Paper Code: 6125 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc.,Degree Examinations, Mar/Apr 2018

(At the end of 6th Semester)

Regular (2015-18 batch)

MATHEMATICS Paper – VIII (CE-2)

(Special Functions)

Date: 02.04.2018 FN Max Marks:75

Duration: 3 hrs

PART-I

Answer any FIVE questions , each question carries FIVE marks. **5 x 5 =25M**

1. Prove that $H_n'(x) = 2n\, H_{n-1}(x)$ for n >1.
2. Prove that if m<n $\frac{d^m}{dx^m}(H_n(x) = \frac{2^m n!}{(n-m)!} H_{n-m}(x)$..
3. Prove that $l_n(0) = 1$.
4. Prove that $x l_n''(x) + (1-x)\, l_n'(x) + n\, l_n(x) = 0$ and hence deduce that $l_n''(0) = -n$.
5. Prove that $(2n+1)p_n = p'_{n+1} - p'_{n-1}$.
6. Prove that $\frac{d}{dx}[x^{-n} J_n(x)]. = x^{-n} J_{n+1}(x)$.
7. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
8. Evaluate $\int_0^\infty x^2 . e^{-x^2} dx$.

PART-II

Answer any FIVE questions. Choosing at least TWO question from each section . Each question carries TEN marks. **5 x 10=50M**

SECTION-A

9. Find the values of $H_0(x), H_1(x), H_2(x), H_3(x), H_4(x), H_5(x)$.
10. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$.
11. Prove that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n . e^{-x})$.
12. Prove that $l_n'(x) = -\sum_{r=0}^{n-1} l_r(x)$.
13. Prove that $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2-1)^n$

SECTION-B

14. Prove that $\int_{-1}^{+1}(1-x^2)(p_n')^2\,dx\ \frac{2n(n+1)}{2n+1}$.

15. Prove that $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$.

16. Prove that $\sqrt{\left(\frac{\pi x}{2}\right)\, J_{\frac{3}{2}}(x)} = \frac{1}{x}\sin x - cosx.$

17. Prove that $B(l,m) = \int_0^\infty \frac{y^{l-1}}{(1+y)^{l+m}}\,dy.$

18. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}}\,dx.$

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SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc.,Degree Examinations, Mar/Apr 2019

(At the end of 6th Semester)

Regular (2016 batch)

MATHEMATICS Paper – VIII (CE-II)

(Special Functions)

Date: 16.04.2019 FN Max Marks:75

Duration:3hrs

PART-I

Answer any FIVE questions. Each question carries FIVE marks. 5 x 5 = 25

1. Find the Hermite polynomials $H_0(x), H_1(x), H_2(x)$.

2. Prove that $Hn^1(x) = 2n\,H_{n-1}(x)$.

3. Prove that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$.

4. Prove that $\frac{1}{1-t}e^{-xt/(1-t)} = \sum_{n=0}^{\infty} t^n L_n(x)$.

5. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$.

6. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\,Sinx$.

7. Evaluate $\int_0^2 x(8-x^3)^{1/3}dx$.

8. Evaluate $\pi\left(\frac{-5}{2}\right)$. $\Gamma\left(\frac{-5}{2}\right)$

PART - II

Answer any FIVE questions. Choosing atleast TWO questions from each section carries 10 marks. 5 x 10 = 50

SECTION - A

9. Prove that $e^{2tx-t^2} = \sum_{n=0}^{\alpha} \frac{t^n}{n!} H_n(x)$.

10. Prove that $H_n(x) = 2^n\left\{\exp\left(-\frac{1}{4}\frac{d^2}{dx^2}\right)x^n\right\}$.

11. Prove that
$$\int_0^{\alpha} e^{-x} L_n(x).L_m(x)dx = 0 \quad if \quad m \neq n$$
$$= 1 \quad if \quad m \neq n.$$

Contd...

12. Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(x^n e^{-x}\right)$.

13. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \text{ if } m \neq n$.

SECTION - B

Answer any FIVE Questions.

14. Show that $P_n(x)$ is the coefficient of h^n in the expansion of the ascending powers of $\left(1-2xh+h^2\right)^{-1/2}$.

15. Prove that $J_n(x)$ is the coefficient of z^n in the expansion of $e^{x\left(z-\frac{1}{z}\right)/2}$.

16. Prove that $xJ_n^1(x) = nJ_n(x) - xJ_{n+1}(x)$.

17. Prove that $\beta(\ell,m) = \frac{P(\ell).P(m)}{P(\ell+m)}$. $\beta(l,m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$

18. Prove that $T(m)T\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}T(2m)$. $\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

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Paper Code: 6125 Regd. No

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc., Degree Examinations, September 2020

(At the end of 6th Semester)

Regular (2017 batch), Supplementary (2016 batch)

MATHEMATICS Paper – VIII (CE-2)

(Special Functions)

Date: 19.09.2020 AN Max Marks:75

Duration:3hrs

PART – I

Answer any **FIVE** of the following questions. Each question carries 5 marks. **5x5 = 25 M**

1. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.
2. Prove that $2x\, H_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
3. Prove that $xL_n^{11}(x) + (1-x)L_n^1(x) + nL_n(x) = 0$.
4. Find the Laguerre polynomials $L_0(x), L_1(x), L_2(x), L_3(x)$.
5. Prove that $nP_n(x) = xP_n^1(x) - P_{n-1}^1(x)$.
6. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\, sinx$.
7. Compute a) $\Gamma\left(\frac{-1}{2}\right)$ b) $\Gamma\left(\frac{-3}{2}\right)$.
8. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}$ dx

PART – II

Answer any **FIVE** questions. Choosing atleast **TWO** questions from each section.

Each question carries **10** marks. **5 x 10 = 50 M**

SECTION – A

9. Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$.
10. Prove that $\int_{-\infty}^{\infty} e^{-x^2} H_n(x)H_m(x)\, dx = \begin{cases} 0 & if\ m \neq n \\ \sqrt{\pi}\, 2^n\, n! & if\ m = n \end{cases}$
11. Prove that $\int_0^{\infty} e^{-x} L_n(x)L_m(x) dx = \delta_{mn} = \begin{cases} 0\ if\ m \neq n \\ 1\ if\ m = n \end{cases}$
12. Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})$.
13. Prove that $P_n(x) = \frac{1}{n! 2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$.

SECTION – B

14. Prove that) $P_n(1) = 1$ ii) $P_n(-x) = (-1)^n P_n(x)$.

15. Prove that $x\, J_n^1(x) = n\, J_n(x) - x\, J_{n+1}(x)$.

16. Prove that i) $J_{-n}(x) = (-1)^n J_n(x)$ when 'n' is a positive integer and

ii) $J_n(-x) = (-1)^n J_n(x)$ when 'n' is a positive or negative integer.

17. Prove that $\beta(l, m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$.

18. Prove that $\Gamma(m)\,.\,\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$., where 'm' is an integer.

Paper Code: 6125 Regd. No. | | | | | | | |

SRI Y.N.COLLEGE (AUTONOMOUS)–NARSAPUR, W.G.Dt.

(Affiliated to Adikavi Nannaya University)

III B.Sc/B.Com/B.B.A/B.A., DegreenExaminations, June 2022

(At the end of 6th Semester)

Regular (2019-22 batch), Supplementary (2018,2016 batch)

MATHEMATICS Paper – VIII (CE-2)

(Special Functions)

Date: 21.06.2022 FN

Duration:3hrs

Max Marks:75

PART - I

Answer any Five of the following Questions 5×5=25M

1. Prove that i) $H_{2n}(0) = (-1)^n \cdot \frac{(2n)!}{n!}$ ii) $H_{2n+1}(0) = 0$
2. Prove that $H_n^{1}(x) = 2n\, H_{n-1}(x)$, where n≥1
3. Find $L_1(x)$ and $L_2(x)$
4. Prove that $L_n^{\alpha}(x) = \frac{e^x x^{-\alpha}}{n!} \cdot \frac{d^n}{dx^n}(e^{-x}.x^{n+\alpha})$
5. Prove that $np_n = xP_n^1 - P_{n-1}^1$
6. Prove that $J_2 - J_0 = 2J_0^{II}$
7. Show that $\Gamma(n) = \int_0^1 (\log\frac{1}{y})^{n-1} dy$
8. Show that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta.\cos^{2n-1}\theta d\theta$

PART – II

Answer any Five questions. Choosing at least Two questions from each Section 5×10=50M

SECTION - A

9. Prove that $\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\, dx = \begin{cases} 0, & if\ m \neq n \\ 2^n.\sqrt{\pi}n!, & if\ m = n \end{cases}$
10. Prove that $H_n(x) = 2^n \{\exp(-\frac{1}{4}\frac{d^2}{dx^2})\, x^n\}$
11. Prove that $\frac{1}{1-t}.e^{\frac{-tx}{1-t}} = \sum_{n=0}^{\infty} t^n L_n(x)$
12. Prove that $L_{n-1}^{\alpha}(x) + L_n^{\alpha-1}(x) = L_n^{\alpha}(x)$
13. Prove that i) $\int_{-1}^{1} P_m(x)P_n(x)dx = 0\ if\ m \neq n$

    ii) $\int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}, if\ m = n$

SECTION - B

14. Prove that $\int_{-1}^{1} x^2 P_{n+1}P_{n-1}dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.
15. Prove that $J_{-1/2}^2 + J_{1/2}^2 = \frac{2}{\pi x}$
16. Prove that $\frac{d}{dx}(xJ_nJ_{n+1}) = x(J_n^2 - J_{n+1}^2)$
17. Show that $2^n\Gamma\left(n+\frac{1}{2}\right) = 1.3.5\ldots.(2n-1)\sqrt{\pi}$
18. Show that $B(n,n+1) = \frac{1}{2}.\frac{[\Gamma(n)]^2}{\Gamma(2n)}$ and hence deduce that

    $\int_0^{\frac{\pi}{2}}(\frac{1}{\sin^3\theta} - \frac{1}{\sin^2\theta})^{1/4}\cos\theta d\theta = \frac{[\Gamma(\frac{1}{4})]^2}{2\sqrt{\pi}}$.