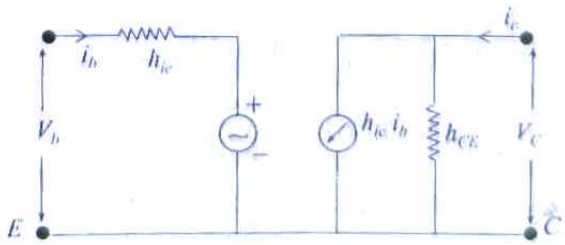
$$h_{22} = h_0 = \left|\frac{\partial i_2}{\partial v_2}\right|_{i_1=0} = \left|\frac{i_2}{v_2}\right|_{i_1=0} = \text{Output admittance with input open}$$

Here h_i, h_f, h_r and h_0 are the hybrid parameters.

Hybrid parameters of a *CE* transistor :

1) **In put impedance (h_{ie}) :**

"The input impedance is defined as the ratio of change base-emitter voltage (ΔV_{BE}) to the change in base current (ΔI_B) at constant collector voltage (V_{CE})



$$h_{ie} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

The units are ohm.

2) **Reverse voltage ratio (h_{re}) :**

"The reverse voltage ratio is the ratio of change in base-emitter voltage (V_{BE}) to the change in collector voltage (ΔV_{CE}) at constant base current (I_B)

$$h_{re} = \left(\frac{\Delta V_{BE}}{\Delta V_{CE}}\right)_{I_B}$$

This has no units.

3) **Forward current gain (h_{fe}) :**

Forward current gain is the ratio of change is collector current (ΔI_C) to the change in base current (ΔI_B) at constant collector voltage (V_{CE})

$$h_{fe} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$$

This has no units.

4) **Out put admittance (h_{oe}) :**

Out put admittance is defined as the ratio of change in collector current (ΔI_C) to the change in collector voltage (ΔV_{CE}) at constant base current (I_B).

$$h_{oe} = \left(\frac{\Delta I_C}{\Delta V_{CE}}\right)_{I_B}$$

This has the units Siemens.

5. Define α, β and γ in the case of a transistor. Derive the expressions for the relations between them. **[ANU 17; BRAU 17; KU 18; SKU 18; SVU 18, 17]**

A. **Current amplification factor (α) :**

"The ratio of the collector current to the emitter current without the application of a signal is called the current amplification factor of a *CB* transistor"

$\alpha = \frac{I_c}{I_E}$ when signal is applied $\alpha = \frac{\Delta I_C}{\Delta I_E}$

α value varies from 0·9 to 0·99

Current amplification factor (β) :

The ratio of collector current to the base current with out the application of a signal is called the current amplification factor (β) of a *CE* transistor.

$\beta = \frac{I_C}{I_B}$ when signal is applied $\beta = \frac{\Delta I_C}{\Delta I_B}$

β value varies from 20 to 500.

Current amplification factor (γ) :

The ratio of emitter current to the base current with out the application of the signal is called the current amplification factor of *CC* transistor.

$$\gamma = \frac{I_E}{I_B}$$

when signal is applied $\gamma = \frac{\Delta I_E}{\Delta I_B}$

Relation between α and β :

We know that $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

$I_E = I_B + I_C$ (or) $I_B = I_E - I_C$

$$\beta = \frac{I_C}{I_E - I_C} = \frac{I_C/I_E}{1-(I_C/I_E)} = \frac{\alpha}{1-\alpha} \; ; \; \beta = \frac{\alpha}{1-\alpha}$$

$$\beta(1-\alpha) = \alpha \; ; \; \beta - \beta\alpha = \alpha \Rightarrow \beta = \alpha(1+\beta) \; ; \; \alpha = \frac{\beta}{1+\beta}$$

It can be seen that $(1-\alpha) = \frac{1}{(1+\beta)}$

Relation between α and γ :

We know that $\gamma = \frac{I_E}{I_B}$ and $\alpha = \frac{I_C}{I_E}$

$I_B = I_E - I_C$

$$\gamma = \frac{I_E}{I_E - I_C} = \frac{1}{(1-I_C/I_E)} = \frac{1}{1-\alpha} \; ; \quad \gamma = \frac{1}{1-\alpha}$$

Relation between α, β and γ :

We know that $(1-\alpha) = \frac{1}{(1+\beta)}$

$$\gamma = \frac{1}{1-\alpha} = (1+\beta)$$

6. Explain the common emitter characteristics of a transistor.

[AdNU 18; BRAU 18, 17; RU 18; SKU 18]

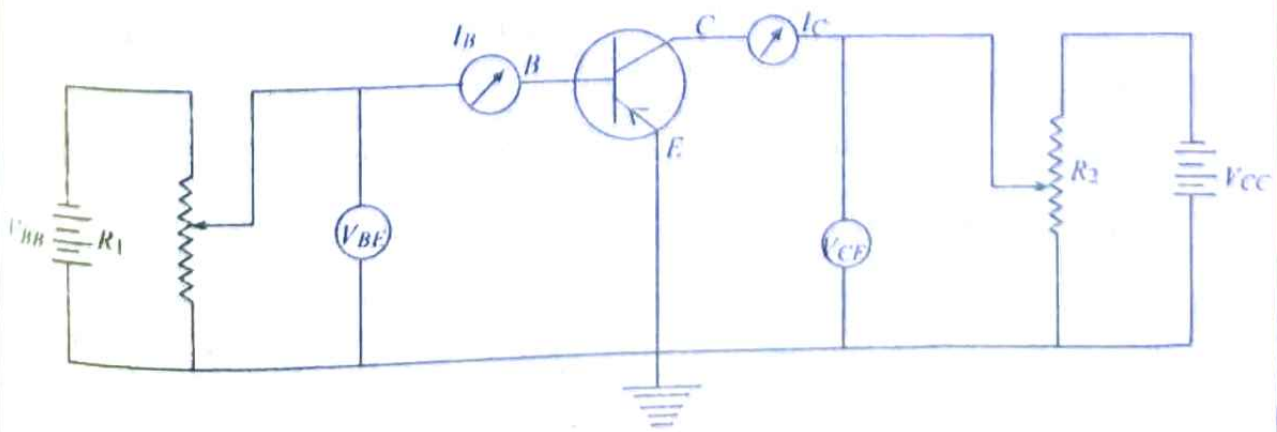
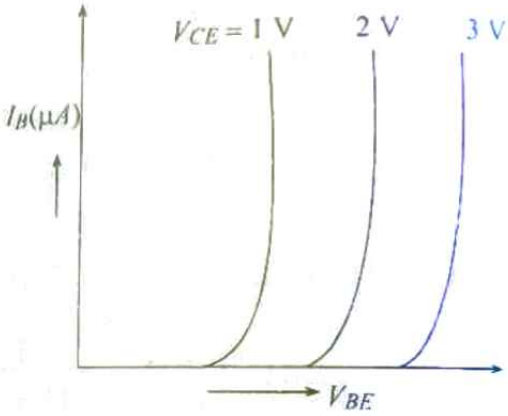


Figure shows common emitter *PNP* transistor. In the circuit, the battery V_{BB} provides forward bias to emitter-base with the help of potential divider R_1. V_{BE} is base-emitter voltage. I_B is the base current. A battery V_{CC} is connected between collector and emitter through a potential divider R_2. The positive terminal of a battery is connected to emitter while the negative terminal is connected to the collector. So that the collector- base junction is reverse biased. V_{CE} is the collector emitter voltage and I_C is the collector current.

Input characteristics :

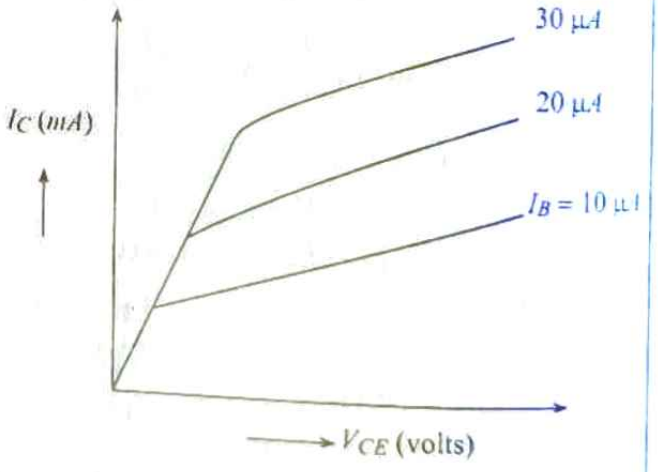
The curve between base current I_B and base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE} represents the input characteristics.



The collector emitter voltage V_{CE} is fixed. the base emitter voltage V_{BE} is varied with the help of potential divider R_1 and the base current I_B is noted for each value of V_{BE}. A graph is drawn between V_{BE} and I_B. The curves so obtained is known as input characteristics

Output characteristics :

The curve between collector current I_C and collector emitter voltage V_{CE} at constant base current I_B represents the output characteristics.



The base current I_B is fixed, the value of V_{CE} is varied in steps and the collector current I_C is noted for each value of V_{CE}. A graph is drawn between I_C and V_{CE}. The curve so obtained is known output characteristics.

SHORT ANSWER TYPE QUESTIONS

1. Write a short note on Zener diode?

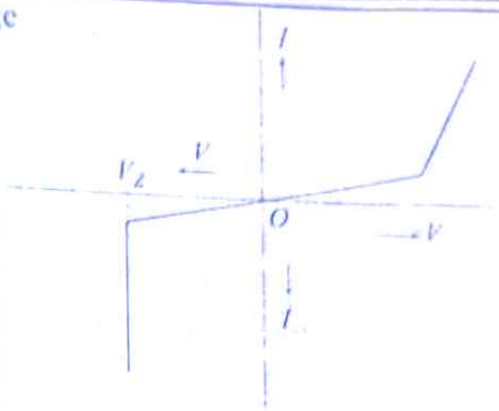
[ANU 18, 17; AdNU 18, 17; AU 18; RU 18, 17; SKU 17; VSU 17; YVU 18, 17]

A. " A properly doped *PN* junction diode which has sharp breakdown voltage when operated in the reverse bias condition is called Zener diode". The symbol of zener diode is shown in figure

V – I characteristics :

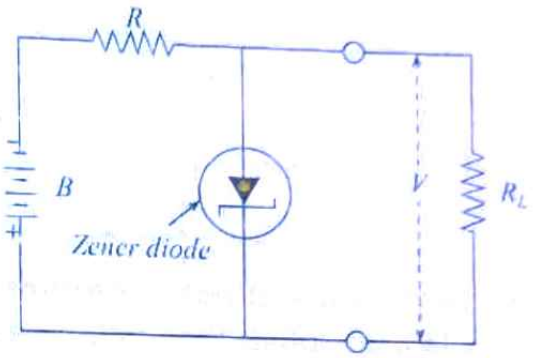
The $V-I$ characteristics of a Zener diode is shown in figure. In forward bias, the characteristics are same as that of ordinary diode. In the reverse bias, as voltage is increased beyond zener voltage (V_Z) (or) the break down voltage, the current increases sharply.

For widely different zener currents, the voltage across zenerdiode remains constant. Hence the diode can be used as a voltage regulator (or) stabilizer.



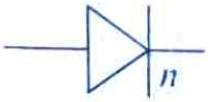
Zener diode as voltage stabilizer :

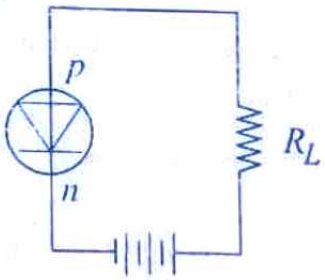
The zener diode is connected to a battery B through resistor R. The battery B reverse-bias the zener diode. The load R_L is connected across the terminals of zener diode. The value of R is selected in such a way that in the absence of load R_L, maximum safe current flows in the zenerdiode. The load draws a current. The current through the diode falls by the same amount but the voltage across the resistance R almost remains constant. In this way variation in load current hardly affect the voltage V supplied to it.



2. Write a short note on LED.

A. LED is a light emitting diode. It is a forward biased $p-n$ junction diode, which converts electrical energy into light energy.



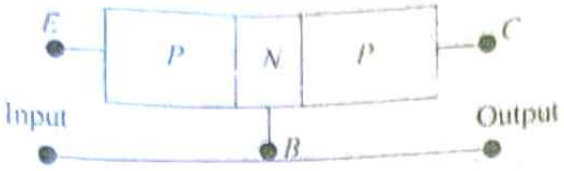


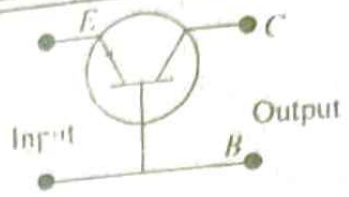
Working : When a junction diode is forward biased, electrons from n – side and holes from p – side move towards the depletion region and recombination takes place. When an electron in the conduction band recombines with a hole in the valence band, energy is released. In case of semi conducting materials like gallium arsenide ($GaAs$), gallium phosphide (GaP) and Gallium-arsenide-phosphide ($GaAsP$) a great percentage of energy is given out in the form of light. If the semi conducting material is translucent, light is emitted and the junction becomes a light source.

3. Explain three types of transistor configurations?

A. 1) **Common base configuration (CB) :**

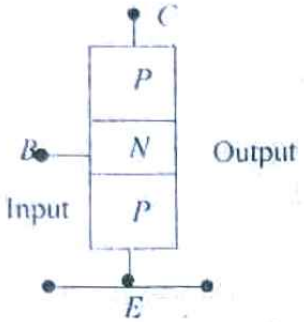
In this configuration base is common to both input and output. Base terminal is earthed and input is given across base-emitter and output is taken across base-collector as shown in figure.

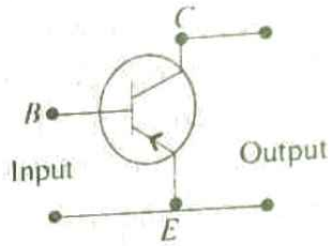




2) **Common emitter configuration (CE) :**

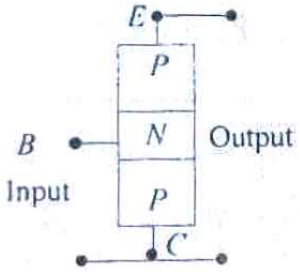
In this configuration emitter is common to both input and output. The emitter is earthed and input is given across base-emitter and output is taken across collector emitter as shown in figure.

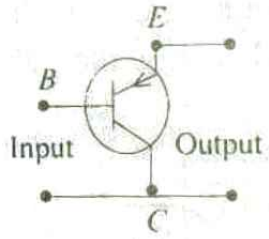




3) **Common collector configuration (CC) :**

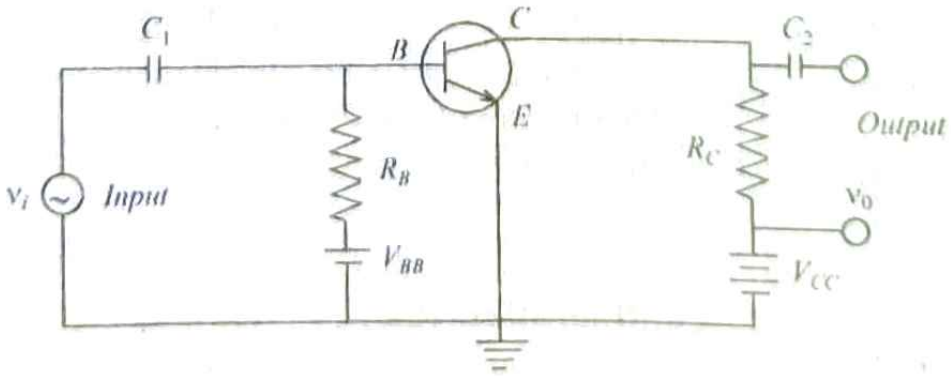
In this configuration collector is common to both input and output. The collector is earthed and input is given across base-collector and output is taken across emitter-collector as shown in figure.





4. Explain how transistor acts as an amplifier? **[ANU 19, 18, 17]**

A. The process of raising the strength of a weak signal is known as amplification and the device which accomplishes this job is called an amplifier. Figure shows the basic transistor amplifier.



The input signal voltage V_i is applied to the input side through a capacitor C_1. So that the biasing dc voltage V_{BB} is blocked from going towards the source of signal. The output is taken from the collector resistance R_C. The dc voltage V_{CC} in

the output is blocked with the help of capacitor C_2. If V_i is applied to the input base-emitter side, it will change I_B. The collector current would also change. Thus transistor is used as an amplifier.

PROBLEMS

1. A half wave rectifier supplies power to a 1K Ω load. The input supply is 220 V_{rms}. Neglecting forward resistance of the diode, calculate i) V_{dc} ii) I_{dc} iii) ripple voltage (rms value).

Sol : i) $V_{dc} = \frac{V_m}{\pi} = \frac{\sqrt{2}V}{\pi}$ $\quad (\because V_m = \sqrt{2}V)$

$= \sqrt{2} \times 0{\cdot}318V = 0{\cdot}45V = 0{\cdot}45 \times 220 = 99$ volt.

ii) $I_{dc} = \frac{V_{dc}}{R_L} = \frac{99}{1 \times 10^3} = 99$ mA

iii) $\gamma = \frac{(V_r)_{rms}}{V_{dc}}$ (or) $(V_r)_{rms} = \gamma \times V_{dc}$

$\therefore (V_r)_{rms} = \frac{1{\cdot}21}{100} \times 99 = 1{\cdot}1979$ volt.

2. The applied input a.c power to a half wave rectifier is 100 W. The d.c output power obtained is 40W. Find the rectifier efficiency.

Sol : $P_{ac} = 100$ W, $P_{dc} = 40$ W

$\eta = \frac{P_{dc}}{P_{ac}} = \frac{40}{100} = 0{\cdot}4 = 40\%$.

3. A $P-n$ diode is used in a half-wave rectifier with load resistance of 1000Ω. If the forward resistance of ideal diode is 10Ω, calculate the efficiency of half-wave rectifier.

Sol : $R_L = 1000\Omega$, $r_f = 10\Omega$

$\eta = \frac{0{\cdot}46\, R_L}{r_f + R_L} = \frac{0{\cdot}406 \times 1000}{10 + 1000} = 0{\cdot}4019$

$\therefore \eta = 40{\cdot}19\%$.

4. A full-wave rectifier uses two diodes with load resistance of 100Ω. Each diode is having negligible forward resistance. Find efficiency of full wave rectifier.

Sol : $R_L = 100\Omega$, $r_f = 0\Omega$

$\eta = \frac{0{\cdot}812\, R_L}{r_f + R_L} = \frac{0{\cdot}812 \times 100}{0 + 100} = 0{\cdot}812$

$\eta = 81{\cdot}2\%$.

5. When the emitter current of a transistor is changed by 1mA, its collector current changes by 0·995 mA. Calculate a) its common base short circuit current gain and b) its common emitter short circuit current gain.

UNIT V

8. DIGITAL ELECTRONICS

LONG ANSWER TYPE QUESTIONS

1. What is the binary system of numbers? Explain the binary arithmetic in detail in connection with addition, subtraction, multiplication and division.

A. **Binary system of numbers :** A system of numbers which makes use of these two digits alone is called the binary system of numbers.

In the binary system of numbers we make use of the digits 0 and 1 only.

E.g. $(10011)_2 = 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 16 + 0 + 0 + 2 + 1 = 19$

Binary addition : In the binary addition, the highest digit is 1 only. When we add two digits in the binary system and get a value more than one then we have to carry the 2 as 1 to the left side and put 0 in the original place.

Rule 1, $0 + 0 = 0$

Rule 2, $0 + 1 = 1$

Rule 3, $1 + 0 = 1$

Rule 4, $1 + 1 = 1$ with a carry of 1

E.g. Add $(1011)_2$ and $(1001)_2$ in binary system.

$$\begin{array}{r} 1011 \\ \hline 1001 \\ \hline 10100 \end{array}$$

i) Start from right side end

ii) $1 + 1 = 2$ and this is to be carried as 1 to left side and 0 is to be placed in the original place.

iii) 1 (carried) + 1 = 2 and this is to be carried as 1 to the left and 0 is to be placed in original place.

iv) 1 (carried) + 0 + 0 = 1. No carry is needed. Place 1 in the original place.

v) $1 + 1 = 2$ is to be carried as 1 to left and 0 is to be placed in original position.

Binary subtraction : Here the subtraction rules of binary system can be stated as follows.

Rule 1, $0 - 0 = 0$

Rule 2, $1 - 0 = 1$

Rule 3, $0 - 1 = 1$ (with a borrow of 1 from next higher column)

Rule 4, $1 - 1 = 0$

E.g.
$$\begin{array}{r} 1101 \\ -\ 1010 \\ \hline 0011 \end{array}$$

i) Start from right side end.

ii) $1 - 0 = 1$. Place one in the original position.

iii) $0 - 1$. To achieve this, borrow 1 from left side (3rd place from right end left wards)

This 1 in 3rd place will have a value 2 in the second place and hence we have in second place, $2 - 1 = 1$.

iv) The one in the 3rd place is already borrowed to the second place and there is only zero remaining. $0 - 0 = 0$

v) $1 - 1 = 0$.

Binary multiplication : In the case of binary multiplication, the partial products are added using binary addition. In adding the partial products, we have to follow the same shift and add procedure that is followed in the decimal multiplication procedure.

Rule 1. $0 \times 0 = 0$

Rule 2. $0 \times 1 = 0$

Rule 3. $1 \times 0 = 0$

Rule 4. $1 \times 1 = 1$

E.g. 1: Let us multiply $(11)_2$ by $(11)_2$

The first number is called the multiplicand and the second number is called the multiplier.

11 → multiplicand

11 → multiplier

i) Write the multiplier just below the multiplicand in a proper way. Start partial multiplications by the last digit on right hand side of the multiplier

```
  11
 11  → shifted
----
1001
```

ii) 11

```
  1
----
 11  → is obtained as the first partial product.
```

iii) Then start the second partial multiplication by the second (from right end left wards) digit of the multiplier.

```
  11
  1
----
 11  → This is to be shifted left wards by one position to become 110.
```

iv) Now add the two binary numbers which are the partial products of multiplication as

```
  11
 110
----
1001 This final product gives us the result of multiplication
```

E.g. 2: Multiply $(1010)_2$ by $(1111)_2$

Here multiplicand is $(1010)_2$ and multiplier is $(1111)_2$

1010 → multiplicand

1111 → multiplier

1010 → (i) First partial multiplication.

1010 → (ii) Second partial multiplication, shifted one position left wards.

1010 → (iii) Third partial multiplication, shifted one more position left wards.

1010 → (iv) Fourth and final multiplication, shifted one more place left wards.

10010110 (v) Add all the above partial products to get the result of multiplication.

Binary division: This method is also called restoring - division method. This is actually a shift and - subtract method.

Rule 1, $\frac{0}{1} = 0$

Rule 2, $\frac{1}{1} = 1$

E.g. Divide $(111)_2$ by $(10)_2$

Here $(10)_2$ is called the divisor and $(111)_2$ is the dividend. After division we get a quotient and remainder.

Dividend : (divisor × quotient) + remainder

Dividend

Divisor 10) 111 (11 quotient

− 10 → i) subtract the divisor from left end on wards.

11 ii) Bring down the digit in the next position.

− 10 iii) Subtract the divisor again result is 1.

1 In quotient we again get 1.

iv) As $(10)_2$ is larger than 1 and there is no further digit in divide

we stop bringing down any digit and write 1 as the remainder.

We want to extend the division.

10) 1 (0·1

− 10 → (i) As 1 is not divisible by 10, we put a binary point. In the quotient and

10 bring down a zero.

− 10 (ii) Then subtract the divisor. We get 1 in quotient and 0 as the result of

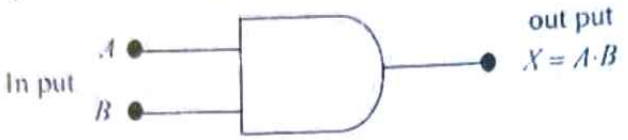
00 subtraction and hence the division is over.

$(111)_2 \div (10)_2 = (11 \cdot 1)_2$ in the binary system.

2. Draw the diagrams for AND, OR and NOT gates using discrete components. Explain their working with truth tables. **[ANU 19, 18, 17; RU 18; SKU 18; SVU 18, 17; VSU 18; YVU 18, 17]**

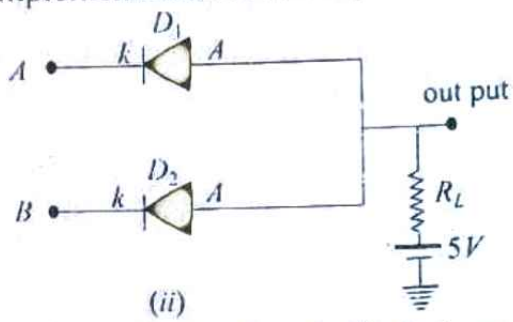
A. **AND gate (Multiplication):**

An AND gate has two (or) more input signals but only one out put signal. It is called an AND gate because the out put voltage is high. when all inputs are high. The symbolic representation of two input AND gate is shown in figure.



(i)

Fig (ii) shows the diode implementation of two input AND gate.



(ii)

i) When A and B are zero, the cathode of each diode is grounded. There fore the positive supply forward biases both diodes in parallel. We know that conducting diode has very low resistance and out put voltage (between anode and earth) will be zero.

Hence A and B are zero, out put is also zero.

ii) When A is zero and $B = 1$ (High), the diode D_1 is forward biased and diode D_2 is reverse biased. Therefore the output will be zero.

iii) When A is 1 and B is 0. The diode D_1 is reverse biased and diode D_2 is forward biased. Hence output is zero.

iv) When $A = 1$ and $B = 1$, Both diodes are reverse biased. Hence the out put is equal to the supply voltage (5v). Therefore out put is high.

A	B	$X = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Note : Here $0V = 0$

$5V = 1$

OR gate (Addition) : An OR gate has two (or) more input signals but only one out put signal. It is called an OR gate because the output voltage is high if any (or) all of the input voltages are high.

The symbolic representation of OR gate is shown in figure.

Figure (ii) shows the diode implementation of two input OR gate.

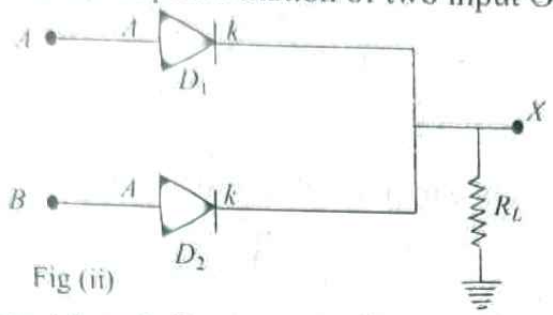


Fig (ii)

i) When both inputs (A and B) are zero, i.e., $A = 0$ and $B = 0$ both diodes are nonconducting, and $X = 0$

ii) When $A = 0$ and $B = 1$, the diode D_2 will conduct and output $X = 1$.

iii) When $A = 1$ and $B = 0$, the diode D_1 will conduct and output $X = 1$.

iv) If $A = 1$ and $B = 1$, both diodes D_1 and D_2 will conduct and output $X = 1$

Truth table

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

NOT gate: The NOT gate is a gate with only one input and one output. When the input voltage is high, the output is low. When the input voltage is low, the output is high. The NOT gate is also called an inverter. Figure (i) shows the symbol for the NOT gate.

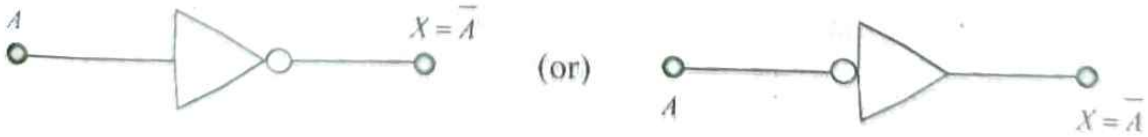
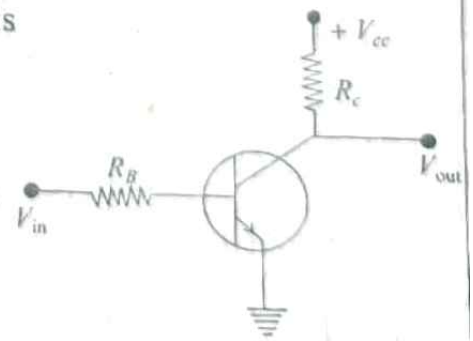


Figure (ii) shows the common emitter circuit which is used as a NOT gate.



i) When V_{in} is high, the transistor is driven into saturation. So that V_{out} is low.

ii) It V_{in} is low, the transistor is in cut off. So that V_{out} is high.

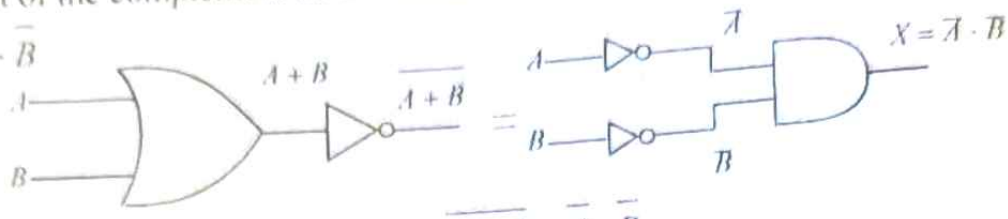
Truth table

A	$X = \bar{A}$
0	1
1	0

3. State and prove De-Morgan's theorems. **[ANU 17; AdNU 18; AU 17; RU 17; BRAU 19; KU 18; RU 18; SKU 18, 17; SVU 17; VSU 17; YVU 18]**

A. **Theorem 1 :** "The complement of the sum of two (or) more variables is equal to the product of the complement of the variables.

$\overline{A+B} = \overline{A} \cdot \overline{B}$



Proof : Here we would like to prove that $\overline{A+B} = \overline{A} \cdot \overline{B}$

We have four cases

i) When $A = 0, B = 0$

L.H.S $= \overline{A+B} = \overline{0+0} = 1$

R.H.S $= \overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{0} = \overline{0} = 1 \cdot 1 = 1$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

ii) When $A = 0, B = 1$

L.H.S $= \overline{A+B} = \overline{0+1} = \overline{1} = 0$

R.H.S $= \overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{1} = 1 \cdot 0 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

iii) When $A = 1, B = 0$

L.H.S $= \overline{A+B} = \overline{1+0} = \overline{1} = 0$

R.H.S $= \overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{0} = 0 \cdot 1 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

iv) When $A = 1, B = 1$

L.H.S $= \overline{A+B} = \overline{1+1} = \overline{1} = 0$

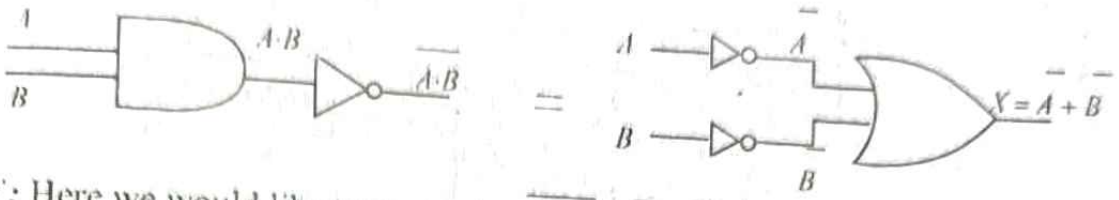
R.H.S $= \overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{1} = 0 \cdot 0 = 0$

Hence $\overline{A+B} = \overline{A} \cdot \overline{B}$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Theorem 2 : The compliment of the product of two (or) more variables is equal to the sum of the complements of the variables.

$\overline{A \cdot B} = \overline{A} + \overline{B}$



Proof : Here we would like to prove that $\overline{A \cdot B} = \overline{A} + \overline{B}$

We have four cases

i) When $A = 0, B = 0$

L.H.S $= \overline{A \cdot B} = \overline{0 \cdot 0} = \overline{0} = 1$

R.H.S $= \overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 + 1 = 1$

Hence $\overline{A \cdot B} = \overline{A} + \overline{B}$

ii) When $A = 0, B = 1$

L.H.S $= \overline{A \cdot B} = \overline{0 \cdot 1} = \overline{0} = 1$

R.H.S $= \overline{A} + \overline{B} = \overline{0} + \overline{1} = 1 + 0 = 1$

Hence $\overline{A \cdot B} = \overline{A} + \overline{B}$

iii) When $A = 1, B = 0$

L.H.S $= \overline{A \cdot B} = \overline{1 \cdot 0} = \overline{0} = 1$

R.H.S $= \overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Hence $\overline{A \cdot B} = \overline{A} + \overline{B}$

iv) When $A = 1, B = 1$

L.H.S $= \overline{A \cdot B} = \overline{1 \cdot 1} = \overline{1} = 0$

R.H.S $= \overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$

Hence $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

4. Discuss the working of half adder and full adder and give their truth tables?
[ANU 19, 17; AdNU 17; AU 18, 17; BRAU 18, 17; KU 18; RU 17; SKU 17; VSU 17; YVU 18, 17]

A. **Half adder :** "A logic circuit that adds two bits producing a sum and a carry to be used in the higher position is called a half adder".

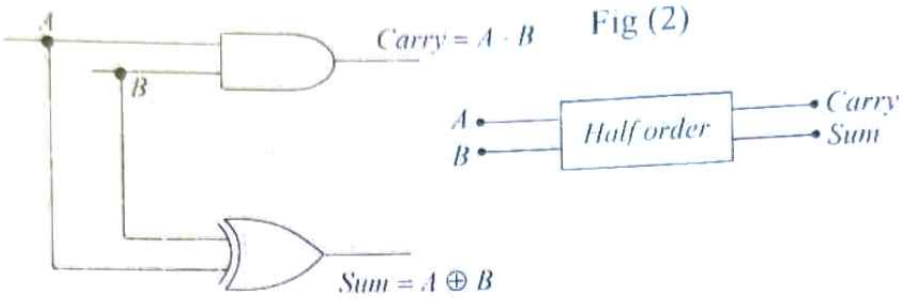


Fig (2)

Fig shows the circuit of a half-adder. It consists of a exclusive – *OR* gate and *AND* gate. The output of a *AND* gate is called the carry while the out put of a exclusive – *OR* gate is called the *SUM*. Figure (2) shows the circuit symbol of a half-adder. We know that the *AND* gate produces a high output only when both inputs are high. Hence the truth table of a Half-adder is developed by writing the truth table output of *AND* gate in carry column and the output truth table of ex – *OR* gate in sum column.

Truth table :

A	*B*	*Carry*	*SUM*
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Full adder : It is defined as a logic circuit that adds three bits - two bits to be added and a carry bit from previous addition, which results in *SUM* and carry.

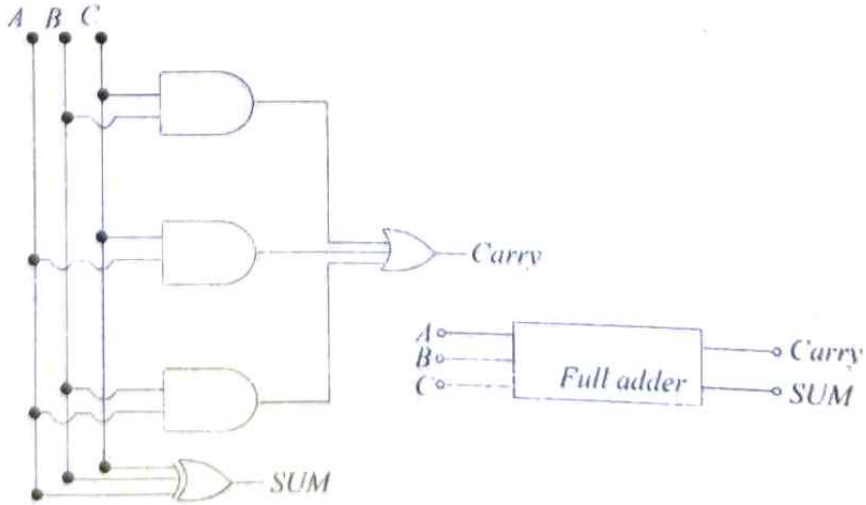


Figure shows a full adder circuit. Figure(2) shows the symbol of a full adder It has two inputs *A* and *B*, plus a third input *C*. Input *C* is also called the CARRY IN as it comes from a lower order column. There are two outputs SUM and carry The output carry is also called the CARRY OUT as it goes to the next higher column. The truth table of a Full adder circuit is shown below.

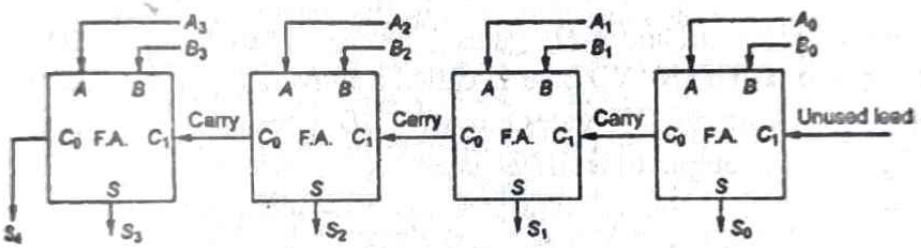
Truth table

A	B	C	Carry	*SUM*
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
1	0	0	0	1
0	1	1	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

5. Describe a parallel binary adder. Explain its operation. **[ANU 19; BRAU 18]**

By cascading a number of full adders, we can get an n - bit binary adder circuit which is generally called a parallel adder.

A four bit binary parallel adder circuit is shown in figure. The two numbers being added are A_3, A_2, A_1, A_0 and B_3, B_2, B_1, B_0. Their sum is S_4, S_3, S_2, S_1 and S_0 as shown in figure.

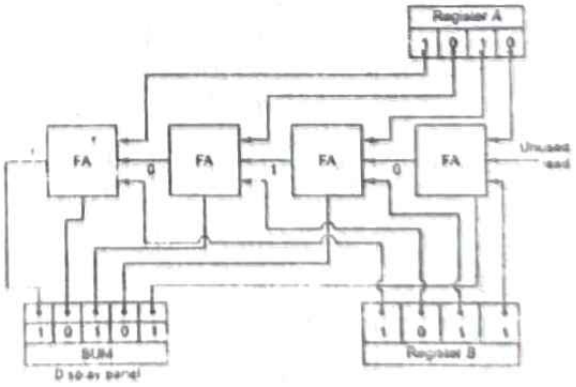


The full adder circuit in each position has three inputs A, B and C (CARRY IN) bits and it produces two out puts "SUM" and "CARRY OUT" bits.

For example. Full adder has inputs A_0, B_0 and C_1 and it produces output S_0 and C_1. The procedure is repeated for other full adders.

Operation:

The actual operation may be understood with the help of figure. Suppose, we want to add the following two four bit numbers.




```
  1010
+ 1011
 10101
```

i) The first adder performs 0 + 1 binary addition. This gives a sum of 1 and a carry of 0. The two bits 0 and 1 are supplied from two registers A and B simultaneously. The sum 1 appears on the display panel and carry 0 is passed to the next full adder.

ii) The second adder 1 + 1 + 0 carry = sum 0 with carry 1.

iii) The third adder performs 0 + 0 + 1 carry = 1 with carry 0.

iv) The fourth adder adds 1 + 1 + 0 carry = sum 0 with carry 1

∴ The final number appears as 10101.

SHORT ANSWER TYPE QUESTIONS

1. What is 1^{rs} complement method? Explain the subtraction with this method.

A. The 1^{rs} complement of a binary number is obtained by changing its each zero (0) into one (1) and each one (1) into zero (0).

E.g. 1^{rs} complement of $(10011)_2 = (01100)_2$

1^{rs} complement subtraction:

1) We first find the 1^{rs} complement of the subtrahend number. This is done by changing all its 1^{rs} to 0^{rs} and all its 0^{rs} to 1^{rs}.

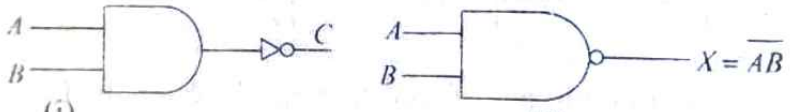
2) Now we add this 1^{rs} complement to the minuend from which the subtraction is required.

3) If there is a 1 carry in the most significant position of the result obtained by addition in step (2), Then this carry is removed and perform the end - around carry of the last 1.

4) If there is 0 carry (No carry), Then the answer is recomplemented and a negative sign is attached to it, to get the final result.

2. What are *NAND*, *NOR* and *X-OR* gates ? **[AdNU 17; SKU 17; YVU 17]**

A. ***NAND* gate :** The *NAND* gate is called a universal gate since any logic circuit can be built by using *NAND* gates only. *NAND* gate can be obtained by connecting a *NOT* gate in the output of an *AND* gate"



(i) (ii)

This gate gives an output of 1, if its both inputs are not 1. In other words, it gives an output 1 if either *A* or *B* or both are 0.

The truth table for a 2-input *NAND* gate is given in table

If $A = 0, B = 0, X = \overline{0 \cdot 0} = \overline{0} = 1$

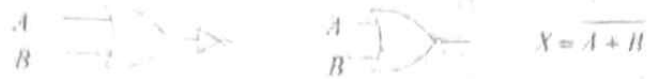
$A = 0, B = 1, X = \overline{0 \cdot 1} = \overline{0} = 1$

$A = 1, B = 0, X = \overline{1 \cdot 0} = \overline{0} = 1$

$A = 1, B = 1, X = \overline{1 \cdot 1} = \overline{1} = 0$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

The *NOR* gate : It is the combination of *OR* and *NOT* gates. When the output of *OR* gate is connected to *NOT* gate the combination is called a *NOR* gate. The output of a *NOR* gate is 1, if and only if both the inputs are each 0. The truth table of *NOR* gate is given in table



If $A = 0, B = 0, X = \overline{0 + 0} = \overline{0} = 1$

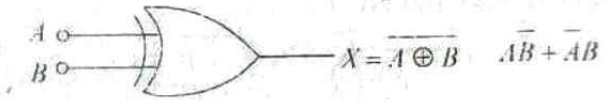
$A = 0, B = 1, X = \overline{0 + 1} = \overline{1} = 0$

$A = 1, B = 0, X = \overline{1 + 0} = \overline{1} = 0$

$A = 1, B = 1, X = \overline{1 + 1} = \overline{1} = 0$

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Exclusive *OR* gate (*XOR* gate) : Digital electronics allows as to build circuits that can add and subtract. For subtraction, the circuits used are called the exclusive *OR* gate.



In this gate, the output is 1 if its either input but not both, is 1. In other words it has an output 1 when its inputs are different. The output is 0 only when inputs are the same. The symbol ⊕ denotes the exclusive *OR* operation.

If $A = 0, B = 0, X = 0 \cdot \overline{0} + \overline{0} \cdot 0 = 0 \cdot 1 + 1 \cdot 0 = 0 + 0 = 0$

$A = 0, B = 1, X = 0 \cdot \overline{1} + \overline{0} \cdot 1 = 0 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$

$A = 1, B = 0, X = 1 \cdot \overline{0} + \overline{1} \cdot 0 = 1 \cdot 1 + 0 \cdot 0 = 1 + 0 = 1$

$A = 1, B = 1, X = 1 \cdot \overline{1} + \overline{1} \cdot 1 = 1 \cdot 0 + 0 \cdot 1 = 0 + 0 = 0$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

3. Explain in detail why NAND and NOR gates are known as universal gates with suitable examples. **[ANU 19; AdNU 18; AU 17, 18; BRAU 17; KU 18; RU 18; SKU 18; YVU 18]**

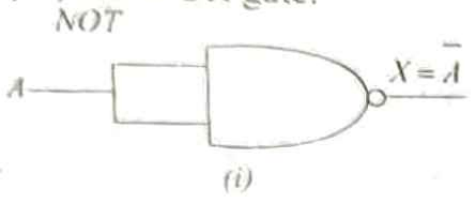
A. **NAND Gate is a universal Gate :** The NAND gate is called a universal gate since any logic circuit can be built by using NAND gates only. NAND gate can be used as NOT gate, AND gate and OR gate.

For example : When all the inputs of NAND gate are connected together, the resulting circuit is a NOT gate.

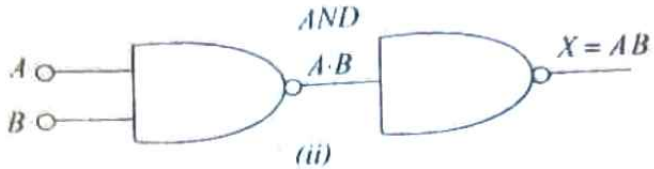
If $A = 1$, the out put of NAND gate is $\overline{A \cdot A} = \overline{1 \cdot 1} = \overline{1} = 0$

If $A = 0$, the out put of NAND gate is $\overline{A \cdot A} = \overline{0 \cdot 0} = \overline{0} = 1$

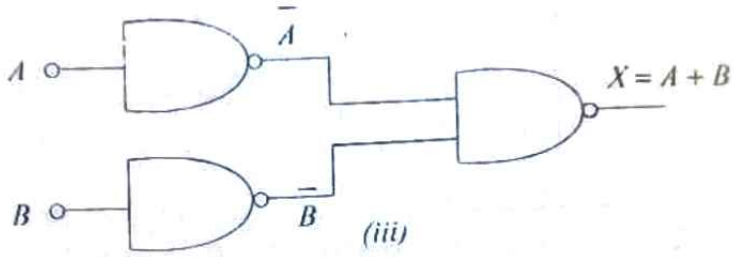
The use of two NAND gates to produce and AND gate is shown fig (ii), fig (iii) show OR gate.



(i)



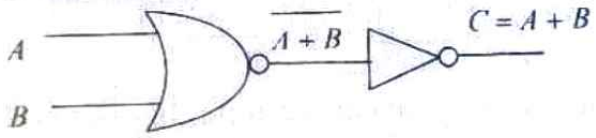
(ii)



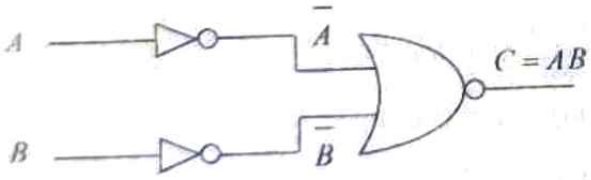
(iii)

NOR Gate is universal gate :

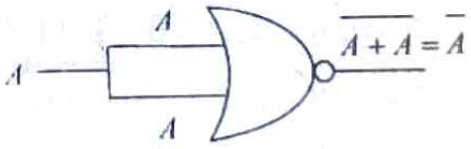
NOR gate is a universal gate because, it can be used to perform the basic logic functions OR, AND and NOT.



(i)



(ii)



(iii)

Fig (i) shows, the output of NOR gate is $\overline{A + B}$. Now using another inverter in the output, the output is inverted and becomes $C = A + B$. This is logic function of a normal OR gate. So NOR gate can be used as OR gate.

Fig (ii) shows, two inverters are used for each input before they are applied to a NOR gate. The out put of two inverters is $\overline{A}, \overline{B}$. When this is applied to NOR gate the out put is AB.

Fig (iii) shows the two inputs have been tied together the out put is $\overline{A + A} = \overline{A}$. So a NOR gate can be used as NOT gate.

4. Differentiate between analog and digital circuits.

A.

Analog circuits	Digital circuits
1) A continous varying signal (voltage (or) current) is called an analog signal.	1) A signal (voltage (or) current) that can have only two discrete values is called a digital signal.

2) For example, a sinusoidal voltage is an analog signal.	2) For example, a square wave is a digital signal.
3) In an analog electronic circuit, the output voltage changes continuously according to the input voltage variations	3) In digital circuits, output voltage can have two values, either low (or)high.
4) The output voltage can have an infinite number of values.	4) An electronic circuit that is designed for two stage operation is called a digital circuit.

5. What is 2^{rs} complement method? Explain the subtraction with this method.

2^{rs} complement : The 2^{rs} complement of a binary number is obtained by adding 1 to its 1^{rs} complement.

Thus 2^{rs} complement = 1^{rs} complement + 1.

Ex: 2^{rs} complement of $(1101)_2$

1^{rs} complement of $(1101)_2 = (0010)_2$

2^{rs} complement = 1^{rs} complement + 1

$= (0010)_2 + 1$

$= (0011)_2$

2^{rs} complemental subtraction :

1) Change all 0^{rs} to 1^{rs} and 1^{rs} to 0^{rs} of subtrahend to get the 1^{rs} complement and then add 1 to get 2^{rs} complement.
2) Now we add the 2^{rs} complemental to minuend.
3) If there is a carry in the result, it is dropped.
4) If there is one (1), then the answer is positive and needs no recomplementing.
5) If there is no carry, we recomplement the result and attach a negative sign to it.

6. Explain the laws of Boolean Algebra ? **[RU 18; SVU 18]**

1) OR law : In Boolean algebra (+) sign indicates OR operation. $\boxed{A+B=C}$

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

The OR laws in general form may be expressed as

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + \overline{A} = 1$ where $\overline{A}$ is complement (or) negative of A.

2) AND law : In Boolenalgebra the AND operation is represented by multiplication. $\boxed{A \cdot B = C}$

$0 \cdot 0 = 0$	In general
$0 \cdot 1 = 0$	$A \cdot 0 = 0$
$1 \cdot 0 = 0$	$A \cdot 1 = A$
$1 \cdot 1 = 1$	$A \cdot A = A$
	$A \cdot \overline{A} = 0$

3) NOT law : It is a complementation operation and its symbol is an over bar

$$\boxed{\overline{\overline{A}} = A}$$

$$\overline{0} = 1$$

$$\overline{1} = 0$$

4) Commutative laws :

$$A + B = B + A \;;\quad A \cdot B = B \cdot A$$

5) Assosiative laws :

$$A + (B + C) = (A + B) + C \;;\; A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

6) Distributive laws :

$$A(B + C) = AB + AC \;;\; (A + B) \cdot (A + C) = A + BC$$

7) Absorptive laws :

$$A + AB = A$$

$$A \cdot (A + B) = A \;;\; A \cdot (\overline{A} + B) = AB$$

8) De Morgan's theorem :

i) $\overline{A + B} = \overline{A} \cdot \overline{B}$; ii) $\overline{A \cdot B} = \overline{A} + \overline{B}$

PROBLEMS

1. Find the binary equivalent of 576.

Sol :

			Remainder
2	576		
2	288	---	0
2	144	---	0
2	72	---	0
2	36	---	0
2	18	---	0
2	9	---	0
2	4	---	1
2	2	---	0
	1	---	0

$\therefore$ Binary equivalent of 576 is $(1001000000)_2$

2. Find the decimal equivalent of 1111.

Sol: $1111 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 2 + 1 = 15$

3. Convert $(110111)_2$ to decimal number ? [KU 2019]

Sol: $(110111)_2 = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 32 + 16 + 0 + 4 + 2 + 1$

$= (55)_{10}$

4. Convert binary number 0·101 to a decimal number ?

Sol: $(0{\cdot}101)_2 = 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= \frac{1}{2} + 0 + \frac{1}{8}$

$= 0{\cdot}5 + 0 + 0{\cdot}125$

$= (0{\cdot}625)_{10}$

5. Convert 0·1100101 to decimal number.

Sol: $(0{\cdot}1100101)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5} + 0 \times 2^{-6} + 1 \times 2^{-7}$

$= \frac{1}{2} + \frac{1}{4} + \frac{0}{8} + \frac{0}{16} + \frac{1}{32} + \frac{0}{64} + \frac{1}{128}$

$= 0{\cdot}5 + 0{\cdot}25 + 0 + 0 + 0{\cdot}0125 + 0 + 0{\cdot}0078125$

$= (0{\cdot}7890625)_{10}$

6. Add binary numbers 110, 111 and 101.

Sol:

Step 1	Step 2
110	1101
111	101
1101	10010

Thus, the result of addition of given three numbers will be 10010.

7. Subtract $(0111)_2$ from $(1001)_2$.

Sol:

1001	1st column, $1 - 1 = 0$
− 0111	2nd column, $0 - 1 = 1$ (with a barrow of 1)
0010	3rd column, $1 - 1 = 0$
	4th column, $0 - 0 = 0$

8. Subtract $(111001)_2$ from $(1110001)_2$.

Sol:

1110001

111001

111000

9. Multiply $(10011)_2$ by $(11011)_2$.

Sol:

10011

× 11011

10011

Rule 1 → $0 \times 0 = 0$

Rule 2 → $0 \times 1 = 0$

Rule 3 → $1 \times 0 = 0$

Rule 4 → $1 \times 1 = 1$

[ANU 2017]

10. Carry out the binary division 11001 ÷ 101

Sol :

```
        101
      ------
101 | 11001
     | 101
     |------
     | 101
     | 101
     |------
     | 000
```

Rule 1 → 0 / 1 = 0
Rule 2 → 1 / 1 = 1

11. Subtract $(101)_2$ from $(111)_2$ using 1^{rs} complement method.

Sol :

111 → Minuend

010 → 1^{rs} complement of subtrahend 101

(1) 001

Add carry ↳ 1 ← End round carry

Final result = 010

12. Subtract $(1101)_2$ from $(1010)_2$ using 1^{rs} complement method.

Sol:

1010 ← Minuend

+ 0010 ← 1^{rs} complement of subtrahend 1101.

No carry 1100

recomplement 0011 ← As there is no end - around carry.

− 0011 ← Final result.

13. Subtract $(101101)_2$ from $(110011)_2$ using 1^{rs} complement method.

Sol:

110011 ← Minuend

+ 010010 ← 1^{rs} complement of subtrahend 101101

(1) 000101

↳ 1 ← End round carry

000110 ← Result

14. Using 2^{rs} complemental subtract $(100111)_2$ from $(110011)_2$.

Sol : 011000 ← 1^{rs} complement of (10011)

1 ← Add one

011001 ← 2^{rs} complement of (10011)

Now 110011 ← Minuend

011001 ← 2^{rs} complemental

Carry (1) 001100

Discord carry 001100 ← Final result.

15. Use 2^{rs} complemental to subtract $(1101)_2$ from $(1010)_2$.

Sol : 0010 ← 1^{rs} complemental of 1101.

1 Add one

0011 ← 2^{rs} complemental

Now 1010 ← Minuend

+ 0011 ← 2^{rs} complemental

No carry 1101

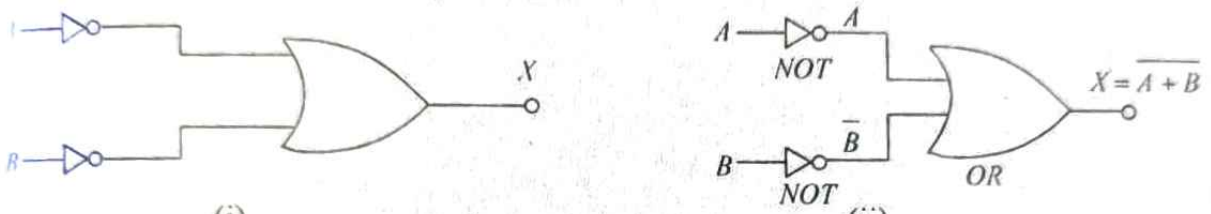
Subtract 1

1100

Recomplement 0011

Final result = – 0011

16. What is the Boolean expression for the output X of fig i ? Compute the value of X, when i) $A = 0, B = 0$ ii) $A = 1, B = 1$



(i) (ii)

Sol : The output is showing figure (ii). As shown in fig (ii) both inputs to OR gates have been inverted. i.e., the outputs of both gates will be $\overline{A}$ and $\overline{B}$.

Now the inputs of OR gate are $\overline{A}$ and $\overline{B}$. The output will be $X = \overline{A} + \overline{B}$

i) when $A = 0, B = 0$, then $X = \overline{0} + \overline{0} = 1 + 1 = 1$

ii) when $A = 1, B = 1$ then $X = \overline{1} + \overline{1} = 0 + 0 = 0$

17. De Morganize the function $\overline{A\overline{B} + C}$.

Sol : **Step 1 :** Complement the given function i.e., $\overline{A\overline{B} + C}$

i.e., $A\overline{B} + C$ which should be as $(A \cdot \overline{B}) + C$

Here put · between the variables

Step 2 : Replace · by + and + by · in step 1

$(A + \overline{B}) \cdot C$

Step 3 : Complement variables

$(\overline{A} + \overline{\overline{B}}) \cdot \overline{C} = (\overline{A} + B) \cdot \overline{C}$

Hence $\overline{AB + C} = (\overline{A} + B) \cdot \overline{C}$

18. Prove this by Boolean algebra and De.Morgan's theorem

$\overline{AB + BC + CA} = \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{C}$

Sol : $\overline{AB + BC + CA} = \overline{AB + (BC + CA)}$