Paper Code: 5123 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

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(At the end of 5th Semester)

Regular (2016 batch), Supplementary (2015 batch)

MATHEMATICS Paper – V

(Ring Theory and Vector Calculus)

Date: 23.10.2018 FN Max Marks:75

Duration:3hrs

PART – I

Answer any FIVE Questions, each question carries FIVE marks. 5 x 5M = 25M

1. Prove that a ring R has no zero divisors if and only if the cancellation laws hold in R.
2. Prove that the characteristic of an integral domain is either a prime or zero.
3. If f is a homomorphism of a ring R into a ring R' then prove that f is an into isomorphism if and only if $Ker\, f = \{0\}$.
4. Prove that an ideal $U \neq R$ of a commutative ring R with unity is a prime ideal if and only if R/U is an integral domain.
5. If $\bar{a} = i \sin t + j \cos t + t\, k, \bar{b} = i \cos t - j \sin t - 3\, k$ and $\bar{c} = 2i + 3j - k$ then find $[\bar{a}\, X\, (\bar{b}\, X\, \bar{c})]'$ at $t = 0$.
6. Find the directional derivative of $\varphi = xy + yz + zx$ at A in the direction of $\overrightarrow{AB}$, where $A = (1,2,-1)$, $B = (-1,2,3)$.
7. Evaluate $\int_C F.dr$ where $F = x^2y^2i + yj$ and the curve C is $y^2 = 4x$ in the xy plane from $(0,0)$ to $(4,4)$.
8. If $F = yi + (x - 2xz)j - xyk$ then evaluate $\int_S (\nabla X\, F).N\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the $xy -$ plane by using Stoke's theorem.

PART – II

Answer any FIVE questions by choosing at least TWO questions from each section. Each question carries 10 marks. **5 x 10M = 50M**

SECTION - A

9. Prove that $Q[\sqrt{2}] = \{a + b\sqrt{2}/a, b \in Q\}$ is a field with respect to ordinary addition and multiplication of numbers.

10. Prove that a commutative ring R with unity element is a field if and only if R have no proper ideals.

11. State and Prove Fundamental Theorem of Homomorphism in rings.

12. Prove that an ideal U of a commutative ring R with unity is a maximal ideal if and only if the quotient ring R/U is a field.

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at (2,-1,2).

SECTION – B

14. If A and B are two differentiable vector functions then prove that

$$curl\ (AXB) = A\ div\ B - B\ div\ A + (B.\nabla)A - (A.\nabla)B$$

15. Find $\int_S F.N\ dS$ over the entire surface of the region bounded by

$$x^2 + z^2 = 9, x = 0, y = 0, z = 0\ and\ y = 8\ if\ F = 6zi + (2x + y)j - xk.$$

16. If $\varphi = 45x^2y$ then evaluate $\iiint_V \varphi\ dV$ where V is the closed region bounded by the planes $4x + 2y + z = 8, x = 0, y = 0, z = 0$.

17. State and Prove Gauss Divergence Theorem.

18. Verify Greens theorem in the plane for $\oint_C (xy + y^2)dx + x^2\ dy$, where C is the closed curve of the region bounded by $y = x\ and\ y = x^2$.