Paper Code: 5102 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc.,Degree Examinations, Oct/Nov 2017

(At the end of 5th Semester)

Regular (2015-18 batch)

MATHEMATICS Paper – VI

(Linear Algebra)

Date: 27.10.2017 FN Duration:3hrs Max Marks:75

PART-I

Answer any FIVE Questions. Each question carries FIVE marks. 5 X 5 = 25M

1. Let p, q, r be the fixed elements of a field F.Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz = 0 is a subspace of $V_3(F)$.
2. If α, β, γ are linearly independent vectors of V(R) then show that $\alpha+\beta,\ \beta+\gamma,\ \gamma+\alpha$ are also linearly independent.
3. Show that {(1,2,1),(2,1,0),(1,-1,2)} forms a basis of $V_3(F)$.
4. The mapping T: $V_3(R) \rightarrow V_1(R)$ defined by $T(a,b,c)=a^2+b^2+c^2$; Can T be a linear transformation?
5. Let U(F) and V(F) be two vector spaces and T: U→V is a linear transformation, then prove that the range set R(T) is a subspace of V(F).
6. Prove that the square matrices A and A^1 have the same characteristic values.
7. State and prove Triangle inequality.
8. Prove that $\{(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}),(\frac{2}{3},\frac{-1}{3},\frac{2}{3}),(\frac{2}{3},\frac{2}{3},\frac{-1}{3})\}$ is an orthonormal set in R^3 with standard inner product.

PART-II

Answer any FIVE Questions. Choosing at least TWO questions from each section.
Each question carries 10 marks. 5 X 10 = 50M

SECTION-A

9. Let V(F)be a vector space. A non empty set $W \subseteq V$.Prove that the necessary and sufficient condition for W to be a subspace of V is $a,b \in F$ and $\alpha,\beta \in V \Rightarrow a\alpha + b\beta \in W$.
10. Let V(F) be a vector space and S = { $\alpha_1, \alpha_2, \ldots \ldots \alpha_n$ } is a finite subset of non-zero vectors of V(F). Then prove that S is linearly independent if and only if some vector $\alpha_k \in S, 2 \leq K \leq n$,can be expressed as a linear combination of its preceding vectors.
11. If V(F) is a finite dimensional vector space then prove that there exists a basis set of V.
12. Let W be a subspace of a finite dimensional vector space V(F), then show that $dim\frac{V}{W} = dimV - dimW$.
13. Let U(F) and V(F) be two vector spaces and T: U→V be a linear transformation .Let U be finite dimensional then prove that rank(T) + nullity(T) = dimU.

SECTION-B

14. Describe explicitly of the linear transformation $T:R^2 \rightarrow R^2$ such that T(2,3)=(4,5), T(1,0)=(0,0).
15. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
16. State and prove Clayey – Hamilton theorem.
17. State and prove Cauchy – Schwartz's inequality.
18. Apply Gram-Schmidt process to the vectors {(1,0,1),(1,0,-1),(0,3,4)} to obtain an orthonormal basis of $V_3(R)$ with the standard inner product.

Paper Code: 5124 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc.,Degree Examinations, Oct/Nov 2018

(At the end of 5^{th} Semester)

Regular (2016 batch), Supplementary (2015 batch)

MATHEMATICS

Paper – VI

(Linear Algebra)

Date: 01.11.2018 FN

Time: 3Hrs

PART-I

Max.Marks:75

Answer any **FIVE** Questions ,Each Question carries **FIVE** marks. **5×5M=25M**

1. If S is a non empty subset of the vector space V(F) then prove that linear span L(S) is a sub space of V(F)

2. Express the vector α= (1,-2,5) as a linear combination of the vectors $\alpha_1 = (1,1,1)$ $\alpha_2 = (1,2,3)$ $\alpha_3 = (2,-1,1)$ in $V_3(F)$.

3. If α, β, γ are linearly independent vectors in the Real vector space V (R) then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent vectors.

4. If T is a linear transformation from a vector space U(F) into a vector space V(F) Then Prove that the null space N(T) is a subspace of U(F).

5. Show that the mapping T: $V_2(R) \rightarrow V_3(R)$ is defined by T(a, b)= (a + b, a- b, b) is a linear transformation from $V_2(R)$ in to $V_3(R)$.

6. Show that the system of Equations $x + y + z = -3$, $3x + y - 2z = -2$, $2x + 4y + 7z = 7$ are inconsistent.

7. State and prove Parallelogram Law.

8. If α, β are two vectors in an inner product space V(F) such that $|| \alpha || = || \beta ||$ then Prove that $\alpha + \beta$, $\alpha - \beta$ are orthogonal.

PART -II

Answer any FIVE questions .Choosing at least TWO from each section. Each question carries 10 marks. 5×10=50M

Section -A

9. Let W_1 and W_2 be two Subspaces of a vector space V(F) then Prove that $W_1 \cup W_2$ is a subspace of V(F) iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

10. Prove that every finite dimensional vector space has a basis

11. If W_1 and W_2 are two sub spaces of a finite dimensional vector space V(F) then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

12. If W is subspace of a finite dimensional vector space V(F) then Prove that $\dim(V/W) = \dim V - \dim W$.

13. Verify Rank – Nullity theorem for a linear Transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (x - y, 2y + z, x + y + z)$.

Section – B

14. Find the linear transformation $T: R^2 \to R^3$ such that

$$T(1, 2) = (3, -1, 5) \quad T(0,1) = (2, 1, -1)$$

15. For what values of λ, the equations $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$ have solution? Solve them completely in each case.

16. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

17. State and prove Schwarz's Inequality

18. Apply the Gram-Schmidt's process to the vectors { (2,1,3),(1,2,3) (1,1,1) } to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.