ADIKAVI NANNAYA UNIVERSITY

Department of Mathematics

Rajamahendravaram – 533 296

M.Sc. Mathematics

Syllabus { w.e.f. 2019 Admitted Batch }

Course Structure

Code	Subjects	No.of Periods/ Week	Credits
I SEMESTER			
M101	ALGEBRA - 1	6	5
M102	REAL ANALYSIS – 1	6	5
M103	DIFFERENTIAL EQUATIONS	6	5
M104	TOPOLOGY	6	5
M105	DISCRETE MATHEMATICS	6	5
		30	25
II SEMESTER			
M201	ALGEBRA – II	6	5
M202	REAL ANALYSIS-II	6	5
M203	COMPLEX ANALYSIS –I	6	5
M204	LINEAR ALGEBRA	6	5
M205	PROBABILITY THEORY & STATISTICS	6	5
		30	25
III SEMESTER			
M301	FUNCTIONAL ANALYSIS	6	5
M302	LEBESGUE THEORY	6	5
M303	ANALYTICAL NUMBER THRORY	6	5
M304	PARTIAL DIFFERENTIAL EQUATIONS	6	5
M305	ELECTIVE – 1	6	5
		30	25
IV SEMESTER			
M401	MEASURE THEORY	6	5
M402	NUMERICAL ANALYSIS	6	5
M403	GRAPH THEORY	6	5
M404	LINEAR PROGRAMMING	6	5
M405	ELECTIVE – 1I	6	5
		30	25

Note: For each subject, One Additional Hour per week shall be allotted for Seminar and Tutorial

ELECTIVE – 1	ELECTIVE – 1I
LATTICE THEORY	DISCRETE DYNAMICAL SYSTEMS
COMMUTATIVE ALGEBRA	OPERATOR THEORY
COMPLEX ANALYSIS -II	ADVANCED DIFFERENTIAL
SEMI GROUPS-I	NONLINEAR FUNCTIONAL ANALYSIS
Any other Subject with the approval of BoS	Any other Subject with the approval of BoS

Instructions for evaluation

1. Each theory subject is evaluated for 100 Marks out of which 75 Marks through end examination and internal assessment would be for 25 Marks. The minimum marks for qualifying in theory subject shall be 40% subject to securing minimum of 40% in the end examination.
2. End Examination Question Paper Pattern is as follows:

Sl. No.	Questions	Units of the Syllabus	Marks
1	Question1 and Question2	Form UNIT-I	15
2	Question3 and Question4	Form UNIT-II	15
3	Question5 and Question6	From UNIT-III	15
4	Question7 and Question8	From UNIT-IV	15
5	Question 9 Short answers from (a) to (e) (Three out of Five should be answered, each question is of 5 Marks)	Covers All Four Units of the Syllabus	3X5=15
Total:			75

3. Internal assessment for 25 Marks is as follows:
   - i) Mid Examinations : 10 Marks
     (Two mid examinations shall be conducted and average of two should be considered as mid examinations marks).
   - ii) Assignments / Seminar : 5 Marks
   - iii) Swachhata activity : 5 Marks
   - iv) Attendance : 5 Marks
     (% of attendance < 80 : 4 Marks
     % of attendance ≥ 80 : 5 Marks)

M101 ALGEBRA – I

Unit I

Automorphisms - Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. (Section 3 and 4 of Chapter 5, Sections 1,2,3 of Chapter 6)

Unit II

Structure theorems of groups: Direct product- Finitely generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p2 ,pq .
(Sections 1 to 5 of Chapter 8)

Unit III

Ideals and homomorphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma (Sections 1 to 6 of Chapter 10)

Unit-IV

Unique factorization domains - Principal ideal domains- Euclidean domains-

Polynomial rings over UFD. (Sections 1 to 4 of Chapter 11)

TEXT BOOK: Basic Abstract Algebra, Second Edition by P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul.

Reference: [1] Topics in Algebra by I.N. Herstein.

M102 REAL ANALYSIS-I

UNIT–I

Basic Topology: Finite-Countable- and Uncountable Sets- Metric spaces- Compact sets-Perfect Sets-Connected sets.

(Chapter 2 of the text book)

UNIT–II

Numerical Sequences and Series: Convergent Sequences- Subsequences -Cauchy Sequences-Upper and Lower limits- Some Special Sequences- Series- Series of Non-negative Terms-The number e -The Root and Ratio tests- Power series -Summation by parts - Absolute Convergence-Addition and Multiplication of series- Rearrangements.

(Chapter 3 of the text book)

UNIT–III

Continuity: Limits of Functions- Continuous Functions- Continuity and Compactness-Continuity and Connectedness- Discontinuities- Monotonic Functions- Infinite Limits and Limits at Infinity.

(Chapter 4 of the text book)

UNIT–IV

Differentiation: The Derivative of a Real Function -Mean Value Theorems - The Continuity of Derivatives- L' Hospital's Rule- Derivatives of Higher order- Taylor's theorem-Differentiation of Vector- valued Functions.

(Chapter 5 of the text book)

TEXT BOOK: Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3rd Edition, 1985.

REFERENCE: Mathematical Analysis by Tom M. Apostal, Narosa Publishing House, 2nd Edition,1985.

M103 - DEFFERENTIAL EQUATIONS

UNIT-I

Second order linear differential equations: Introduction-general solution of the homogeneous equation - Use of a known solution to find another - Homogeneous equation with constant coefficients - method of undetermined coefficients - method of variation of parameters.

Chapter 3 (Sec 14-19)

UNIT-II

Oscillation theory and boundary value problems: Qualitative properties of solutions - The Sturm comparison theorem - Eigen values, Eigen functions and the vibrating string.

Chapter 4 (Sec 22-24, Appendix A)

UNIT-III

Power series solutions: A review of power series-series solutions of first order equations-second order linear equations - ordinary points-regular singular points.

Chapter 5 (Sec 25-29)

UNIT-IV

Systems of first order equations: Linear systems - Homogeneous linear systems with constant coefficients - Existence and Uniqueness of solutions - successive approximations - Picard's theorem - Some examples.

Chapter 7 (Sec 36-38) and Chapter 11(Sec 55-56)

TEXT BOOK: George F. Simmons, Differential Equations, Tata McGraw-Hill Publishing Company Limited, New Delhi

M 104 - TOPOLOGY

UNIT-I

Sets and Functions: Sets and Set inclusion – The algebra of sets – Functions – Products of sets – Partitions and equivalence relations – Countable sets – Uncountable sets – Partially ordered sets and lattices. (Chapter I: Sections 1 to 8.)

UNIT-II

Metric spaces: The definition and some examples – Open sets – Closed sets – Convergence,

Completeness and Baire's theorem – Continuous mappings. (Chapter 2: Sections 9 to 13.)

UNIT-III

Metric spaces (Continued): Spaces of continuous functions – Euclidean and unitary spaces.

Topological spaces: The definition and some examples – Elementary concepts – Open bases and open sub bases – Weak topologies – The function algebras C(X, R) and C(X, C).

(Chapter 2: Sections 14,15 and Chapter 3: 16 to 20.)

UNIT-IV

Compactness: Compact spaces – Product of Spaces – Tychonoff's theorem and locally Compact spaces – Compactness for metric spaces – Ascoli theorem.

(Chapter 4: Sections 21 to 25.)

TEXT BOOK: Introduction to Topology by G.F.Simmons, Mc.Graw-Hill book company.

M 105 - DISCRETE MATHEMATICS

UNIT –I

Relations and ordering: Relations- properties of binary relations in a set-Relation matrix and the graph of a relation, partition and covering of a set, equivalence relations, compatability relation, composition of binary relations- partially ordereing- Partially ordered sets - representation and associated terminology. [2-3.1 to 2-3.9 of Chapter 2 of the Text Book]

UNIT- II

Lattices: Lattices as partially ordered sets - some properties of Lattices - Lattices as algebraic systems - sub-Lattices - direct product and homomorphism some special Lattices. [4-1.1 to 4-1.5 of Chapter 4 of the Text Book]

UNIT- III

Boolean Algebra: Sub algebra - direct product and Homorphism - Boolean forms and free Boolean Algebras - values of Boolean expressions and Boolean function. [4-2.1,4-2.2,4-3.1, 4-3.2 of Chapter 4 of the Text Book]

UNIT- IV

Representations and minimization of Boolean Function: Representation of Boolean functions – minimization of Boolean functions- Finite State Machines - Introductory Sequential Circuits - Equivalence of Finite-State Machines. [4-4.1,4-4.2,4-6.1, 4-6.2 of Chapter 4 of the Text Book]

Text Book:

Discrete Mathematical structures with applications to Computer Science by J.P.Trembly and R. Manohar, Tata McGraw-Hill Edition.

Reference Book:

Discrete Mathematics for Computer Scientists and Mathematicians by J.L.Mott, A.Kandel and T.P. Baker, Prentice-Hall India.

M201 ALGEBRA – II

UNIT - I

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion- Adjunction of roots- Algebraic extensions-Algebraically closed fields. (Sections 1 to 4 of Chapter 15)

UNIT - II

Normal and separable extensions: Splitting fields- Normal extensions- Multiple roots- Finite fields- Separable extensions (Sections 1 to 5 of Chapter 16)

UNIT - III

Galois theory: Automorphism groups and fixed fields- Fundamental theorem of Galois theory- Fundamental theorem of Algebra (Sections 1 to 3 of Chapter 17)

UNIT - IV

Applications of Galoes theory to classical problems: Roots of unity and cyclotomic polynomials- Cyclic extensions- Polynomials solvable by radicals - Ruler and Compass constructions. (Sections 1 to 3 and 5 of Chapter 18)

TEXT BOOK: Basic Abstract Algebra , Secound Editionby P.B. Bhattacharya, S.K. Jain and S.R. Nagpanl

REFERENCE: Topics in Algrbra By I. N. Herstein.

M202 REAL ANALYSIS-II

UNIT-I

Riemann-Stieltjes Integral: Definition and existence of the Riemann Stieltjes Integral, Properties of the Integral, Integration and Differentiation, the fundamental theorem of calculus – Integral of Vector- valued Functions, Rectifiable curves.

(Chapter 6)

UNIT-II

Sequences and Series of the Functions: Discussion on the Main Problem, Uniform Convergence, Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinuous families of Functions, the Stone-Weierstrass Theorem.

(Chapter 7)

UNIT-III

Power Series: (A section in Chapter 8 of the text book)

Functions of Several Variables: Linear Transformations, Differentiation, The Contraction Principle, The Inverse Function theorem.

(First Four sections of chapter 9 of the text book)

UNIT-IV

Functions of several variables Continued: The Implicit Function theorem, The Rank theorem, Determinates, Derivatives of Higher Order, Differentiation of Integrals.

(5 th to 9 th sections of Chapter 9 of the text book)

TEXT BOOK: Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3 rd Edition, 1985.

REFERENCE: Mathematical Analysis by Tom M. Apostal, Narosa Publishing House, 2 nd Edition, 1985.

M203 COMPLEX ANALYSIS -I

UNIT-I

Elementary properties and examples of analytic functions: Power series- Analytic functions- Analytic functions as mappings, Mobius transformations.

(1,2,3 of chapter-III)

UNIT-II

Complex Integration: Riemann- Stieltjes integrals- Power series representation of analytic functions- zeros of an analytic functions- The index of a closed curve.

(1,2,3,4 of chapter-IV)

UNIT-III

Cauchy's theorem and integral formula- the homotopic version of Cauchy's theorem and simple connectivity- Counting zeros; the open mapping theorem.

(5,6,7of chapter-IV)

UNIT-IV

Singularities: Classifications of singularities- Residues- The argument principle.

(1,2,3 of chapter-V)

TEXT BOOK: Functions of one complex variables by J.B.Conway : Second edition,

Springer International student Edition, Narosa Publishing House, New Delhi.

M204 LINEAR ALGEBRA

UNIT-I

Elementary Canonical Forms : Introduction – Characteristic Values – Annihilating Polynomials –invariant subspaces – Simultaneous Triangulation – Simultaneous Diagonalization.

(Sections 6.1,6.2,6.3,6.4,6.5 of chapter-6)

UNIT-II

Direct – sum Decompositions – invariant direct sums – the primary decomposition theorem – cyclic subspaces and Annihilators – cyclic decompositions and the rational form.

(Sections 6.6,6.7,6.8 of chapter-6 and Sections 7.1,7.2 of chapter - 7)

UNIT-III

The Jordan Form – Computation of Invariant Factors – Semi Simple Operators.

(Sections 7.3,7.4,7.5 of chapter - 7)

UNIT-IV

Biliear Forms : Bilinear Forms – Symmetric Bilinear Forms – Skew Symmetric Bilinear Forms – Group Preserving Biliear Forms.

(Sections 10.1,10.2,10.3,10.4 of chapter - 10)

TEXT BOOK: Linear Algebra second edition By Kenneth Hoffman and Ray Kunze, Prentice Hall of india Private Limited, New Delhi.

M205 - PROBABILITY THEORY & STATISTICS

UNIT-I

Sample Space & Events, Axioms of Probability, Some theorems on Probability, Boole's inequality, Conditional probability, Multiplication theorem on probability, Independent events, Multiplication theorem on probability for Independent Events, Extension of Multiplication theorem on probability to n events, Pair-wise Independent Events, Baye's theorem
[Section 3.8 to 3.15 , Page no: 3.2 to 3.98 & Section 4.2, Page no: 4.4 to 4.20]

UNIT-II

Distribution function, Discrete Random variables, Continuous random variables, Mathematical Expectation, Expected value of function of a random variable, Properties of Expectation, Properties of variance, Covariance, Moment Generating Function, Characteristic function, Binomial Distribution, Poisson Distribution, Normal Distribution, Uniform Distribution
[Section5.2 to 5.4 , Page no: 5.2 to 5.31, Section 6.2 to 6.6, Page no: 6.1 to 6.22, Section 7.1, 7.3(7.3.1&7.3.2 only), Page no: 7.2 to 7.6, Page no: 7.9 to 7.15, Section 8.4, 8.5, Page no: 8.4 to 8.47, Section 9.2.1 to 9.2.11 and 9.2.14, 9.3, Page no: 9.2 to 9.12, 9.14 to 9.28, and 9.3 to 9.37]

Unit III

Correlation: Introduction, meaning of correlation, scatter diagram, Karl Pearson's Coefficient of Correlation, Rank Correlation, Linear and Curvilinear Regression: Introduction, linear regression, curvilinear regression
[Section 10.1 to 10.4(10.4.1, 10.4.2 only & in 10.7 –10.7.1 only), Page no: 10.1 to 10.16 and 10.23 – 10.25, Section 11.1 to 11.3, Page no: 11.1 to 11.19]

Unit IV

Large Sampling theory: Introduction, types of sampling, parameters and statistic, tests of significance, procedure for testing of hypothesis, tests of significance for large samples
[Section 14.1 – 14.6, Page no: 14.1 to 14.22]

Text Book: Fundamentals of Mathematical Statistics, S.C.Gupta,V.K.Kapoor
Eleventh Thoroughly Revised Edition
Published by: Sultan Chand & Sons, NEW-DELHI

M301 – FUNCTIONAL ANALYSIS

UNIT-I

Banach spaces: the definition and some examples, continuous linear transformation, the Hahn-Banach theorem, the natural imbedding of N in N^{**}, The open mapping theorem.

(Sections 46 – 50 of chapter 9)

UNIT-II

The conjugate of an operator, **Hilbert spaces**: The definition and some simple properties, orthogonal complements, orthonormal sets.

(Sections 51 of chapter 9 and Sections 52- 54 of chapter 10)

UNIT-III

The Conjugate space H^*, the ad joint of an operator, Self- ad joint operators, Normal and Unitary operators, Projections.

(Sections 55 - 59 of chapter 10)

UNIT-IV

Finite- dimensional spectral theory: Matrices, determinants and the spectrum of an operator, the spectral theorem, A survey of the situation.

(Sections 60 - 63 of chapter - 11)

TEXT BOOK: Introduction to Topology and Modern Analysis by G.F.Simmons, McGraw Hill Book Company, Inc-International student ed.

M302 – LEBSESGUE THEORY

UNIT-I

Algebra of sets, Lebesgue measure, Outer measure, Measurable set and Lebesgue measure, a non-measurable set, measurable function, Little woods's Three principles.(Chapter 3)

UNIT-II

The Riemann integral, the Lebesgue integral of a bounded function over a set of finite measures, the integral of a non-negative function, the general Lebesgue integral convergence in measure. (Chapter 4)

UNIT-III

Differentiation of monotonic functions, functions of bounded variation, differentiation of an integral, absolute continuity. (Chapter 5)

UNIT-IV

Lp- Spaces the Holder's and Minkowski inequalities, convergence and completeness

(Chapter 6)

TEXT BOOK: H.L.Royden, Real Analysis, Macmillan Publishing Company, New York, Third Edition, 1988.

M303 – ANALYTICAL NUMBER THRORY

UNIT-I

ARITHMETICAL FUNCTIONS AND DIRICHLET MULTPLICATION :-Introduction – The Mobius function μ (n).-The Euler quotient function φ(n)-A relation connecting φ and μ- A product formula for φ(n)-The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The mangoldt function Λ(n)- multiplicative functions- multiplicative function and Dirichlet multiplication – The inverse of a completely multiplicative function- Liouville's function λ(n)- The divisor functions σ_α(n). Generalized convolutions.

(Sections 2.1 – 2.14 of chapter 2)

UNIT-II

AVERAGES OF ARITHMETICAL FUNCTIONS:- Introduction- The big oh notation. Asymptotic equality of functions – Euler's summation formula – Some elementary asymptotic formulas – The average order of d(n)-The average order of the divisor functions σ_α(n)- The average order of φ(n)-An application to the distribution of lattice points visible from the origin – the average order of μ(n) and Λ(n) – The partial sums of a Dirichlet product Applications to μ(n) and Λ(n) – Another identity for the partial sums of a Dirichlet product.

(Sections 3.1 – 3.12 of chapter 3)

UNIT-III

SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OFPRIME NUMEBRS:- Introduction – chebyshev's function $\psi(x)$ and $\vartheta(x)$- Relations connecting $\vartheta(x)$ and $\pi(x)$ – Some equivalent forms of the prime number theorem - Inequalities for $\pi(n)$ and p_n – Shapiro's Tauberian theorem – Applications of Shapiro's theorem – An asymptotic formula for the partial sums $\sum_{p\le x}(1/p)$ - The partial sums of the Mobius function.

(Sections 4.1 to 4.9 of Chapter 4)

UNIT-IV

CONGRUENCES :- Definition and basic properties of congruences – Resudue classes and complete residue systems – linear congruences – Reduced residue systems and theEuler-Fermat theorem – Polynomial congruences modulo p. Lagrange's theorem –Applications of Lagrage's theorem – Siultaneous linear congruences. The Chinese remainder Theorem- Applications of the Chinese remainder Theorem – Plnomial congruences with prime powermoduli.

(Sections 5.1 – 5.9 of chapter 5)

TEXT BOOK : Introduction to Analytic Number Theory – By T.M.APOSTOL – Springer

Verlag New York, Heidlberg – Berlin – 1976.

M304 - PARTIAL DIFFERENTIAL EQUATIONS

UNIT I

Introduction, Methods of Solution of dx/P = dy/Q = dz/R, Orthogonal trajectories of a system of curves on a surface, Pfaffian Differential forms and equations, Solutions of Pfaffian differential equations in three variables, Cauchy's problem for first order partial differential equations. (Sections 3 to 6 of Chapter 1, Sections 1 to 3 of Chapter 2)

UNIT II

Linear Equations of the first order, Integral surfaces, orgonal surfaces, non linear partial differential equations of the first order, Cauchy's method of characteristics, Compatible systems of first order equations, Charpit's Method, Special types of first order equations, Jacobi's method.(Sections 4 to 13 of Chapter 2)

UNIT III

Partial Differential Equations of the second order, Their origin, Linear partial Differential equations with constant and variable coefficients, Solutions of linear hyperbolic equations, Method of separation of variables, Monger's method.

(Sections 1 to 5 and Sections 8,9,11 of Chapter 3)

UNIT IV

Laplace Equation, elementary solutions, families of equipotential surfaces, Boundary value problems, Method of separation of variables of solving Laplace equation, problems with axial symmetry, Kelvin's inversion theorem, The wave equat, Elementary solution in one dimensional form, Riemann-Volterra solution of one dimensional wave equation.

(Sections 1 to 7 pf Chapter 4 and Sections 1 to 3 of Chapter 5)

TEXT BOOK:

[1] Elements of Partial Differential Equations by I.N.Sneddon, Mc Graw Hill, International Edition, Mathematics series.

REFERENCE BOOK:

1 Fritz John, Partial Differential Equations, Narosa Publishing House, New Delhi, 1979

M305.1 – LATTICE THEORY

UNIT-I:

Partially Ordered sets – Diagrams – Special subsets of a poset – length – lower and upper bounds – the minimum and maximum condition – the Jordan Dedekind chain conditions – dimension functions.

(Sections 1 – 9 of Chapter 1)

UNIT-II:

Algebras – lattices – the lattice theoretic duality principle – semi lattices – lattices as posets – diagrams of lattices – semi lattices, ideals – bound elements of Lattices – atoms and dual atoms – complements, relative complements, semi complements – irreducible and prime elements of a lattice – the homomorphism of a lattice – axioms systems of lattices.

(Sections 10 - 21 of Chapter 2).

UNIT-III:

Completer lattices – complete sub lattices of a completer lattice – conditionally complete lattices – lattices – compact elements, compactly generated lattices – sub algebra lattice of an algebra – closure opoerations – Galois connecitons, Dedekind cuts – partilally ordered sets as topological spaces..

(Sections 22 - 29 of Chapter 3)

UNIT-IV

Distributive lattices – infinitely distributive and completely distributive lattices – modular lattices – characterization of modular and distributive lattices by their sub lattices – distributive sublattices of modular lattices – the isomorphism theorem of modular lattices, coveing conditions- meet representations in modular and distributive lattices – some special subclasses of the class of modular lattices – preliminary theorems – modular lattices of locally finite length – the valuation of a lattice, metric and quasi metric lattices – complemented modular lattices.

(Sections 30 – 40 of Chapter 4)

TEXT BOOK: Introduction to Lattice Theory by Gabor Szasz, Academic Press, New York

REFERENCE :

General Lattice theory by G.Gratzer, Academic Press, New York.

M305.2 – COMMUTATIVE ALGEBRA

UNIT-I

Rings and ring homomorphism, ideals, quotient rings, zero divisors, Nilpoent elements, units, prime ideals and Maximal ideals, nil radical and Jacobson radical, operations on ideals, Extensions and contractions.

UNIT-II

Modules and module homomorphisms, Sub modules and quotient modules, operations on submodules, direct sum and product, finitely generated modules, exact sequences, Tensor product of modules, Restriction and extension of scalars, Exactness properties of the tensor product, algebras, tensor product of algebras.

UNIT-III

Local Properties, Extended and Contracted ideals in rings of fractions..

UNIT-IV

Primary decompostitions.

(Content and extent of chapters 1 to 4 of the prescribed text book)

TEXT BOOK: Introduction to commutative algebra, M.F.ATIYAH and I.G.MACDONALD,

Addision – Wesley publishing Company, London.

M305.3 - COMPLEX ANALYSIS -II

UNIT-I

The maximum modulus theorem: The maximum principle – Schwarz's lemma – convex function's and hadamard's three circles theorem – Phargmem – Lindelof theorem.

(1,2,3,4 of chapter-VI)

UNIT-II

Compactness and Convergence in the Space of Analytic Functions: The space of continuous function C (G, Ω) – Spaces of Analytic functions – spaces of meromorhic functions – The Riemann Mapping Theorem – Weierstrass factorization theorem – Factorization of sine functions. .

(1,2,3,4,5,6 of chapter-VII)

UNIT-III

Runge's Theorem : Runge's Theorem – Simple connectedness – Mittang – Leffler's Theorem, Analytic Continuatin and Riemann Surfaces, Schwarz Reflection Principle – Analytic Continuatin Along A Path – Mondraomy Theorem..

(1,2,3 of chapter-VIII and 1,2,3 of chapter IX)

UNIT-IV

Harmonic Functions : Basic properties of Harmonic functions – Harmnic functions on a disk. Jenson's formula, the genus and the order of an entire function Hadamard's factorization theorem. .

(1,2, of chapter-X and 1,2,3 of chapter XI)

TEXT BOOK: Functions of one complex variables by J.B.Conway : Second edition,

Springer International student Edition, Narosa Publishing House, New Delhi.

M 305.4- SEMI GROUPS- I

UNIT-I

Basic definition, monogenic semigroups, ordered sets, semilatttices and lattices, binary relations,equivalences and congruences.

UNIT-II

Free semigroups, Ideals and Rees' congruences, Lattices of equivalences and congruences. Green's equivalences, the structure of D-classes, regular semigroups.

UNIT-III

Simple and 0-simple semigroups, Principal factors, Rees' theorem, Primitive idempotents.

UNIT-IV

Congruences on completely 0-simple semi groups, The lattice of congruences on a completely 0-simple semigroup, Finite congruence free semigroups.

Contents of the syllabus-Chapters 1,2 and 3 of the text book.

TEXT BOOK: An introduction to semi group theory by J.M. Howie, 1976, Academic press,

New York.

M401 - MEASURE THEORY

UNIT- I

Convergence and Completeness, Measure spaces, Measurable functions, Integration, General convergence Theorems.

[Section 3 of Chapter 6, Section 1 to 4 of Chapter 11 of the text book]

UNIT- II

Signed Measures, The Raydon-Nikodym Theorem, the Lp spaces.

[Sections 5 to 7 of Chapter 11 of the text book]

UNIT- III

Outer Measure and Measurability, The Extension theorem, The Lebesgue - Stieltjes Integral, Product measures.

[Sections 1 to 4 of Chapter 12 of the text book]

UNIT- IV

Integral Operators, Inner Measure, Extension by sets of measure zero, caratheodory outer measure, Hausdroff Measure.

[Sections 5 to 9 of Chapter 12 of the text book]

TEXT BOOK:

Real Analysis by H. L. Royden, Macmillan Publishing Co. Inc. 3 rd Edition, New York, 1988.

M 402 – NUMERICAL ANALYSIS

UNIT I

Transcendental and polynomial equations: Introduction, Bisection method, Iteration methods based on first degree equation; Secant method, Regulafalsi method, Newton- Raphson method, Iteration method based on second degree equation; Mullers method, Chebyshev method, Multipoint iterative method, Rate of convergence of secant method, Newton Raphson method,
(Section 1 of the Text Book pages 1 to 52 above specified methods only)

Unit II

System of linear algebraic equation: Direct methods, Guass elimination method, Triangularization method, Cholesky method, Partition method, Iteration method: Gauss seidel Iterative method, OR method.
(Section 2 of the Text Book pages 53 to 169 above specified methods only)

UNIT III

Interpolation and Approximation: Introduction, Lagrange and Newton's divided difference interpolation, Finite difference operators, sterling and Bessel interpolation, Hermite interpolation, piecewise and Spline Interpolation, least square approximation.
(Section 3 of the Text Book pages 210 to 300 above specified methods only)

UNIT IV

Numerical Differentiation: methods based on Interpolation, methods based on Finite difference operators Numerical Integration: methods based on Interpolation, Newton's cotes methods, methods based on Undetermined coefficients, Gauss Legendre Integration method, Numerical methods ODE: Single step methods: Euler's method, Taylor series method, Runge kutte second and forth order methods, Multistep methods: Adam Bash forth method, Adam Moulton methods, Milne-Simpson method.
(Section 4 of the Text Book pages 320 to 495 above specified methods only)

Text Book: [1] Numerical Methods for Scientific and Engineering computation by M.K. Jain, S.R.K. Iyengar, R.K. Jain, New Age Int. Ltd., New Delhi.

Reference: [1] Introduction to Numerical Analysis, by S.S. Sastry,Prentice Hall Flied.

M403 - GRAPH THEORY

UNIT-I

Basic concepts, Paths and Circuits, Trees and Fundamental Circuits.

[Chapters 1, 2, 3 of the text book]

UNIT-II

Cut Sets and Cut Vertices: Cut sets, Some properties of a cut set, All cut sets in a graph, Fundamental circuits and cut sets, Connectivity and Separability, Network Flows, 1-Isimorphsm, 2-Isomorphsm; Planar and Dual Graphs: Combinatorial Vs Geometric graphs, Planar graphs, Kuratowski's Two graphs, Different Representations of Planar Graphs, Detection of Planarity, Geometric Dual, Combinational Duals of a Graph,

[Chapter 4 and Sections 5.1 to 5.7 of Chapter 5 of the text book]

UNIT-III

Matrix Representation of graphs: Incident matrix of a Graph, Sub Matrices of A(G), Circuit Matrix, Fundamental Circuit Matrix and Rank of B, An Applications to a Switching Network, Cut set matrix, Relationship among A_f, B_f and $C_{f,}$, Path Matrix and Adjacency Matrix.

[Chapter 7 of the text book]

UNIT-IV

Coloring, Covering and Partitioning: Chromatic Number, Chromatic Partitioning, Chromatic Polynomial, Matching's, Coverings, The four color Problem; Graph Theory in Operation Research: Transport networks, Extensions of Max-flow Mi cut theorem, Minimal cost flows.

[Chapter 8 and Sections 14.1 to 14.3 of Chapter 14 of the text book]

TEXTBOOK: [1] Graph Theory with applications to Engineering and computer Science by Narsingh Deo; Prentice-Hall of India.

REFERENCES:

[1] Graph Theory with applications by Bond JA and Murthy USR, North Holland, NewYork.

[2] Introduction to Graph Theory by Donglas B. West. Prentice Hall of India.

M404 - LINEAR PROGRAMMING

UNIT I

Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method.

[Sections 3.1 to 3.7 of Chapter 3 and Section 5.4 of Chapter 5 of the text book]

UNIT II

Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions. Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

[Sections 5.5, 5.7, 5.8, 5.12, 5.13 of Chapter 5 and Sections 7.1, 7.7 of Chapter 7]

UNIT III

Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.

[Sections 12.1 to 12.4, 12.9 of Chapter 12 of the text book]

UNIT IV

Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

[Sections 11.1 to 11.5 and 11.8 to 11.12 of Chapter 11 of the text book]

TEXT BOOKS:

[1] S. D. Sharma, Operations Research.

REFERENCE BOOKS:

[1] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.

[2] H. A. Taha, Operations Research – An Introduction.

M405.1 - DISCRETE DYNAMICAL SYSTEMS

UNIT I

Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's theorem, Differentiability and its Implications [Hyperbolic, Attractive and Repelling Periodic Points]

[Chapters 1,4,5,6]

UNIT II

Parameterized Families of Functions and Bifurcations; The Logistic Function Part I [Cantor Sets], Symbolic Dynamics and Chaos.

[Chapters 7,8,9]

UNIT III

The Logistic Function Part II Topological Conjugacy, The Logistic Function Part III [Period Doubling Cascade], newton's Method

[Chapters 10,11,12]

UNIT IV

Numerical solutions of Differential Equations, The Dynamics of Complex functions [newton's Method in Complex Plane], the Quadratic Family and Mandelbrot Set

[Chapters 13, 15 and Sections 14.3, 14.5]

TEXT BOOK : Richard M. Holmgren, A First Course in Discrete Dynamical Systems,

Springer Verlag

M405.2 OPERATOR THEORY

UNIT I

Banach fixed point theorem- application of Banach's theorem to linear equations - application of Banach's theorem to differential equations-application of Banach's theorem to integral equations.

[Chapter 5 of the text book]

UNIT II

Approximation in normed spaces-Uniqueness, strict convexity-uniform approximation Chebyshev polynomials – Splines.

[Sections 6.1 to 6.4 and 6.6 of Chapter 6 of the text book]

UNIT III

Spectral theory in finite dimensional Normed spaces-basic concepts-spectral properties of bounded linear operators-further properties of Resovent and spectrum-use of complex analysis in spectral theory.

[Sections 7.1 to 7.5 of Chapter 7 of the text book]

UNIT IV

Compact linear operator of normed spaces-Further properties of compact linear operators Spectral properties of compact linear operators on normed spaces-further spectral properties of compact linear operators.

[Sections 8.1 to 8.4 of Chapter 8 of the text book.]

TEXT BOOK:

Introductory Functional Analysis and Applications by Kreyszig, John Wiley and Sons, Delhi, 2001.

M405.3 - ADVANCED DIFFERENTIAL EQUATIONS

UNIT I

Boundary value problems: Preliminaries – Sturm – Liouville Problem – Green's function – Application of Boundary Value Problem – Picard's theorem.

[Chapter 7 of prescribed text book.]

UNIT II

Oscillations of second order equations: Fundamental results – Sturm's Comparisons theorem – Elementary linear oscillations – Comparisons theorem of Hille – Wintner – oscillations of x" + a (t) x = 0..

[Chapter 8 of prescribed text book.]

UNIT III

Stability of linear and nonlinear systems: preliminaries – Elementary critical points – system of equations with constant coefficients – Linear equation with constant coefficients – Lyapunov stability – stability of quasi linear systems – second order linear differential equations.

[Chapter 9 of prescribed text book.]

UNIT IV

Equations with deviating arguments: Preliminaries – equations with constant delay – Equations with piecewise constant delay – a few other types of delay equations.

[Chapter 11 of prescribed text book.]

TEXT BOOK:

S.G. Deo, V. Lakshmikantham and V. Raghavendra: Text book of ordinary Differential equations, Second edition, Tata McGraw-Hill Publishing Company Limited, New Delhi, 1997.

M405.4 NONLINEAR FUNCTIONAL ANALYSIS

UNIT-I

Various forms of continuity- Geometry in normed spaces and duality mapping, Nemytskii, Hammerstein and Urysohn operators.

Chapter 1 of the textbook

UNIT-II

Gateaux and Frechet derivative, Properties of derivative, Taylor's theorem, Inverse function theorem and Implicit function theorem, Sub differential of convex functions.

Chapapter 2 of the text book

UNIT-III

Banach's contraction principle and its genrerliation, Nonexpansive mappings, Fixed point theorems of Brouwer and Schauder.

Secions 4.1 to 4.3 of Chapter 4 of the text book.

UNIT-IV

Fixed point theorems for multifunctions, common fixed point theorems, Sequences of contractions, generalized contractions and fixed points.

Sections 4.4 to 4.6 of Chapter 4 of the textbook.

TEXT BOOK:

Joshi, Mohan C., and Ramendra K. Bose. *Some topics in nonlinear functional analysis*. John Wiley & Sons, 1985.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M101: ALGEBRA-1

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. (a) If N be a normal subgroup of the group G, then prove that the mapping $\phi : G \to G/N$ defined by $x \mapsto xN$ is an epimorphism with $Ker\phi = N$

   (b) Show that every permutation can be expressed as the product of transpositions.

Or

2. (a) let H and K be normal subgroups of a group G and $K \subset H$. Prove that
   $$\frac{G/K}{H/K} \cong G/H$$

   (b) Define an alternating group A_n. Show that the alternating group A_n is generated by the set of all 3 – cycles in S_n

3. (a) Define a finite abelian group.
   Let A be a finite abelian group of order $p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$ (p_i's are distinct primes, $e_i > 0$). Show that $A = S(p_1) \oplus S(p_2) \oplus \ldots \oplus S(p_k)$ where $|S(p_i)| = p_i^{e_i}, 1 \le i \le k$

   (b) State and prove first Sylow theorem

Or

4. State and prove 2nd and 3rd Sylow theorems

5. (a) state and prove the fundamental theorem of homomorphism

   (b) if K is an ideal in a ring R, then show that each ideal in R/K is of the form A/K where A is an ideal in R containing K

Or

6. (a) In a non – zero commutative ring with unity, prove that an ideal M is maximal if and only if R/M is a field

   (b) If R is a commutative ring, then prove that an ideal P in R is prime if and only if $ab \in P$ implies either $a \in P$ or $b \in P$

7. (a) if R is a non – zero ring with unity and I is an ideal in R such that $I \neq R$, then prove that there exists a maximal ideal M of R such that $I \subseteq M$

(b) Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime.

Or

8. (a) show that every Euclidean ring is a principal ideal domain
   (b) State and prove Gauss lemma

9. Answer any THREE questions of the following $5 \times 3 = 15$

   (a) Define the following and give one example for each
      (i) Aut(G)
      (ii) Eisenstein Criteria of irreducibility
   (b) (i) Define invariants of a group
      (ii) Define p – group and give an example

   (c) (i) define an ideal and give two examples
      (ii) Define a principal ideal and given an example of a principal ideal ring

   (d) (i) define nilpotent ideal and give an example
      (iii) Show that every nilpotent ideal is nil. What about the converse? Justify?
   (e) (i) write Zorn's lemma and give an application
      (ii) Define a Euclidean domain and give an example

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ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M102: REAL ANALYSIS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. If X is a metric space, and E is a subset of X, then

(a) $\overline{E}$ is closed

(b) $E = \overline{E}$ if and only if E is closed

(c) $\overline{E}$ is a subset of F for every closed subset F of X such that E is a subset of F.

(or)

2. Prove that " if $K \subset Y \subset X$, then K is compact relative to X if and only if K is compact relative to Y".

3. Prove that (a) if $\{p_n\}$ is a sequence in a compact metric space X, then some subsequence of $\{p_n\}$ converges to a point of X

(b) Every bounded sequence in R^k contains a convergent subsequence

Or

4. Prove that (a) in any metric space X, every convergent sequence is a Cauchy sequence.
(b) If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then $\{p_n\}$ converges to some point of X.
(c) In R^k , every Cauchy sequence converges

5. Prove that "a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

Or

6. Prove that " if f is a continuous mapping of a compact metric space X_ into a metric space Y, then f is uniformly continuous on X"

7. Prove that "if f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a,b]$, g is defined on an interval I which contains the range of f, and g is differentiable at the point $f(x)$, $h(t) = g(f(t))$, $a \le t \le b$, then h is differentiable at x and

$h'(x) = g'(f(x))f'(x)$".

Or

8. Prove that "if f and g are continuous real functions on $[a, b]$ which are differentiable in (a, b), then there is a point $x \in (a,b)$ at which $\left[f(b)-f(a)\right]g'(x)=\left[g(b)-g(a)\right]f'(x)$

Answer any three questions of the following. Each question carries 5 marks

(i) State and prove Weierstrass theorem

(ii) Give an example of a Cantor set

(iii) If $\{p_n\}$, $\{q_n\}$ are Cauchy sequences in a metric space, then show that $\{d(p_n,q_n)\}$ converges

(iv) Prove that "if f is a continuous mapping of a compact metric space X into a metric space Y, then $f(X)$ is compact"

(v) Prove that "if $\mathbf{f}$ is a continuous mapping on $[a, b]$ into R^k and $\mathbf{f}$ is differentiable in (a, b), then there exists $x \in (a,b)$ such that $|\mathbf{f}(b)-\mathbf{f}(a)| \leq (b-a)|\mathbf{f}'(x)|$"

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M103: DIFFERENTIAL EQUATIONS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. if $y_1(x)$ and $y_2(x)$ are the linearly independent solutions of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ on the interval [*a, b*], then prove that $c_1y_1(x) + c_2y_2(x)$ is a solution of the differential equation. Also prove that the Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on [*a, b*].

(OR)

2. If one of the solutions $y_1(x)$ for $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ is given, then how would you find the 2^{nd} solution to it by explaining the method? Using this technique, if $y_1(x) = x$ is the solution for the differential equation $x^2y'' + xy' - y = 0$, then find the 2^{nd} solution and thus the general solution of the differential equation

3. State and prove Sturm comparison Theorem

(OR)

4. Find the Frobenius series solution and the general solution for the differential equation $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$

5. (i) Define the regular singular point
   (ii) Locate and classify the singular points on the x – axis for $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$
   (iii) Determine the nature of the point $x = 0$ for $x^2y'' + (\sin x)y = 0$

(OR)

6. Solve the Legendre differential equation

7. Derive the generating function of the Bessel polynomial

(OR)

8. Prove that if $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ are two solutions for the homogeneous system $\begin{cases} dx/dt = a_1(t)x + b_1(t)y \\ dy/dt = a_2(t)x + b_2(t)y \end{cases}$ on [*a, b*], then prove that $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is the general solution of this system.

9. Answer any **THREE** questions of the following

a) Explain the method of undetermined coefficients $y'' + p(x)y' + q(x)y = \sin bx$ and solve $y'' + 10y' + 25y = 14e^{-5x}$

b) Prove that 'if $q(x) < 0$, and $u(x)$ is a nontrivial solution of $u'' + q(x)u = 0$, then $u(x)$ has at most one zero'?

c) Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in *x*.

d) Write two independent Frobenius series solutions for $xy'' + 2y' + xy = 0$

e) Solve the linear system $\begin{cases} dx/dt = x + y - 5t + 2 \\ dy/dt = 4x - 2y - 8t - 8 \end{cases}$

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ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M104: TOPOLOGY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours | Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. a) If $\{A_i\}$ and $\{B_i\}$ are two classes of sets such that $\{A_i\} \subseteq \{B_i\}$. Show that $\bigcup A_i \subseteq \bigcup B_i$ and $\bigcap B_i \subseteq \bigcap A_i$.

b) Give an example of a relation which is

i) Reflexive but not symmetric or transitive.

ii) Symmetric but not reflexive or transitive.

iii) Transitive but not reflexive or symmetric.

iv) Reflexive and symmetric but not transitive.

(OR)

2. a) Define countable set. Prove that non – empty subset of countable set is countable.

b) Prove that countable union of countable sets is countable.

3. a) Define metric space. Prove that any metric space X each open sphere is an open set.

b) Let X be metric space and let A be a subset of X. If x is a limit point of A. Show that each open sphere centered on x contains an infinite number of distinct points of A.

(OR)

4. Define open set and closed set. Let (X, d) be a metric space. Let A is subset of X. Then the following hold.

a) $b(A) = \bar{A} \cap \bar{A}^{1}$

b) b(A) is a closed set.

c) A is closed $\Leftrightarrow$ b (A) $\subseteq A$.

5. a) State and prove Minkowski's inequality.

b) Define Topological space. Let X be a topological space and A is an arbitrary sub set of X.

Then $\bar{A}$ = {x ∈ X | each neighborhood of x intersects A}

$\bar{A} = \{x \in X / G \cap A \neq \phi$, for any neighborhood G of $x\}$.

(OR)

6. a) State and Prove Lindelof's theorem.

   b) Show that every separable metric space is second countable.

7. a) Prove that any closed subspace of a compact space is compact.

   b) Prove that a metric space is sequentially compact if it has the Bolzano Weierstrass property.

(OR)

8. State and prove Ascoli's theorem.

9. Answer any THREE questions from the following **(3x 5 = 15)**

   a) Prove that set of all integers is countable.

   b) Prove that a sub set F of metric space is closed iff its's complement F* is open.

   c) Prove that intersection of two topological spaces is a topology.

   d) Prove that any continuous image of compact space is compact.

   e) Prove that every compact metric space is separable.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M105: DISCRETE MATHEMATICAL STRUCTURES

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1) Let $X = \{1, 2, 3\}$ if $R = \{(x, y)/\ x \in X \wedge y \in X \wedge ((x - y)$ is an integral non-zero multiple of 2)$\}$, $S = \{(x, y)/\ x \in X \wedge y \in X \wedge ((x - y)$ is an integral non-zero multiple of 3)$\}$
(a) Find $R \cup S$ and $R \cap S$ (b) If $X = \{1, 2, 3, \ldots.\}$, What is $R \cap S$ for R and S as defined in (a)

(OR)

2) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hasse diagrams of $(\rho(A), \subseteq)$ for (a) A= {a}; (b) A={a, b}; (c) A= {a, b, c} (d) A= {a, b, c, d}

3) Let (L, ≤) be a lattice . For any a, b, c ∈ L show that the following holds:
$a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

(OR)

4) Show that in a lattice (L,≤), for any a , b, c$\in L$, the distributive inequalities hold:

$$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$$
$$a * (b \oplus c) \geq (a * b) \oplus (a * c)$$

5) Write the following Boolean expression in an equivalent sum of products canonical form in three variables x_1, x_2 *and* x_3 (a) $x_1 * x_2$ (b) $x_1 \oplus x_2$ (c) $(x_1 \oplus x_2)' * x_3$

(OR)

6) Obtain the values of the Boolean forms $x_1 * (x_1' \oplus x_2)$, $x_1 * x_2$and $x_1 \oplus (x_1 * x_2)$ over the ordered pairs of the two-element Boolean algebra.

7) Prove that if for some integer k , $p_{k+1} = p_k$, then $p_k = p$ and conversely.

(OR)

8) Draw the karnaugh map for one variable, two variables, 3-variable, 4-variable

9) Answer any **THREE** of the following:

a) Draw Hasse diagram of the set x = {2, 3, 6, 12, 24, 36} and the relation $\leq$ be such that $x \leq y$ if x divides y

b) Write some properties of lattices

c) Define subalgebra, Direct product and Homomorphism.

d) Obtain the product of sums canonical forms of the Boolean expression $x_1 * x_2$

(e) Write about finite state machines.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M201: ALGEBRA-II

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. a) Let $F \subseteq E \subseteq K$ be fields. If $[K : E] < \infty$ and $[E : F] < \infty$, then show that
   i) $[K : F] < \infty$
   ii) $[K : F] = [K : E]\,[E : F]$

   b) State and prove Gauss lemma

(OR)

2. a) Let E be an extension field of F and let $u \in E$ be algebraic over F. Let $p(x) \in F[x]$ be a polynomial of the least degree such that $p(u) = 0$. Then prove that,
   i) $p(x)$ is irreducible over F.
   ii) If $g(x) \in F[x]$ is such that $g(u) = 0$, then $p(x) | g(x)$
   iii) There is exactly one monic polynomial $p(x) \in F[x]$ such that $p(u) = 0$.

   b) Let K and K′ be algebraic closures of a field F. Then prove that $K \cong K'$ under an isomorphism that is an identity on F.

3. a) State and prove Uniqueness of splitting field theorem.

   b) If $f(x) \in F[x]$ is irreducible over F, then show that all roots of f(x) have the same multiplicity.

(OR)

4. a) Show that the prime field of a field F is either isomorphic to □ or $Z/(p)$, p prime.

   b) Show that if E is a finite separable extension of a field F. Then E is a simple extension of F.

5. Let H be a finite subgroup of the group of automorphisms of a field E. Then show that $[E : E_H] = |H|$.

(OR)

6. State and prove Fundamental theorem of a Galois theory.
7. Show that $\Phi_n(x) = \pi(x - w)$, w is primitive n^{th} root in □, is an irreducible polynomial of degree $\Phi(n)$ in $Z[x]$.

(OR)

8. a) Let F contain a primitive n^{th} root w of unity. Then show that the following are equivalent.
   i) E is a finite cyclic extension of degree n over F.

ii) E is the splitting field of an irreducible polynomial $x^n - b \in F[x]$.

b) Let F be a field and let U be a finite subgroup of the multiplicative group $F^* = F - \{0\}$. Then show that U is cyclic.

9. Answer any **THREE** of the following.

   a) Let $f(x) = a_0 + a_1x + a_nx^{n-1} + x_n \in Z[x]$ be a monic polynomial. If f(x) has a root $a \in$ □, then prove that $a \in Z$ and $a|a_0$.

   b) Show that the degree of the splitting field of $x^3 - 2 \in Q[x]$ is 6.

   c) Show that the multiplicative group of nonzero elements of a finite field is cyclic.

   d) If E is a finite extension of a field F, then prove that $|G(E/F)| \leq [E : F]$.

   e) Show that if $a > 0$ is constructible, then $\sqrt{a}$ is constructible.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M202: REAL ANALYSIS II

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1 a) State and prove necessary and sufficient condition for Riemann integrability of a real valued function f of $[a, b]$.

b) Suppose $f \epsilon (\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and □$(x) = \phi(f(x))$ on $[a, b]$. Then □(α)on $[a, b]$.

(OR)

2 a) State and prove integration by parts formula.

b) i) If γ' is continuous on$[a, b]$, then γ is rectifiable and $\wedge (\gamma) = \int_a^b |\gamma'(t)|dt$.

ii) If $f \epsilon (\alpha)$ on $[a, b]$ then $cf \epsilon(\alpha)$ for every constant c, and $\int^b cf d\alpha = c \int_a^b f d\alpha$.

3 a) State and prove Cauchy's criterion for uniform convergence of sequence of function.

b) State and prove Weirstrass M- Test.

(OR)

4 a) State and prove Stone-Weirstrass Theorem.

b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

5 a) State and prove Abel's Theorem.

b) State and prove Taylor's Theorem.

(OR)

6 a) State and prove contraction principle.

b) Suppose that f maps a convex open set $E \subset R^m$, f is differentiable in E, and there is a real number M such that $\|f'(x)\| \leq M$ for every $x \epsilon E$.

Then $|(b) - f(a)| \leq M|b - a|$ for all $a \epsilon E$, $b \epsilon E$.

7 State and prove the linear version of implicit function Theorem.

(OR)

8 a) Take $n = 2$, $m = 3$ and consider the mapping $f = (f_1, f_2)$ of R^5 on R^2 given by $f_1(x_1, x_2, y_1, y_2, y_3) = 2e^{x_1} + x_2y_1 - 4y_2 + 3$ and $f_2(x_1, x_2, y_1, y_2, y_3) = x_2 cos x_1 - 6x_1 + 2y_1 - y_3$. If $a = (0,1)$ and $b = (3,2,7)$ then $f(a, b) = 0$.

b) Take $(x, y) = \{ \frac{(x^2-y^2)}{x^2+y^2} \quad if\ (x, y) \neq (0,0);$

$= 0 \qquad if\ (x, y) = (0,0).$

Prove that i) f, D_1f, D_2f are continuous in R^2 ;

ii) $D_{12}f$ and $D_{21}f$ exist at every point of R^2 and are continuous except at (0,0);

iii) $D_{12}(0,0) = 1$ and $D_{21}f(0,0) = -1$.

9 Answer any **THREE** of the following

a) Suppose f is a real, continuously differentiable function on $[a, b]$, $f(a) = f(b) = 0$. Then prove that $\int_a^b xf(x)\, f'(x)dx = \frac{-1}{2}$ and $\int_a^b [f'(x)]^2\, dx \,.\, \int_a^b x^2 f^2(x)\, dx > \frac{1}{4}$.

b) Define $(x) = \int_x^{x+1} sin\,(t^2)\, dt.$ i) Prove that $|(x)| < \frac{1}{x}$, $if\ x > 0$;

ii) Prove that $2(x) = \cos(x^2) - \cos((x+1)^2) + \gamma(x)$, where $|\gamma(x)| < \frac{c}{x}$ and c is a constant; iii) Find the upper and lower limits of $xf(x),\ as\ x \to \infty$.

c) If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E. Prove that $\{f_n + g_n\}$ converges uniformly on E. If, in addition $\{f_n\}$ and $\{g_n\}$ are sequence of bounded function then prove that $\{f_n g_n\}$ converges uniformly on E.

d) Define $(x, y) = \begin{cases} \frac{x^3}{x^2+y^2} & if\ (x, y) \neq (0,0); \\ 0 & if\ (x, y) = (0,0). \end{cases}$

Prove that D_1f and D_2f are bounded function in R^2.

e) Define $(x, y) = \begin{cases} e^{1/x^2} & if\ x \neq 0; \\ 0\ if\ x = 0. \end{cases}$

Prove that f has derivatives of all orders at $x = 0$ and that $(0) = 0$ for all $n = 1,2,3, \ldots$.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M203: COMPLEX ANALYSIS I

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours | Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks

1. a) Define radius of convergence. If $\sum_{n=0}^{\infty} a_{n(z-a)^n}$ is a given power series with radius of convergence R then $R = \lim_{n\to\infty} |\frac{a_n}{a_{n+1}}|$, if this limit exists.

   b) State and prove Chain Rule.

(OR)

2. a) State and prove Cauchy- Riemann Equations.

   b) For any four distinct points $z_1, z_2, z_3$and z_4 . Prove that the cross ratio z_1, z_2, z_3, z_4 is a real number iff all the four points lie on a circle.

3 a) State and prove Leibniz's Theorem.

b) If γ be a piecewise smooth and f be a continuous on$[a, b]$. Then $\int_a^b f d\gamma = \int_a^b (t)'(t)dt$.

(OR)

4 a) State and prove Fundamental Theorem of Algebra .

b) State and prove Maximum Modulus Theorem.

5 State and prove Cauchy's Integral Formula Second Version

(OR)

6 a) State and prove Morera's Theorem.

b) Let G be a region and let f be an analytic function on G with zeros $a_1, a_2, \dots, a_m$ (repeated according to multiplicity). If γ be a closed rectifiable curve in G which does not passing through any of the a_k with γ = 0. Prove that $\frac{1}{2\pi i}\int \frac{f'(z)}{f(z)} dz = \sum_{k=1}^{m} \eta(\gamma; a_k)$.

(OR)

7 a) State and prove Casorarti-Weirstrass Theorem.

b) Show that $\int_0^\infty \frac{\log x}{1+x^2} dx = 0$.

8 a) State and prove Rouche's Theorem.

b) Obtain the Laurent Series Expansion for the function $f(z) = \frac{1}{(z-1)(z-2)}$

following annuli i) $a(0; 0,1)$ ii) $ann(0; 1,2)$ iii)$ann(0; 2, \infty)$

9. Answer any **THREE** of the following

a) Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi$, for $|z| < 1$.

b) Evaluate $\int_0^{\pi} \frac{1}{(a + \cos\theta)^2} d\theta$.

c) Let G be a region and f: $G \to C$ be an anlytical function such that there is a point in $a \in$ C with $|f(z)|<|f(a)|$ for all $z \in G$. Prove that f is constant.

d) Let G be open and connected and $f: G \to$ □ is differentiable with $f'(z) = 0$ for all $z \in G$. Prove that is constant.

e) Prove that any Mobius Transformation maps circle onto circle.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M204: LINEAR ALGEBRA

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours   Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. a) Let T be a linear operator on a finite – dimensional space V and c be a scalar . Then prove that the following are equivalent
   - i) c is a characterstic value of T.
   - ii) The operator (T-Ci) is singular (not invertible)
   - iii) det (T- cI) = 0

   b) Let T be a linear operator on a finite – dimensional space V. Then prove that the characteristic and minimal polynomials for T have the same roots , except for mulitiplicities.

(OR)

2. State and prove Cayley- Hamilton Theorem
3. Let V be a finite – dimensional space . let w_1 , ………….., w_k be subspaces of V and let W = W_1+ W_2 +…………+W_K , then prove that the following are equivalent :
   - i) W_1 , ……….., W_K are independent
   - ii) For each j ,$2\leq j \leq k$, we have $W_j \cap (W_1 + \ldots\ldots\ldots\ldots W_{j-1}) = \{0\}$
   - iii) If $Ɓ_i$ is an orderd basis for W_i , $1 \leq i \leq k$, then $\mathcal{B} = (Ɓ_i, \ldots\ldots\ldots\ldots Ɓ_k)$ is an orderd basis for W.

(OR)

4. State and prove primary Decomposition theorem.
5. a) How many possiable Jordan form are there for a 6x6 complex matrix with characteristic polynomial $(x+2)^4$ $(x-1)^2$.

   b) Let M and N be an mxn matrices with entries in the polynomial algebra F[X]. Then prove that N is row – equivalent to M if and only if N =PM , where P is an invertible mxm matrix with entries in F[X].
6. a) Let A be an nxn matrices with entries in the field F , and let P_1,…….P_r be the invariant factors for A. Then prove that the matrix Xi- A is equivalent to the nxn diagonal matrix with diagonal entries P_1 , ……….., p_r 1 , 1 , …….., 1.

b) Let T be a linear operator on V , and suppose that the minimal polynomial for T is irreducible over the scalar field F. Then show that T is semi-simple.

7. a) Let V be a finite-dimensionalvecor space over the field F. Then prove that for each ordered basis Ɓ of V , the function which associates with each bilinear form on V its matrix in the ordered basis Ɓ is an isomorohism of space L(V,V,F) onto the space of nxn matrices over the field F.

b) If bilinear form on the n-dimensional vector space V , then prove that the following are equivalent:

i) Rank (f) = n

ii) For each non-zero α in V , there is a β in V such that $f(\alpha, \beta) \neq 0$

iii) For each non-zero β in V , there is a α in V such that $f(\alpha, \beta) \neq 0$

(OR)

8. a) Let V be a finite-dimensionalvecor space over the field of complex numbers . let f be a symmetric bilinear form on V which has rank r . Then prove that there is an ordered basis Ɓ = (β_i, ………….β_n) for V such that

i) The matrix of f in the ordered basis $\mathcal{B}$ is diagonal

ii) $f(\beta_j, \beta_j) = \begin{cases} 1, \square = 1, \dots\dots\dots\dots\dots, \square \\ 0, \square > \square \end{cases}$

b) Let f be a non-degenarate bilinear form on a finite-dimensional vecorspace V. Then prove that the set of all linear operators on V which preserve f is a group under operation of composition.

9. Answer any **Three** of the following

a) Prove that similar matrices have the same characterstic Polynomial.

b) Let E_1,……………E_k be linear operators on the space V such that $E_1 + \dots\dots\dots + E_k = I$. Then prove that if $E_iE_j = 0$ for $i \neq j$, then $\square_{\square}^2 = E_i$ for each i.

c) Define a Smith Normal form and give an example .

d) Define a bilinear form and give an example .

e) Let V be a finite-dimensionalvecor space over the field F and f a symmetric bilinear form on V . For each subspace W of V , let X be the set of all vectors a in V suchthat $f(\alpha, \beta) = 0$ for every β in then show that X is a subspace.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M205: PROBABILITY THEORY & SATISTICS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1a) If A and b are any two events(subsets of sample space S) and are not disjoint, then P(A U B) = P(A) + P(B)- P(A U B).

b) If two dice are thrown, what is the probability that the sum is

(i) greater than 8, and (ii) neither 7 or 11?.

(OR)

2a) A and B alternately cut a pack of cards and the pack is shuffled after each cut. If A starts and the game is continued until one cuts a diamond, what are the respective chances of A and B first cutting a diamond?

b) State and prove Bayes' Theorem.

3a) The probability distribution of a random variable X is: f(x) = k $\sin\frac{1}{5}\pi x$, $0 \le x \le 5$.

Determine

The constant k and obtain the median and quartiles of the distribution.

b) If t is any positive real number, show that the function defined by p(x) = $e^{-t}(1-e^{-t})^{x-1}$ represent a probability function of a random variable X assuming the values 1,2,3, … Find the E(X) and Var(X) of the distribution.

(OR)

4a) The following data due to Weldon shows the result of throwing 12 fair dice 4,096 times; a throw of 4,5, or 6 being called success.

Success	Frequency	Success	Frequency
0	-	7	847
1	7	8	536
2	60	9	257
3	198	10	71
4	430	11	11
5	731	12	-
6	948	-	-

Fit a binomial Distribution and find the expected frequencies.

b) In a distribution exactly normal, 10.03% of the items are under 25 kilogram weight and 89.97% of the items are under 70 kilogram weight. What are the mean and standard deviation of the distribution?

5a) Calculate the correlation coefficient for the following heights(in inches) of fathers (X) and (Y):

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

b) The ranks of same 16 students in Mathematics and Phisics are as follows: Two numbers within brackets denote the ranks of the students in Mathematics and Phisics:

(1,1) (2,10) (3,3) (4,4) (5,5) (6,7) (7,2) (8,6) (9,8) (10, 11) (11,15) (12, 9) (13,14) (14,12) (15,16) (16,13).

(OR)

6a) In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible:

Variance of X = 9. Regression equations: $8X - 10Y + 66 = 0$, $40X - 18Y = 214$.

What are: (i) the mean values X and Y, (ii) the correlation between X and Y, and (iii) the standard deviation of Y?

b) Let (X, Y) have the joint p.d.f. given by:

$$f(x,y) = 1, \text{ if } |y|<x,\ 0<x<1$$

= 0, otherwise

Show that the regression of Y on X is linear but regression of X and Y is not linear.

7a) Write the procedure for testing of hypothesis.

b) A die is thrown 9,000 times and a throw of 3 or 4 is observed 3,240 times. Show that the die cannot be regarded as an unbiased one and find the limitsbetween which the probability of a throw of 3 or 4 lies.

(OR)

8a) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survial rate, if attacked by this disesse, is 85% in favour of the hypoyhesis that is more, at 5% level(Use large sample test).

8b) Before an increase in excise duty on tea, 800 persons out of a sample of 1,000 persons were found to be tea drinkers. After an increse in dutyt, 800 people were tee drinkers in a sample of 1,200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty?

9. **Answer any THREE of the following:**

a) For any two events A and B, we have (i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$.

b) The contents of urns I, II, and III are as follows:

I: 1 white, 2 black and 3 red balls

II: 2 white, 1 black and 1 red balls and

III: 4 white, 5 black and 3 red balls.

One turn is chosen at random and two balls drawn from it. They happen to be white and Red. What is the probability that they come from urns I, II, or III?

c) If the distribution function of a random variable X is symmetrical about zero, i.e., if

1-F(x) = F(-x) $\Rightarrow$ f(-x) = f(x), then $\phi_X(t)$ is real valued and even function of t.

d) Show that correlation coefficient is independent of change of origin and scale.

e) Explain about one-tailed and two-tailed tests.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M301: FUNCTIONAL ANALYSIS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. a) Define a Banach Space. Prove that the real linear space R^n is a Banach Space.

   b) In a Banach Space B, prove that the vector addition and scalar multiplication are jointly continuous.

(OR)

2. a) State and prove Hahn-Banach theorem.

   b) Prove that the mapping $x \to F_x$: $N \to N^{**}$ where $F_x(f) = f(x)$ $\forall f \in N^*$ is an isometric isomorphism of N in to N^{**}.

3. a) Stata and prove open mapping theorem.

   b) Stata and prove uniform boundedness theorem.

(OR)

4. a)If M and N are closed linear subspaces of a Hilbert space H such that M □ N , then prove that the linear subspace M + N is also closed.

   b) Stata and prove Bessel's Inequality (finite case) .

5. a) Prove that the mapping $y \to f_y$ is a norm preserving mapping of H into H^* where f_y (x) = <x,y> for all x ϵ H.

   b) If T is an operator on a Hilbert space H for which <Tx,x> =0 for all x ϵ H ,then prove that T = 0 on H .

(OR)

6 a) If T is an operator on a Hilbert space H then prove that the following conditions are all equivalent to one another .

   i) $T^*T = I$

   ii) $\langle Tx,Ty\rangle = \langle x,y\rangle$ for all x and y.

   iii) $\|Tx\|=\|x\|$ for all x

   b) Prove that a closed linear subspace M of H is invariant under T an opearator T on H if and only if $M^{\perp}$ is invariant under T*.

7. a) Prove that two matrices of A_n are similar if and only if they are the matrices of a single opearator on H relative to (possibly) different bases.

b) Prove that an operator T on H is singular if and only if θ ε $\sigma(T)$.

(OR)

8. State and prove Spectral theorem.

9. Answer any THREE of the following questions

a) Prove that every normal linear space is a metric space.
b) State and prove Schwartz inequality.
c) Define an orthogonal set in a Hilbert space H and given an example.
d) Define Eigen value and Eigen vector.
e) Let T be an operator on a Hilbert space H be such that adjoint T* of T is a polynomial in T, then prove that T is Normal.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M302: LEBESGUE THEORY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. Define measurability of a set and prove that the set of all measurable sets $\mathfrak{m}$ is a σ -algebra

(OR)

2. Every Borel set is measurable. In particular each open set and each closed set is measurable.

3. State and prove Lebesgue convergence theorem

(OR)

4. State and prove bounded convergence theorem

5. State and prove Fatou's lemma and hence prove the monotone convergence theorem.

(OR)

6. State and prove Vitali Lemma

7. State and prove Minkowski Inequality in L^P

(OR)

8. State and prove Riesz – Fischer theorem.

9. Answer any **THREE** questions of the following

   a) Define the outer measure of a set and prove that if E_1 & E_2 are measurable, then $E_1 \cup E_2$ is measurable
   b) Describe the invariance property
   c) Define convergence in measure
   d) If *f* and *g* are non negative measurable functions, then show that $\int_E cf = c\int_E f, c > 0$ and $\int_E (f+g) = \int_E f + \int_E g$
   e) State bounded convergence theorem and monotone convergence theorem

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ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M303: ANALYTICAL NUMBER THEORY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours   Max. Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1) If ϕ(n) is an Euler totientfunction,then for n$\geq$1, prove that $\phi(n) = n\prod_{p/n}(1-\frac{1}{p})$. Prove that

$\phi(p^{\alpha}) = p^{\alpha} - p^{\alpha-1}$ for prime p and

(OR)

2) Show that if both g and f*g are multiplicative, then f is also multiplicative.

3) State and prove Euler's summation formula.

(OR)

4) Define Lattice point. Show that the set of lattice points visible from the origin has density $6/\prod^2$.

5) Show that the following relations are logically equivalent:

(i) $\lim_{x\to\infty}\frac{\pi(x)\ \log\ x}{x} = 1$

(ii) $\lim_{x\to\infty}\frac{\theta(x)}{x} = 1$

(iii) $\lim_{x\to\infty}\frac{\psi(x)}{x} = 1.$

(OR)

6) For every integer n$\geq$2 prove that $\frac{1}{6}\frac{n}{\log\ n} < \pi(n) < 6\ \frac{n}{\log\ n}$

7) State and prove Lagrange's theorem.

(OR)

8) State and prove Chinese Remainder Theorem.

9) Answer any **Three** of the following. Eachcarries 5 marks.

(i) Define Mobius function $\mu(n)$ and show that if n$\geq$ 1 then $\sum_{d/n} \mu(d) = \left\lfloor \frac{1}{n} \right\rfloor$

(ii) For x>1 prove that the average order of $\phi(n) is {}^{3n}/_{\pi^{2}}$.

(iii) For x>0 prove that $0 \leq \frac{\psi(x)}{x} - \frac{\theta(x)}{x} \geq \frac{(\log x)^2}{2\sqrt{x}\log 2}$.

(iv) State and prove Abel's identity.

(v) Solve the congruence 5x$\equiv$3(mod24).

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ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M304: PARTIAL DIFFERENTIAL EQUATIONS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks

1. prove that a Pfaffian differential equation in two variables always possesses an integrating factor

(OR)

2. Find the integral curves of the equations $\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}$

3. Prove that the general solution of the linear partial differential equation $Pp+Qq=R$ is $F(u,v)=0$ where F is an arbitrary function and $u(x,y,z)=c_1$ and $v(x,y,z)=c_2$ form a solution of the equations $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$

(OR)

4. Prove that if $u_i(x_1,x_2,...,x_n,z)=c_i, i=1,2,3,...,n$ are independent solutiosn of the equations $\frac{dx_1}{P_1}=\frac{dx_2}{P_2}=...=\frac{dx_n}{P_n}=\frac{dz}{R}$, then the relation $\Phi(u_1,u_2,...,u_n)=0$ in which the function Φ is arbitrary is a general solution of the linear partial differential equation $P_1\frac{\partial z}{\partial x_1}+P_2\frac{\partial z}{\partial x_2}+...+P_n\frac{\partial z}{\partial x_n}=R$

5. Prove that if $\alpha_r D+\beta_r D'+\gamma_r$ is a factor of $F(D,D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then $\alpha_r\neq 0$, $u_r=\exp\left(\frac{-\gamma_r x}{\alpha_r}\right)\phi_r(\beta_r x-\alpha_r y)$ is a solution of the equation $F(D,D')z=0$

(OR)

6. Solve the equation $r+4s+t+rt-s^2=2$

7. Show that the surfaces $(x^2+y^2)^2-2a^2(x^2-y^2)+a^4=c$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

(OR)

8. State and prove Kelvin's inversion theorem

9. Answer any **THREE** questions of the following

a) Solve $\frac{dx}{y+\alpha z}=\frac{dy}{z+\beta x}=\frac{dz}{x+\gamma y}$

b) Show that the orthogonal trajectories of the hyperboloid $x^2+y^2-z^2=1$ of the conics in which it is cut by the system of planes $x+y=c$ are its curves of intersection with the surface $(x-y)z=k$

c) Solve $\frac{adx}{(b-c)yz}=\frac{bdy}{(c-a)zx}=\frac{cdz}{(a-b)xy}$

d) Prove the necessary and sufficient condition for a Pfaffian differential equation $\overline{x}\cdot d\overline{r}=0$ to be integrable is $\overline{x}\cdot \text{curl}\,\overline{x}=0$

e) Solve $y^2p-xyq=x(z-2y)$

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ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M305: LATTICE THEORY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1) Define Partly Ordered Set. State maximum and minimum condition for partly ordered set. Give an example of a poset satisfying minimum condition but failing to satisfy maximum condition.

   Or

2) State and prove Kuratowski-Zorn Lemma.

3) In a lattice L if we define the order relation "≤ " by a ≤ b ⇔ a ∧ b = a for all a, b ∈ L, show that every finite subset {a1,a2, a3,……….. ,an} of L has an infimum and supremum.

   Or

4) i) In a complemented lattice, show that every join prime element except the least one is an atom.

   ii) Show that, if a homomorphism of a lattice has a kernel, then the kernel is an ideal of the lattice.

5) (i) Prove that every order preserving mapping of a complete lattice into itself has a fix element. (ii)Show that, if we affix bound elements to a conditionally complete lattice, then we obtain a complete lattice.

   (Or)

6) Show that every lattice is isomorphic to some sub-lattice of a complete lattice.

7) If H is a non-void subset of a modular lattice L. For the sublattice H of L generated by H to be distributive, it is necessary and sufficient that for every finite distributive, it is necessary and sufficient that for every finite subsystem $\{x_1, x_2, \ldots x_m, y_1, y_2, \ldots y_n\}$ of H

$$\bigcup_{j=1}^{m} x_j \cap \bigcap_{k=1}^{n} y_k = \bigcup_{j=1}^{m} (x_j \cap \bigcap_{k=1}^{n} y_k)$$

   (Or)

8) (i) Show that a lattice is distributive if and only if every triplet of its elements has a median.

(ii)Show that every element of a distributive lattice has at most one irredundant meet representation.

9) Answer any three of the following .Each question carries 5 marks.

(a) Define a Lattice. Give two examples.

(b) Show that every uniquely complemented lattice is weakly complemented.

(c) Define the concepts of (i) an atom (ii) a dual atom and (iii) an atomic lattice

(d) Show that every complete lattice is bounded.

(e) A lattice L is modular if and only if every triplet a, b, c of L satisfies

$$a \cup (b \cap (a \cup c)) = (a \cup b) \cap (a \cup c)$$

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M401: MEASURE THEORY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours | Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. Define a measurable space and give an example in detail. Prove that, if $A \subset B \in \mathscr{B}$, then $\mu(A) \le \mu(B)$ where μ is the measure on *X*.

(OR)

2. State and prove Fatou's Lemma

3. State and prove Lebesgue Convergence Theorem

(OR)

4. State and prove Hahn Decomposition Theorem

5. State and prove Radon – Nikodym Theorem.

(OR)

6. Prove that if $\mathscr{B}$ of μ^* - measurable sets is a σ - algebra, $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$, then $\bar{\mu}$ is a complete measure on $\mathscr{B}$
7. Prove that if the class $\mathscr{B}$ of μ^*- measurable sets is a σ- algebra, $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$, then $\bar{\mu}$ is a complete measure on $\mathscr{B}$

(OR)

8. State and prove Riesz representation theorem.

9. Answer any **THREE** questions of the following

   b) Define the signed measure, positive set, distinguish between the null set and a set of measure zero, and the inner measure
   c) Prove that the countable union of positive sets is positive
   d) State Caratheodory theorem
   e) Define Caratheodory outer measure and Haussdorff measure
   f) State Fubini theorem, product measure and define cross section of a set *E*.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M402:NUMERICAL ANALYSIS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

SECTION-A

Answer all questions and each question carries 15 marks.

1. Explain Muller's method. Use it to find the smallest root of the equation x^3-x-1 = 0.

(OR)

2. Using Regula-Falsi metod, compute a real root of the equation xe^x-sin x = 0 correct to three decimal places.

3. Find the inverse of the matrix A = $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ by LU factorization method. Hence solve the system of equations AX = b, where b = $[4,3,4]^T$ and X = $[x_1,x_2,x_3]^T$.

(OR)

4. Solve equations
$10x_1-2x_2-x_3-x_4 = 3$
$-2x_1+10x_2-x_3-x_4 = 15$
$-x_1-x_2+10x_3-2x_4 = 27$
$-x_1-x_2-2x_3+10x_4 = -9$, by Gauss-Seidel iteration method.

5. Given the data values of tan θ:

θ^0	0	5	10	15	20	25	30
tan θ	0	0.0875	0.1763	0. 2679	0. 3640	0.4663	0.5774

Use Stirling's formula, find tan 16^0.

6. Using

(i) Lagrange's formula

(ii) Newton's divided difference formula

X	5	7	11	13
f(x)	150	392	1452	2366

(OR)

7. Evaluate numerically the value of the integral $\int_0^1 \frac{□□}{4□^2+5}$ Using Gauss-Legendre one point, two point and three point formulas. Also compare with the exact solution.

(OR)

8. Given $\frac{dy}{dx}$ = x^2(1+y) and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, evaluate y(1.4) and Y(1.5) using Adams-Bashforth method.

SECTION-B

9. Answer any THREE of the following.

a) Explain bisection method. What are the merits and demerits of this method?

b) Use partition method to find the inverse of $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

c) Explain spline interpolation.

d) Fit a least square line y = a+bx to the following data:

X:	1.1	1.2	1.3	1.4
Y:	2	6	9	4

e) Find y(0.2) and y(0.4) using taylor series method, given $\frac{dy}{dx}$ = 1+y, y(0) 1.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M403: GRAPH THEORY

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1. a) Define i) Graph

ii) Tree

iii)Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree.

b) Show that in a complete graph with n- vertices there are n(n-1)/2 edge disjoint Hamiltonian circuits, if 'n' is an odd number ≥ 3.

(OR)

2. a) Define i) Free Tree

ii) Binary Tree

iii) Show that the number of labeled trees with 'n' vertices $(n \geq 2)$ is n^{n-2}.

(OR)

b) Show that the maximum flow possible between two vertices 'a' and 'b' in a network is equal to the minimum of the capacities of all cut sets with respect to 'a' and 'b'.

3. a) Define

i) Planar Graph

ii) Euler Graph

iii) Prove that any planar graph with 'v' vertices and 'e' edges satisfies $e \leq 3v-6$.

(OR)

4. a) Explain detection of Planarity.

b) Show that the set of cut set vectors corresponding to the set of fundamental cut sets with respect to any spanning tree form a basis for the cut set sub space W_5.

5. a) Define i) Incidence Matrix

ii) Binary Matrix

iii) If A(G) is an incidence matrix of a connected graph G with 'n' vertices then show that the rank of A(G) is n-1.

(OR)

b) Show that if B is a circuit matrix of a connected graph G with 'e' edges and 'n' vertices then rank of B is equal to e-n+1.

6. a) Define i) Directed graph

ii) Out degree

iii) Sketch all different (non-isomorphic) simple digraphs with 1,2, and 3 vertices.

b) Prove that the determinant of every square sub-matrix of A, the incidence matrix of digraph is 1, -1, or 0.

7. a) Show that a graph with at least one edge is 2-chromatic iff it has no circuits of odd length.

b) Show that the maximal number of vertices in a set V_1 that can be matched into V_2 is equal to number of vertices in $V_1 - \delta$ (G).

(OR)

8. a) Show that a covering 'g' of a graph is minimal iff 'g' contains no paths of length 3 or more.

b) Explain multiple sources and sinks.

9. Answer any THREE of the following:

a) Explain Travelling sales-man problem.

b) Explain some properties of trees.

c) Write about edge connectivity with an example.

d) Show that $K_{3,3}$ is non-planar.

e) Describe linear programming formulation.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M404: LINEAR PROGRAMMING

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours   Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

SECTION-A

Answer all questions and each question carries 15 marks.

1. Solve the L.P. Problem graphically
   Maximize $z = 3x_1+2x_2$
   Subject to $-2x_1+3x_2 \leq 9$
   $x_1 - 5x_2 \geq -20$ and $x_1, x_2 \geq 0$.

(OR)

2. Solve the L.P. Problem by simplex method,

   Maximize $z = 2x_1+4x_2+x_3+x_4$

   Subject to $x_1+3x_2+x_4 \leq 4$
   $2x_1 + x_2 \leq 3$
   $x_2+4x_3+x_4 \leq 3$ and $x_1, x_2, x_3, x_4 \geq 0$.
3. Use big-M-method to solve the L.P. Problem

   Minimize $z = 5x_1-6x_2-7x_3$

   Subject to $x_1+5x_2-3x_3 \geq 15$

   $5x_1 -6x_2+10x_3 \geq 0$
   $x_2+x_2+x_3=5$; and $x_1, x_2, x_3, x_4 \geq 0$.

(OR)

4. Apply the principle of duality to solve the L.P. Problem:
   Maximize $z = 3x_1-2x_2$
   Subject to $x_1+ x_2 \leq 5$
   $x_1 \leq 4$
   $1 \leq x_2 \leq 6$ and $x_1, x_2 \geq 0$.
5. Determine an optimum assignment schedule for the following assignment problem. The cost matrix is given as

Job	Machine					
	1	2	3	4	5	6
A	11	17	8	16	20	15
B	9	7	12	6	15	13
C	13	16	15	12	16	8
D	21	24	17	28	26	15
E	14	10	12	11	15	6

(OR)

6. Solve the following travelling salesman problem so as to minimize the cost per cycle.

From	To				
	A	B	C	D	E
A	-	3	6	2	3
B	3	-	5	2	3
C	6	5	-	6	4
D	2	2	6	-	6
E	3	3	4	6	-

7. Find the optimal solution to the following transportation problem

	D1	D2	D3	D4	Supply
S1	23	27	16	18	30
S2	12	17	20	51	40
S3	22	28	12	32	53
Demand	22	35	25	41	123

(OR)

8. Solve the following transportation problem

	1	2	3	4	5	6	Supply
A	7	5	7	7	5	3	60
B	9	11	6	11	-	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

9. Answer any Three of the following:
   a) Define and explain the terms:
      i) Optimum solution.
      ii) Feasible solution.
   b) Formulate the dual of the following linear programming problem
      Maximize $z = 5x_1+3x_2$
      Subject to $3x_1+5x_2 \leq 15$,
      $5x_1+2x_2 \leq 10$, $x_1 \geq 0$, $x_2 \geq 0$.

c) Obtain the basic solutions for the following system of linear equations:
$x_1 + 2x_2 + x_3 = 4$
$2x_1 + x_2 + 5x_3 = 5.$

d) Give mathematical formulation of travelling salesman problem.

e) Explain briefly Vogel's approximation method for finding an initial basic feasible solution.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Mathematics

I-SEMESTER

M405: DISCRETE DYNAMICAL SYSTEMS

[W.E.F. 2019 A.B]

(MODEL QUESTION PAPER)

Time: 3 Hours   Max . Marks : 75

Answer **ALL** questions. Each question carries 15 marks.

1) (a) Define fixed point, periodic point, attracting and repelling fixed points.

(b)Let f be a $C^{'}$ function and p be a fixed point of f such that $|f^{'}(p)|_{<1}$ Show that there exists a neighbourhood of p which is contained in $W^{s}(p)$.

(OR)

(2) Let f : □ → □ be a continuous function having a periodic point of period three. Show that f has periodic points of all periods.

3) (a) Define the Shift Map and show that the shift map is continuous, it has 2^{n} periodic points of period n and there is an element with dense orbit.

(b) Define Bifurcation, Saddle-node bifurcation, Pitch-fork bifurcation.

(OR)

4) (a) Let $f : X \to X$ be a topologically transitive and suppose that periodic points of f are dense in X. If X is infinite then f exhibits sensitive dependence on initial conditions.

(b) Explain Period doubling bifurcation with an example.

5) Let D and E be metric spaces, f :D$\to$D, g : E$\to$E and τ : D$\to$E be a topological conjugacy of f and g. Then

(a) τ^{-1} : E$\to$D is a topological conjugacy.

(b) $\tau \; o \; f^{n} = g^{n} \; o \; \tau$ for all natural numbers n

(c) f is topologically transitive on D if and only if g is topologically transitive on E.

(OR)

6) Suppose $p(x)$ is a polynomial if we allow cancellation of common factors in the expression of

$N_p(x) = x - \dfrac{p(x)}{P^{'}(x)}$, then $N_p(x)$ is always defined at roots of $p(x)$, a number is a fixed point of $N_p(x)$ if and only if it is a root of the polynomial, and all fixed points of $N_p(x)$ are attracting.

7) (a) Show that all complex quadratic polynomials are topologically conjugate to a polynomial of the form $q_c(z) = z^2 + c$.

(b) Prove that the orbit of a complex number under iteration of a complex quadratic polynomial is either bounded or the number is in the stable set of infinity.

(OR)

8) Let $f(z) = e^{\theta i} z$ and z_0 is a nonzero complex number. Show that

(a) z_0 is a periodic point of f if θ is a rational multiple of π

(b) if θ is a rational multiple of π then z_0 is not a periodic point of f and its orbit is dense in the circle containing z_0 .

9) Answer any THREE of the following:

(a) Define Discrete Dynamical system and give three examples.

(b) Explain the concept of Phase Portrait with the help of an example.

(c) Define Sensitive dependence, Devany Chaos and give an example of a dynamical system which is chaotic in the sence of Devany

(d) Define Topological Transitivity and show that the existence of a dense orbit implies topological transitivity.

(e) Define topological conjugacy and prove that the periodicity and period of a periodic point is preserved by topological conjugacy.

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