II

Paper Code: 3101 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

II B.Sc., Degree Examinations, Oct/Nov 2016

(At the end of 3rd Semester)

(For 2015-18 batch)

Part–II **MATHEMATICS** Paper–II A

(Abstract Algebra (Group Theory))

Date: 01.11.2016 FN

Duration: 3hrs

Max Marks:75

PART - I

Answer any FIVE questions, each question carries FIVE marks. 5 x 5 = 25M

1. If $G = Q - \{1\}$ and $*$ is defined on G as $a * b = a + b - ab$ then show that $(G, *)$ is an abelian group.

2. Prove that a non empty complex H of a group G is a subgroup of G if and only if $a,b \in H \Rightarrow ab^{-1} \in H$.

3. Prove that a subgroup H of a group G is normal if and only if $xHx^{-1} = H \forall x \in G$.

4. Prove that the intersection of any two normal subgroups of a group is a normal subgroup.

5. If f is a homomorphism from a group G into a group G^1 then prove that Kerf is a normal subgroup of G.

6. Let G, G^1 be two groups with identity elements e, e^1 respectively. If $f: G \to G^1$ is a homomorphism then prove that $(1) f(e) = e^1$, $(2) f(a^{-1}) = [f(a)]^{-1}$.

7. Write down the product (1 3 2) (5 6 7) (2 6 1) (4 5) as disjoint cycles.

8. Show that $G = \{1, -1, i, -i\}$ the set of all fourth roots of unity is a cyclic group.

PART - II

SECTION - A

Answer any FIVE questions. Choosing atleast TWO questions from each section. Each question carries 10 marks.

5 x10=50M

9. Prove that in a group G for $a,b,x,y \in G$ the equation $ax = b$ and $ya = b$ have unique solutions.

10. Prove that the order of every element of a finite group is finite and is less than or equal to order of a group.

11. Let H, K be any two subgroups of a group G then prove that HK is a subgroup of G if and only if HK = KH.

12. State and prove Legrange's theorem for cosets.

13. Prove that subgroup H of a group G is a normal subgroup of G if and only if each left coset of H in G is a right coset of H in G.

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SECTION - B

14. State and prove fundamental theorem of homomorphism for groups.

15. Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G^{I} with Kernel K to be an isomorphism of G into G^{I} is that $K = \{e\}$.

16. If $f = (1\,2\,3\,4\,5\,8\,7\,6)$ $g = (4\,1\,5\,6\,7\,3\,2\,8)$ are cyclic permutations then show that $(fg)^{-1} = g^{-1} f^{-1}$.

17. If p is a prime number then prove that every group of order p is a cyclic group.

18. Prove that the order of a cyclic group is equal to the order of its generators.

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II B.Sc.,Degree Examinations, Oct/Nov 2017

(At the end of 3rd Semester)

Regular (2016 batch), Supplementary (2015 batch)

MATHEMATICS Paper – III

(Abstract Algebra)

Date: 28.10.2017 FN Max Marks:75

Duration: 3hrs

PART – I

Answer any *FIVE* questions, each question carries *FIVE* marks. **5x5=25M**

1. Show that in a group G for a,b∈G,$(ab)^2 = a^2b^2$ ⇔ G is abelian.
2. Prove that inverse element in a group is unique.
3. A necessary and sufficient condition for a non – empty subset H of a group G to be a subgroup of G is that $HH^{-1} = H$.
4. Prove that intersection of two subgroups of a group G is a subgroup of G.
5. The intersection of any two normal subgroups of a group is a normal subgroup.
6. If f:G→ $\bar{G}$ defind f(x) = 1 if x>0 and -1 if x<0where G=[set of non zero real numbers] and $\bar{G}$ = [1,-1] are groups prove that f is a homomorphism and find kernel
7. If f = (1 2 3 4 5) is cyclic permutation, find its order
8. Show that every cyclic group is abelian.

PART – II

Answer any *FIVE* questions choosing at least TWO questions from each section.

Each Question carries *10* Marks. **5x10=50M**

SECTION – A

9. Show that set Q_+ of all positive rational numbers from an abelian group under the composition defined by '0' such that (aob) = (a b)/3 for a,b ∈Q_+
10. Prove that the set of n^{th} roots of unity under multiplication form a finite group
11. If H and K are two subgroups of a group G, then HK is a subgroup of G iff HK =KH

12. State and prove lagranges theorem

13. A subgroup H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G

SECTION – B

14. State and prove fundamental theorem on homomorphism of groups

15. The necessary and sufficient condition for a homomorphism F of a group G onto a group G^1 with Kernel K to be an isomorphism of G into G^1 is that K={e}

16. State and prove Cayleys theorem.

17. Prove that every subgroup of a cyclic group is cyclic.

18. Every Isomorphic image of a cyclic group is cycle.

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II B.Sc.,Degree Examinations, Oct/Nov 2018

(At the end of 3rd Semester)

Regular (2017 batch), Supplementary (2016 batch)

MATHEMATICS Paper – III

(Abstract Algebra)

Date: 02.11.2018 FN Max Marks:75

Duration: 3 hrs

PART-I

Answer any five questions each question carries **Five** marks: 5X5=25M

1. Prove that the set Z of all integers forms an abelian group with respect to the operation * defined by $a * b = a + b + 2, \forall a, b \in Z$.
2. If H is any subgroup of G, then prove that $H = H^{-1}$.
3. Prove that a subgroup H of a group G is normal $\Leftrightarrow xHx^{-1} = H$.
4. If M and N are two normal subgroups of a group g such that $M \cap N = \{e\}$, then prove that every element of M commutes with every element of N.
5. Prove that every homomorphic image of an abelian group is abelian.
6. Show that the mapping $f: G \to G$ such that $f(a) = a^{-1}\ \forall a \in G$ is an automorphism of a group G$\Leftrightarrow$G is abelian.
7. If $f = (1\,2\,3\,4\,5\,8\,7\,6), g = (4\,1\,5\,6\,7\,3\,2\,8)$ are cyclic permutations, then show that $(fg)^{-1} = g^{-1}.f^{-1}$
8. Show that the group (G={1,2,3,4,5,6},X_7) is cyclic.Also write down all its generators

PART-II

Answer any five questions .choosing at least two questions from each section.
Each question carries 10 marks. 5X10=50M

SECTION-A

9. Prove that every finite semi group (G,.) satisfying the cancellation laws is a group
10. Define order of an element in a group G. In a group G, if $a \in G$, then show that $O(a) = O(a^{-1})$
11. Prove that the necessary and sufficient condition for afinite complex H of a group G to be a subgroup of G is $a, b \in H \Rightarrow ab \in H$.
12. State and prove Lagrange's theorem.
13. Prove that a subgroup H of a group G is a normal subgroup of G $\Leftrightarrow$ each left coset of H in G is a right coset of H in G.

SECTION-B

14. Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G^1 with kernel K to be an isomorphism of G into G^1 is that K={e}.
15. State and prove fundamental theorem of homomorphism of group G.
16. State and prove Cayley's theorem.
17. Prove that every subgroup of a cyclic group is cyclic.
18. Find the number of generators of cyclic groups of order 5, 6, 8, 12, 15, 60.