

Paper Code: 4101 Regd. No

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SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

II B.Sc., Degree Examinations, Mar/Apr 2017

(At the end of 4<sup>th</sup> Semester)

(For 2015-18 batch)

Part – II

**MATHEMATICS**

Paper – II B

(Real Analysis)



Date: 06.04.2017 FN

Max Marks: 75

Duration: 3hrs

### PART-I

Answer any FIVE questions, each questions carries FIVE marks. 5 X 5 = 25M

1. If  $S_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)}$  <sup>Show that</sup> then  $\{S_n\}$  is increasing and bounded.
2. Prove that every convergent sequence is bounded.
3. Test for the convergence of the series  $\sum_{n=1}^{\infty} (\sqrt[3]{n^3+1} - n)$ .
4. Test for convergence of  $\sum \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2 \cdot 4 \cdot 6 \dots 2n} x^{n-1}$
5. Examine whether the function  $f$  defined by  $f(x) = x \sin\left(\frac{1}{x}\right)$  when  $x \neq 0$  and  $f(0) = 0$  is continuous at the origin.
6. Show that  $f(x) = |x| + |x-1|$  is not derivable at  $x = 0$  and  $x = 1$ .
7. Verify Cauchy's mean value theorem for  $f(x) = x^2$  and  $g(x) = x^3$  in  $[1, 2]$ .
8. If the function  $f$  is defined by  $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} - \mathbb{Q} \end{cases}$  then show that  $f$  is not Riemann Integrable over any interval of  $\mathbb{R}$ .

## PART-II

Answer any FIVE questions from the following, choosing atleast TWO questions from each section. Each question carries TEN marks. 5X10 = 50M

### SECTION-A

9. If  $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$  then show that  $\{s_n\}$  is not convergent by using Cauchy's general principle of convergence theorem.
10. Prove that a sequence is convergent iff it is a Cauchy sequence.
11. State and prove Cauchy's  $n^{\text{th}}$  root test.
12. Define Alternating series. State and Prove Leibnitz's test.
13. Prove that the function  $f$  defined on  $\mathbb{R}^+$  as  $f(x) = \sin\left(\frac{1}{x}\right)$  for every  $x > 0$  is continuous but not uniformly continuous on  $\mathbb{R}^+$ .

### SECTION-B

14. State and prove Rolle's Theorem.
15. Find  $c$  of Lagrange's mean value theorem for  $f(x) = (x-1)(x-2)(x-3)$  on  $[0, 4]$ .
16. If  $f$  is continuous on  $[a, b]$  then prove that  $f$  is R-Integrable on  $[a, b]$ .
17. Show that  $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$ .
18. State and prove fundamental theorem of integral calculus.



Date: 31.03.2018 FN

Max Marks: 75

Duration: 3hrs

## PART - I

Answer any FIVE questions :

5x5=25

1. If  $\{S_n\}$  is a sequence such that  $S_n > 0 \forall n \in \mathbb{Z}^+$  and  $\lim \frac{S_{n+1}}{S_n} = l$  then  $\lim \sqrt[n]{S_n} = l$
2. Prove that  $\lim_{n \rightarrow \infty} \left\{ \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right\} = 1$
3. Examine the convergence of  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$
4. Test for convergence  $\sum_{n=1}^{\infty} 3^{-n-(-1)^n}$
5. Show that  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 1$  if  $x \in \mathbb{Q}$  and  $f(x) = -1$  if  $x \in \mathbb{R} - \mathbb{Q}$  is discontinuous for all  $x \in \mathbb{R}$
6. If  $f: [a, b] \rightarrow \mathbb{R}$  is derivable at  $c \in [a, b]$  then prove that  $f$  is continuous at  $C$
7. Prove that  $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1} 0.6 < \frac{\pi}{6} + \frac{1}{8}$
8. Prove that  $f(x) = \sin x$  is integrable on  $\left[ 0, \frac{\pi}{2} \right]$  and  $\int_0^{\pi/2} \sin x dx = 1$

## PART - II

Answer any FIVE questions choosing atleast TWO questions from each section : 5x10=50

## SECTION - A

9. If  $S_n = \left( 1 + \frac{1}{n} \right)^n$  then show that  $\{S_n\}$  is convergent
10. Discuss the nature of the sequence  $\{r^n\}$  for all  $r \in \mathbb{R}$
11. Test for convergence of  $\sum \left( \sqrt[3]{n^3+1} - n \right)$

12. Define alternating series. State and prove Leibnitz's test
13. State and prove Bolzano's intermediate value theorem

### SECTION - B

14. State and prove Rolle's theorem
15. Show that  $\frac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \frac{v-u}{1+u^2}, 0 < u < v$  and deduce that  $2 < \pi < 4$
16. A bounded function  $f:[a,b] \rightarrow R$  is Riemann integrable on  $[a,b]$  iff for each  $\epsilon > 0$  there exists a partition  $P$  of  $[a,b]$  such that  $0 \leq U(P,f) - L(P,f) < \epsilon$
17. State and prove first mean value theorem
18. Show that  $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$





Date: 15.04.2019 FN

Duration:3hr

Max Marks:75

**PART - I****Answer any FIVE questions, each question carries Five marks****5×5=25M**

1. Prove that every convergent sequence is bounded
2. Prove that the sequence  $\{S_n\}$  defined by  $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$  is convergent.
3. Test for convergence of  $\sum_{n=1}^{\infty} (\sqrt{n^4 + 1} - \sqrt{n^4 - 1})$ .
4. Test for convergence of  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$
5. Show that the function  $f$  defined by  $f(x) = x^3$  is uniformly continuous in  $[-2, 2]$ .
6. If  $f: [a, b] \rightarrow \mathbb{R}$  is derivable at  $c \in [a, b]$  then prove that  $f$  is continuous at  $C$ .
7. Prove that  $\frac{x}{1+x^2} < \tan^{-1}x < x$ , when  $x > 0$
8. If  $f(x) = x^2$  on  $[0, 1]$  and  $p = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$  find  $L(p, f)$  and  $U(p, f)$ .

**PART - II****Answer any FIVE questions. Choosing at least TWO questions****from each section. Each question carries 10 Marks 5×10=50M****SECTION - A**

9. State and prove Cauchy's first theorem on limits.
10. Prove that a monotone sequence is convergent iff it is bounded.
11. State and prove D-Alembert's test.
12. State and prove liebnitz test.
13. Find the constants  $a, b$  so that the function  $f$  defined by  $f(x) = 2x + 1$  if  $x \leq 1$ ,  $f(x) = ax^2 + b$  if  $1 < x < 3$ ,  $f(x) = 5x + 2a$  if  $x \geq 3$  is continuous at  $x = 1, x = 3$ .

**SECTION - B**

14. State and prove Rolle's theorem.
15. Prove that  $\frac{\pi + \sqrt{3}}{6} < \sin^{-1} 0.6 < \frac{\pi + 1}{6}$  by using lagranges mean value theorem.
16. A bounded function  $f: [a, b] \rightarrow \mathbb{R}$  is Riemann integrable on  $[a, b]$  iff for each  $\epsilon > 0$  there exists a partition  $P$  of  $[a, b]$  such that  $0 \leq U(p, f) - L(p, f) < \epsilon$ .
17. If  $f: [a, b] \rightarrow \mathbb{R}$  is monotonic on  $[a, b]$  then prove that  $f$  is integrable on  $[a, b]$
18. Prove that  $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5 + 3\cos x} dx \leq \frac{\pi^3}{6}$ .



SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

II B.Sc., Degree Examinations, October 2020

(At the end of 4<sup>th</sup> Semester)

Regular (2018-21 batch)

**MATHEMATICS**

Paper – IV

(Real Analysis)



Date: 03.11.2020 FN

Duration:3hrs

Max Marks:75

**PART - I**

Answer any **FIVE** questions, Each question carries **FIVE** marks.

5 x 5 = 25 M

1. State and prove sandwich theorem.
2. If  $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$  then show that  $\{S_n\}$  converges.
3. Test for convergence of  $\sum (\sqrt{n^4+1} - \sqrt{n^4-1})$
4. State and prove Leibnitz's Test.
5. If  $f$  is continuous on  $[a, b]$  then  $f$  is bounded on  $[a, b]$ . *Prove that*
6. Find 'C' of Lagrange's theorem for  $f(x) = x(x-1)(x-2)$  on  $[0, 1/2]$ .
7. The Maclaurin's Expansion of  $\sin x$ . *Prove that*
8. If  $f: [a, b] \rightarrow \mathbb{R}$  is continuous on  $[a, b]$  then  $f$  is R - Integrable on  $[a, b]$  *Prove that*

**PART - II**

Answer any **FIVE** questions. Choosing atleast TWO questions from Each section.

Each Question carries **TEN** marks.

5 x 10 = 50 M

**SECTION - A**

9. State and prove cauchy's First theorem on Limits.
10. *Prove that* A monotonic sequence is convergent iff it is bounded.
11. State and prove P-Test.
12. State and prove limit comparison test.

13. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be such that  $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & \text{for } x < 0 \\ C & \text{for } x = 0 \\ \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & \text{for } x > 0 \end{cases}$

Determine the values of  $a, b, c$  for which the function is continuous at  $x = 0$ .

SECTION - B

14. State and prove Rolle's theorem.
15. State and prove Cauchy's mean value theorem.
16. <sup>Prove that</sup> A bounded function  $f : [a, b] \rightarrow \mathbb{R}$  is Riemann integrable on  $[a, b]$  iff for each  $\epsilon > 0$  there exists a partition  $p$  of  $[a, b]$  such that  $0 \leq U(p, f) - L(p, f) < \epsilon$ .
17. State and prove Fundamental theorem of integral calculus.
18. Prove that  $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$

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Date: 29.06.2022 FN

Duration: 3hrs

Max Marks: 75

SECTION-A

Answer any FIVE Questions, each question carries FIVE marks

5 x 5 = 25M

1. Prove that every convergent sequence is bounded.
2. Prove that  $\lim \left[ \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(n+n)^2} \right] = 0$ .
3. Test the convergence of  $\sum_{n=1}^{\infty} \frac{2^n - 2}{2^{n+1}} x^n$ , ( $x > 0$ ).
4. Test the convergence of  $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \dots$
5. Examine the continuity of the function  $f(x) = \frac{1 - \cos x}{x^2}$ ,  $x \neq 0$ ,  $f(0) = 1$  at  $x = 0$ .
6. If  $f: [a, b] \rightarrow R$  is derivable at  $c \in [a, b]$ , then prove that  $f$  is continuous at  $c$ .
7. Verify Cauchy's mean value theorem for  $f(x) = x^2$ ,  $g(x) = x^3$  in  $[1, 2]$ .
8. Prove that  $f(x) = \sin x$  is integrable on  $[0, \frac{\pi}{2}]$  and  $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$ .

Answer any FIVE Questions from sections B and C choosing at least Two questions from each section. Each question carries 10 marks.

5 x 10 = 50M

SECTION-B

9. Prove that a sequence is convergent if and only if it is a Cauchy sequence.
10. If  $s_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$  then Show that  $\{s_n\}$  is convergent.
11. State and prove p-test.
12. State and prove Cauchy's  $n^{\text{th}}$  root test.
13. Prove that if  $f$  is continuous on  $[a, b]$  then prove that  $f$  is bounded on  $[a, b]$ .

SECTION-C

14. State and prove Rolle's theorem.
15. using Lagrange's theorem, show that  $x > \log(1+x) > \frac{x}{1+x} \forall x > 0$ .
16. If  $f: [a, b] \rightarrow R$  is continuous on  $[a, b]$ , then prove that  $f$  is R- integrable on  $[a, b]$ .
17. State and prove fundamental theorem of integral calculus.
18. Prove that  $\frac{\pi^2}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} \, dx \leq \frac{\pi^2}{6}$ .





Date: 29.06.2022 FN

Duration: 3hrs

Max Marks: 75

SECTION-A

Answer any FIVE Questions, each question carries FIVE marks

5 x 5 = 25M

1. Prove that every convergent sequence is bounded.
2. Prove that  $\lim \left[ \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(n+n)^2} \right] = 0$ .
3. Test the convergence of  $\sum_{n=1}^{\infty} \frac{2^n - 2}{2^{n+1}} x^n$ , ( $x > 0$ ).
4. Test the convergence of  $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \dots$
5. Examine the continuity of the function  $f(x) = \frac{1 - \cos x}{x^2}$ ,  $x \neq 0$ ,  $f(0) = 1$  at  $x = 0$ .
6. If  $f: [a, b] \rightarrow R$  is derivable at  $c \in [a, b]$ , then prove that  $f$  is continuous at  $c$ .
7. Verify Cauchy's mean value theorem for  $f(x) = x^2$ ,  $g(x) = x^3$  in  $[1, 2]$ .
8. Prove that  $f(x) = \sin x$  is integrable on  $[0, \frac{\pi}{2}]$  and  $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$ .

Answer any FIVE Questions from sections B and C choosing at least Two questions from each section. Each question carries 10 marks.

5 x 10 = 50M

SECTION-B

9. Prove that a sequence is convergent if and only if it is a Cauchy sequence.
10. If  $s_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$  then Show that  $\{s_n\}$  is convergent.
11. State and prove p-test.
12. State and prove Cauchy's  $n^{\text{th}}$  root test.
13. Prove that if  $f$  is continuous on  $[a, b]$  then prove that  $f$  is bounded on  $[a, b]$ .

SECTION-C

14. State and prove Rolle's theorem.
15. using Lagrange's theorem, show that  $x > \log(1+x) > \frac{x}{1+x} \forall x > 0$ .
16. If  $f: [a, b] \rightarrow R$  is continuous on  $[a, b]$ , then prove that  $f$  is R- integrable on  $[a, b]$ .
17. State and prove fundamental theorem of integral calculus.
18. Prove that  $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} \, dx \leq \frac{\pi^3}{6}$ .